Jan 2012 C1

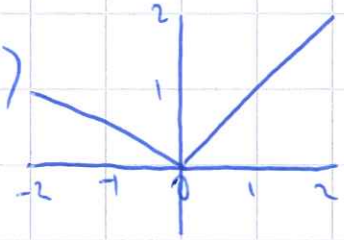
(1) $\frac{15+\sqrt{3}}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{(15+\sqrt{3})(3+\sqrt{3})}{9-3}$

$= \frac{45+18\sqrt{3}+3}{6}$

$= \frac{48+18\sqrt{3}}{6}$

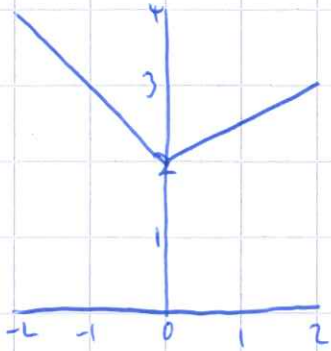
$= 8+3\sqrt{3}$

(2) i)



$y = f(-x)$

ii)



$y = f(x) + 2$

(3) $5x^2 + px - 8 = q(x-1)^2 + r$

$5\left(x^2 + \frac{p}{5}x\right) - 8$

$= 5\left(x + \frac{p}{10}\right)^2 - \frac{p^2}{100} - 8$

$= 5(x-1)^2 - 9$

$\frac{p}{10} = -1$

$p = -10$

$q = 5 \quad p = -10 \quad r = -9$

③ $5(x-1)^2 + r$

$= 5(x^2 - 2x + 1) + r$

$= 5x^2 - 10x + 5 + r$

$5 + r = -8$

$r = -13$

$\therefore 5x^2 - 10x - 8 = 5(x-1)^2 - 13$

$q = 5 \quad p = -10 \quad r = -13$

④ i) $3^{-2} = 1/9$

ii) $16^{3/4} = (\sqrt[4]{16})^3$

$= 2^3$

$= 8$

iii) $\frac{\sqrt{200}}{\sqrt{8}} = \frac{10\sqrt{2}}{2\sqrt{2}}$

$= 5$

⑤ $\frac{3}{y^4} - \frac{10}{y^2} - 8 = 0$ let $x = \frac{1}{y^2}$

$\therefore 3x^2 - 10x - 8 = 0$

$(3x + 2)(x - 4) = 0$

$3x + 2 = 0$ $\quad x - 4 = 0$

$x = -2/3$ $\quad x = 4$

$\frac{1}{y^2} = -2/3$ $\quad \frac{1}{y^2} = 4$

$1 = \frac{-2y^2}{3}$ $\quad 1 = 4y^2$

$3 = -2y^2$ $\quad y^2 = 1/4$

$\pm\sqrt{\frac{3}{-2}} = y$ $\quad y = \pm 1/2$

real roots only

$y = \pm \frac{1}{2}$

(6) $f(x) = \frac{4}{x} - 3x + 2$

$= 4x^{-1} - 3x + 2$

$f'(x) = -4x^{-2} - 3$

$f''(x) = 8x^{-3}$

$f''\left(\frac{1}{2}\right) = 8\left(\frac{1}{2}\right)^{-3}$

$= 8 \times 2^3$

$= 64$

(7) $y = (x+2)(x^2 - 3x + 5)$

$= x^3 - 3x^2 + 5x$

$+ 2x^2 - 6x + 10$

$= x^3 - x^2 - x + 10$

@ min point $\frac{dy}{dx} = 0$ $\quad \frac{dy}{dx} = 3x^2 - 2x - 1$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$x = -1/3 \quad x = 1$

@ min point $\frac{d^2y}{dx^2} > 0$

$\frac{d^2y}{dx^2} = 6x - 2$ $\quad x = 1$, $\frac{d^2y}{dx^2} = 6(1) - 2$

$= 4$

$x = -1/3$ $\quad \frac{d^2y}{dx^2} = 6(-1/3) - 2$

$= -4$

Hence min point @ $x = 1$

$y = 1^3 - 1^2 - 1 + 10$

$= 9$

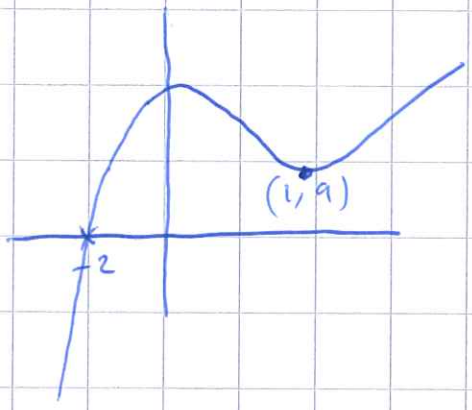
min point $\equiv (1, 9)$

ii) $b^2 - 4ac \Rightarrow (-2)^2 - 4(1)(5)$

$= 9 - 20$

$= -11$

iii) As discriminants is ~~equal~~ less than 0, there are not real roots hence, ~~it~~ the graph will not cross the x-axis when $x > -2$



(8)

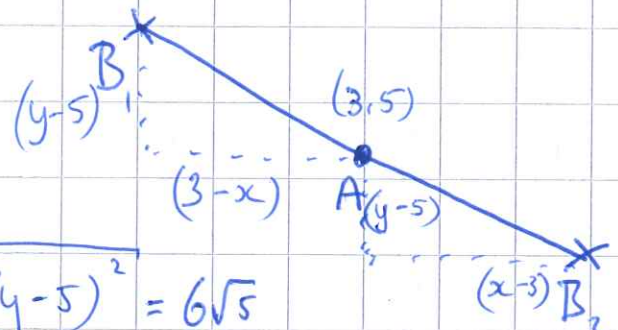
$l \Rightarrow m = -2 \quad y = -2x + c$

@ (3, 5) $\quad 5 = -2(3) + c$

$5 = -6 + c$

eqⁿ of $l \Rightarrow \underline{y = -2x + 11} \quad c = 11$

$AB = 6\sqrt{5}$



$\sqrt{(3-x)^2 + (y-5)^2} = 6\sqrt{5}$

$(3-x)^2 + (y-5)^2 = 180$

* See next page

$y = -2x + 11$

$(3-x)^2 + (-2x+11-5)^2 = 180$

$(3-x)^2 + (-2x+6)^2 = 180$

$9 - 6x + x^2 + 4x^2 - 24x + 36 = 180$

$5x^2 - 30x + 45 = 180$

$x^2 - 6x + 9 = 36$

$x^2 - 6x - 27 = 0$

$(x-9)(x+3) = 0$

$x = 9 \quad x = -3$

@ $x = 9$

$y = -2(9) + 11$

$= -7$

$(9, -7)$

@ $x = -3$

$y = -2(-3) + 11$

$= 17$

$(-3, 17)$

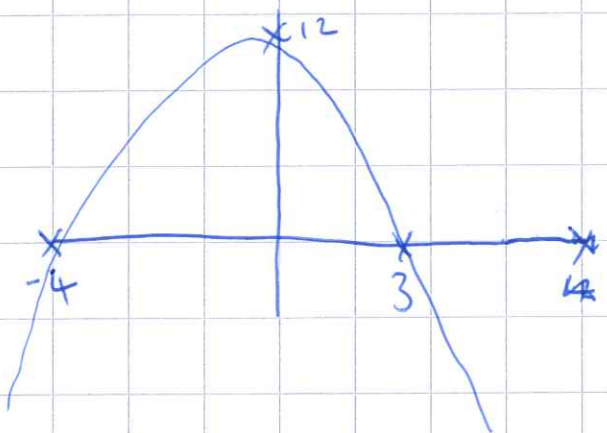
(9) i) $y = 12 - x - x^2$

@ $y = 0$ rearrange to give $x^2 + x - 12 = 0$

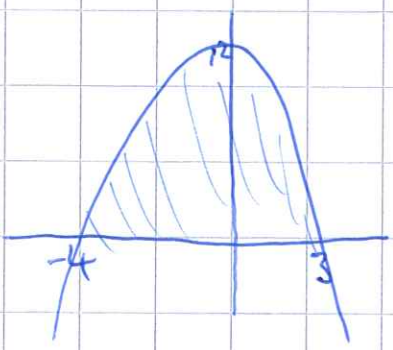
$(x+4)(x-3) = 0$

$x = -4 \quad x = 3$

y intercept $\Rightarrow$ 12



ii)



$12 - x - x^2 > 0$

$-4 < x < 3$

iii) $3x + y = 4$

$y = 4 - 3x \rightarrow$

$y = 12 - x - x^2$

$4 - 3x = 12 - x - x^2$

$0 = 8 + 2x - x^2$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4 \quad x = -2$

$b^2 - 4ac$

$= 4 - 4(1)(-8)$

$= 4 + 32$

$= 36$ Hence 2 real roots/points of intersection.

② $x = 4$

$y = 4 - 3(4)$

$y = 4 - 12$

$y = -8$

$(4, -8)$

② $x = -2$

$y = 4 - 3(-2)$

$= 10$

$(-2, 10)$

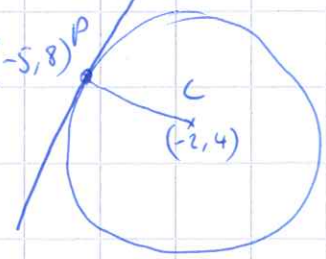
(10) $C(-2, 4)$ $r = 5$

i) $(x+2)^2 + (y-4)^2 = 25$

$x^2 + 4x + 4 + y^2 - 8y + 16 = 25$

$x^2 + y^2 + 4x - 8y - 5 = 0$

ii)



$m_{cp} = \frac{-4}{3}$

$m_{tangent} = \frac{3}{4}$

$y = \frac{3}{4}x + c$

$8 = \frac{3}{4}(-5) + c$

$8 = \frac{-15}{4} + c$

$32 = -15 + 4c$

$47 = 4c$

$y = \frac{3}{4}x + \frac{47}{4}$

$4y = 3x + 47$

$0 = 3x - 4y + 47$

iii) $T(3, 14)$ $x = 3$ $y = 14$

* sub into eq^n of tangent

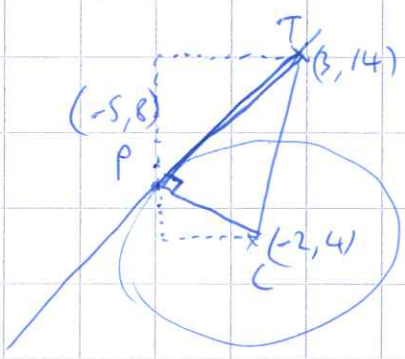
$3(3) - 4(14) + 47$

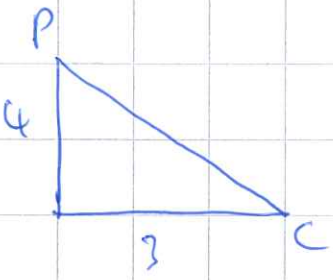
$= 9 - 56 + 47$

$= 0$

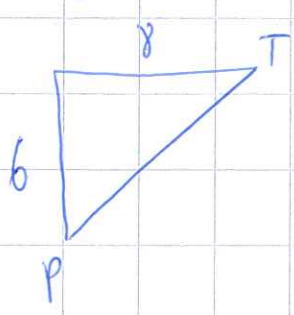
Hence T is on the tangent.

iv)





$$CP = \sqrt{4^2 + 3^2}$$
$$= 5$$



$$PT = \sqrt{6^2 + 8^2}$$
$$= 10$$

$$\text{Area of } CPT = \frac{5 \times 10}{2} = 25 \text{ units}^2$$