1

Core 1 June 2016 LJB

1(i) $(2x-3)(2x-3) - 2(3-x)(3-x)$

$4x^2 - 12x + 9 \quad -18 + 12x \quad -2x^2$

$\underline{2x^2 - 9}$

(ii) $-6x^3 - 4x^3 = -10x^3$ $\qquad \underline{\underline{-10}}$

(2) $\dfrac{(3+\sqrt{20})}{(3+\sqrt{5})} \times \dfrac{(3-\sqrt{5})}{(3-\sqrt{5})} = \dfrac{9 - 3\sqrt{5} + 3\sqrt{20} - \sqrt{100}}{4}$

$= \dfrac{9 - 3\sqrt{5} + 6\sqrt{5} - 10}{4} = \dfrac{-1 + 3\sqrt{5}}{4} = 1/4 + 3/4\sqrt{5}$

(3) $x^2 + y^2 = 34 \qquad 3x - y + 4 = 0$

$y = 3x + 4$

$x^2 + (3x+4)^2 = 34$

$x^2 + 9x^2 + 24x + 16 = 34$

$10x^2 + 24x - 18 = 0$

$5x^2 + 12x - 9 = 0$

$(5x-3)(x+3) = 0$

$x = 3/5 \quad$ or $\quad x = -3$

$y = 29/5 = 5\,4/5 \qquad y = -5$

④ $2y^{1/2} - 7y^{1/4} + 3 = 0$

let $y^{1/4} = x$ so $2x^2 - 7x + 3 = 0$

$(2x - 1)(x - 3) = 0$

$x = 1/2$ or $x = 3$

$y^{1/4} = 1/2$ $\quad y^{1/4} = 3$

$\underline{\underline{y = 1/16}}$ $\quad \underline{\underline{y = 81}}$

⑤ (i) $2^{-6}$

(ii) $5 \times 4^{2/3} + 3 \times 16^{1/3}$

$5 \times 16^{1/3} + 3 \times 16^{1/3}$

$8 \times 16^{1/3}$

$8 \times (2^4)^{1/3}$

$2^3 \times 2^{4/3} = 2^{13/3}$

⑥ (i) $-2(x^2 - 6x - 2)$

$-2((x-3)^2 - 11)$

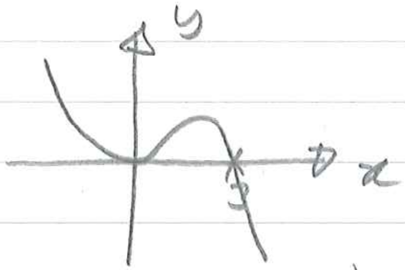
$-2(x-3)^2 + 22$

$a = -2 \quad b = -3 \quad c = 22$

(ii) Max $x = 3 \quad y = 22$

(2) 7 (i) $-ve$ $x^3$ $\cap$

$y = x^2(3 - x)$




intersects $(0,0)$ $(3,0)$

(ii) $y = (x-2)^2(3-(x-2))$

$y = (x-2)^2(5-x)$

(iii) $y = x^2(3-x) \rightarrow \frac{y}{1/2} = x^2(3-x)$

Stretch S.F. $\frac{1}{2}$ in the $y$ direction

8 (i) $y = 2x^2$ $\quad (5, 50)$

$h > 0$ $\quad (5+h, 2h^2 + 20h + 50)$

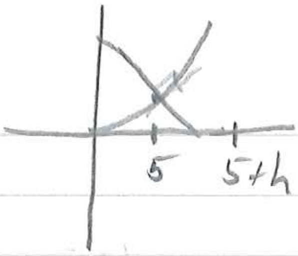
$(5+h)(5+h) = 25 + 10h + h^2$

gradient $= \frac{\Delta y}{\Delta x}$

$= \frac{2h^2 + 20h}{h} = \underline{\underline{2h + 20}}$

(ii) (i) is an approximation to the gradient at A. The smaller $h$ the closer the approximation to the gradient

(8)




$y = 2x^2$

$\frac{dy}{dx} = 4x$ , when $x = 5$ $\frac{dy}{dx} = \underline{\underline{20}}$

Normal gradient $= -1/20$

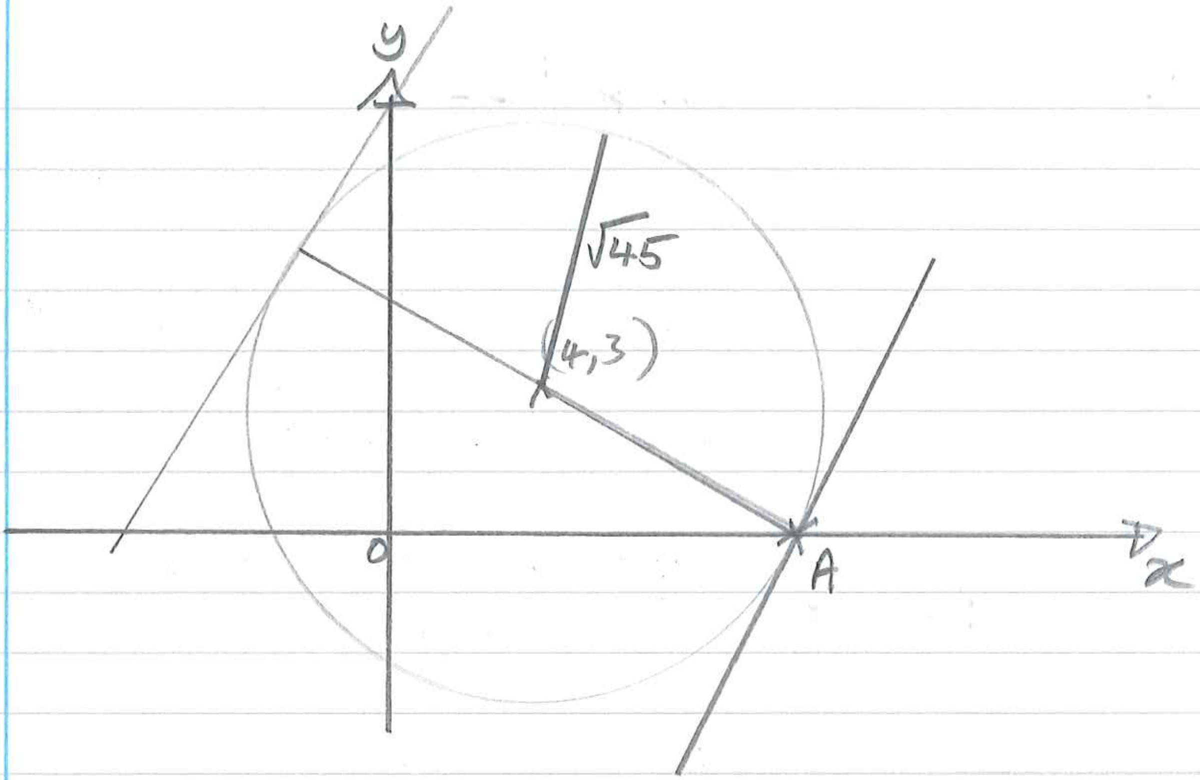
so $\underset{x}{(5}, \underset{y}{50)}$ $m = -1/20$

$y = mx + c$ $50 = 5 \times (-1/20) + c$

$c = 50 1/4$

$y = -\frac{1}{20} x + 50 1/4$

c y co-ordinate is $50 1/4$




at A $\quad y=0$ so $\quad (x-4)^2+(-3)^2=45$

$$x^2-8x+16+9=45$$
$$x^2-8x-20=0$$
$$(x-10)(x+2)=0$$
$$x=10 \quad \text{or} \quad x=-2$$

$(10,0) \rightarrow (4,3)$

gradient $\frac{-3}{6} = -1/2 \Rightarrow$ Normal $\underline{\underline{2}}$

gradient 2 goes through $(10,0)$

$y=mx+c \qquad 0=2\times 10+c \Rightarrow c=-10$

$y=2x-10$

(3)

(9) $x^2 + 2x + 11 = k(2x - 1)$

$x^2 + 2x + 11 = 2kx - k$

$x^2 + 2x - 2kx + 11 + k = 0$

$x^2 + (2 - 2k)x + 11 + k = 0$

$b^2 - 4ac > 0$

$(2 - 2k)^2 - 4 \times 1 \times (11 + k) > 0$

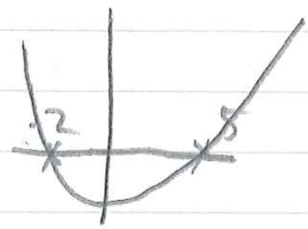
$4 - 8k + 4k^2 - 44 - 4k > 0$

$4k^2 - 12k - 40 > 0$

$k^2 - 3k - 10 > 0$

$(k + 2)(k - 5) > 0$

$k = -2$ or $k = 5$




$k < -2$ or $k > 5$

(10) $(x^2 - 8x) + (y^2 - 6y) - 20 = 0$

$(x - 4)^2 - 16 + (y - 3)^2 - 9 - 20 = 0$

$(x - 4)^2 + (y - 3)^2 - 45 = 0$

$(x - 4)^2 + (y - 3)^2 = (\sqrt{45})^2$

Centre $(4, 3)$ radius $\sqrt{45}$

④ (iii) Co-ordinates (-2, 6)
gradient 2

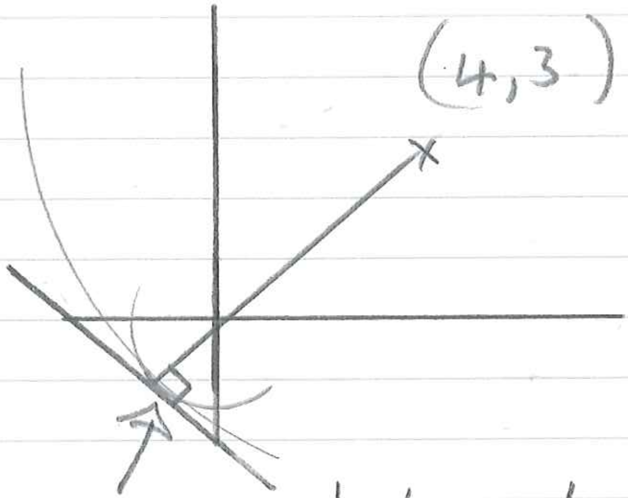
$y = mx + c$

$6 = 2 \times -2 + c$

$c = 10$

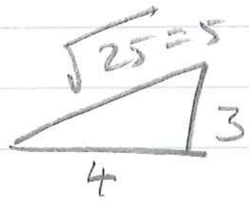
$y = 2x + 10$

(iv)




Key must be at tangent to both the origin at (4, 3) circle, as it has to be shortest distance from both.

So radius $\sqrt{45}$




So distance from origin $\sqrt{45} - 5$

So $r < \sqrt{45} - 5$

(11) $y = 4x^2 + ax^{-1} + 5$

$$\frac{dy}{dx} = 8x - ax^{-2}$$

$$0 = 8x - \frac{a}{x^2} \Rightarrow a = 8x^3$$

$$32 = 4x^2 + 8x^2 + 5$$

$$27 = 12x^2$$

$$x = \pm\sqrt{\frac{9}{4}} = \frac{3}{2}$$

$$a = 8x^3 = 8 \times \left(\frac{3}{2}\right)^3 = \underline{\underline{27}}$$