

Please write clearly in block capitals.

Centre number

Candidate number

Surname Worked Solutions

Forename(s) \_\_\_\_\_

Candidate signature \_\_\_\_\_

# GCSE MATHEMATICS

# H

Higher Tier Paper 3 Calculator

Monday 12 November 2018 Morning Time allowed: 1 hour 30 minutes

### Materials

For this paper you must have:

- a calculator
- mathematical instruments.



### Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

### Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2-3	
4-5	
6-7	
8-9	
10-11	
12-13	
14-15	
16-17	
18-19	
20-21	
22-23	
24-25	
26	
<b>TOTAL</b>	



Answer **all** questions in the spaces provided

Do not write  
outside the  
box

- 1 A shape is translated by the vector  $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$   
In which direction does the shape move?  
Circle your answer.

[1 mark]

up

down

left

right

- 2 What is 1.75 kilometres as a fraction of 700 metres?  
Circle your answer.

[1 mark]

$\frac{5}{2}$

$\frac{1}{4}$

$\frac{4}{1}$

$\frac{2}{5}$

$$\frac{1750}{700} = \frac{5}{2}$$

- 3 The first 4 terms of a linear sequence are

$-5$  | 3    11    19    27  
           $\searrow$   
          8

Circle the expression for the  $n$ th term.

[1 mark]

$8 - 5n$

$n + 8$

$8n + 3$

$8n - 5$



- 4 Work out the lowest common multiple (LCM) of 20, 30 and 40  
Circle your answer.

[1 mark]

10                      120                      240                      24 000

*20 fits into 40*                      *30, 60, 90, 120*

- 5 The length of a table is 110 cm to the nearest cm  
Complete the error interval.

[2 marks]

109.5 cm  $\leq$  length  $<$  110.5 cm

Turn over for the next question

Turn over ►



6

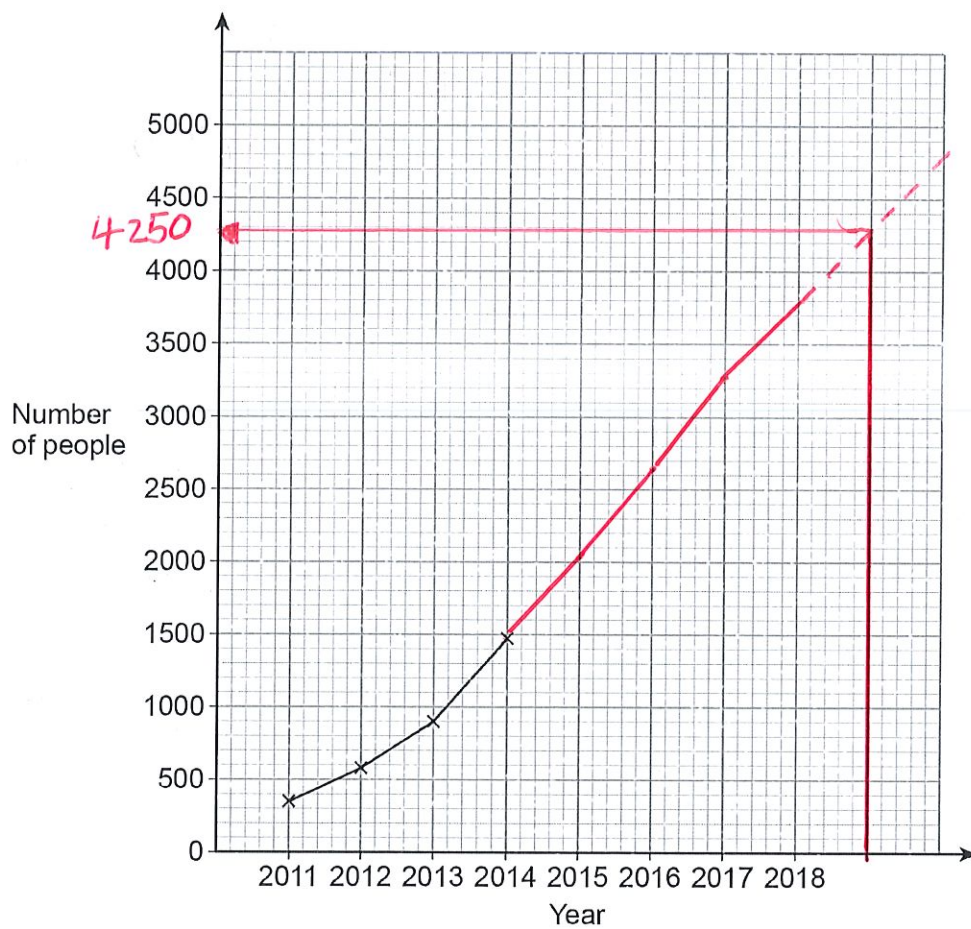
A music festival has taken place each year from 2011

The table shows the number of people who attended each year.

Year	2011	2012	2013	2014	2015	2016	2017	2018
Number of people	350	583	906	1471	2023	2612	3251	3780

The festival organisers draw a time series graph to represent the data.

The first four years have been plotted.



Do not write  
outside the  
box

6 (a) Complete the graph.

[2 marks]

6 (b) Use the graph to estimate the number of people who will attend the festival in 2019

[2 marks]

Answer

4250

Turn over for the next question

Turn over ►



7

$$k = n^2 + 9n + 1$$

Mo says,

" $k$  will be a prime number for all integer values of  $n$  from 1 to 9"

Show that Mo is wrong.

You **must** show that your value of  $k$  is **not** prime.

[3 marks]

- ①  $1 + 9 + 1 = 11$
- ②  $4 + 18 + 1 = 23$
- ③  $9 + 27 + 1 = 37$
- ④  $16 + 36 + 1 = 53$
- ⑤  $25 + 45 + 1 = 71$
- ⑥  $36 + 54 + 1 = 91 \rightarrow \text{Not prime}$   
 $7 \times 13$
- ⑦  $49 + 63 + 1 = 113$
- ⑧  $64 + 72 + 1 = 137$
- ⑨  $81 + 81 + 1 = 163$
- ⑩  $100 + 90 + 1 = 191$
- ⑪  ~~$121 + 99 + 1 =$~~



8

Doug owes an amount of £600

He wants to pay off this amount in five months.

He says,

"Each month, I will pay back 20% of the amount I still owe."

Show working to check if his method is correct.

[3 marks]

$$\textcircled{1} \quad 600 \times 0.8 = 480$$

$$\textcircled{2} \quad 480 \times 0.8 = 384$$

$$\textcircled{3} \quad 384 \times 0.8 = 307.2$$

$$\textcircled{4} \quad 307.2 \times 0.8 = 245.76$$

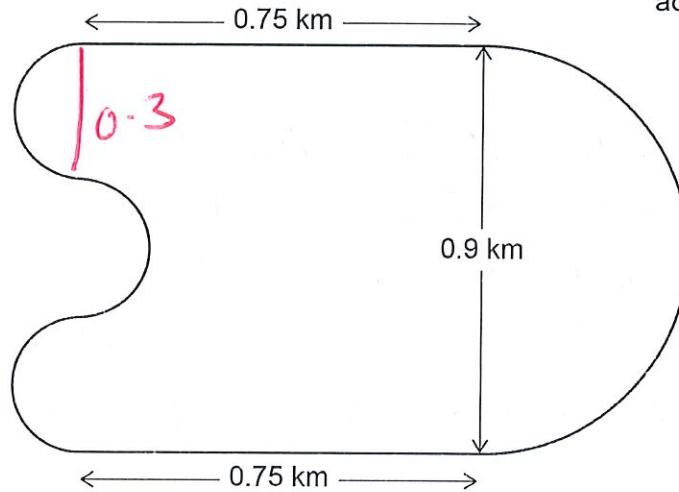
$$\textcircled{5} \quad 245.76 \times 0.8 = \underline{\underline{196.608}}$$

Turn over for the next question



9

A motor racing circuit consists of  
 two parallel straight sections, each of length 0.75 km  
 a semicircle of diameter 0.9 km  
 three equal, smaller semicircles.

Not drawn  
accurately

The length of a motor race must be greater than 305 km

What is the lowest number of **full** laps needed at this circuit?

You **must** show your working.

[5 marks]

Each lap

$$0.75 \times 2 + \frac{0.9 \times \pi}{2} + \frac{0.3 \times \pi}{2} \times 3$$

$$= 4.327433388$$

$$305 \div 4.327433388 = 70.48057651$$

71 smallest number above this

Answer \_\_\_\_\_





10

Solve  $8 > 3 - \frac{1}{2}x$

[2 marks]

$$16 > 6 - x$$

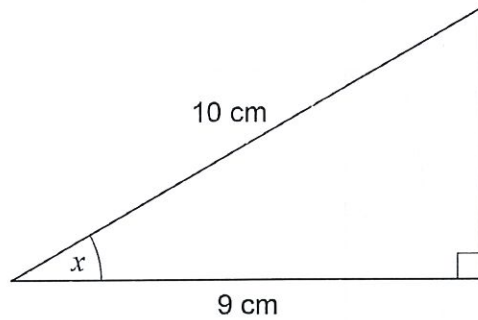
$$x > -10$$

Answer \_\_\_\_\_

11

Use trigonometry to work out the size of angle  $x$ .

[2 marks]

Not drawn  
accuratelySOH (CAH) TOA

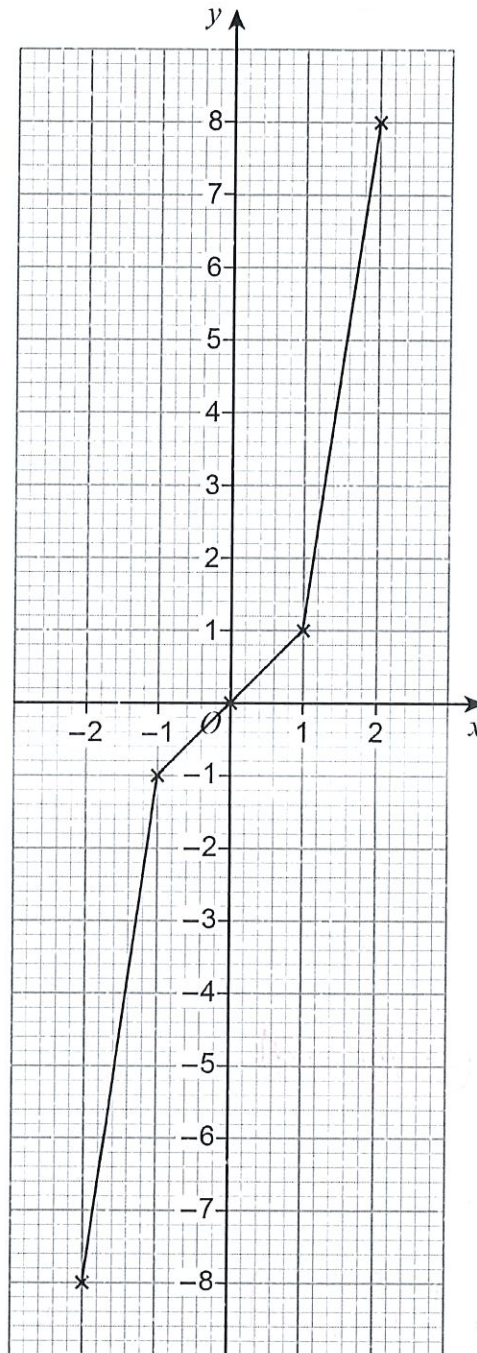
$$\cos \theta = \frac{A}{H} \quad \text{so} \quad \cos x = \frac{9}{10}$$

$$x = \cos^{-1}\left(\frac{9}{10}\right) = 25.84193276$$

Answer 25.8 degrees

12

Lewis wants to draw the graph  $y = x^3$  for values of  $x$  from  $-2$  to  $2$   
Here is his graph.



Make one criticism of his graph.

[1 mark]

Points should be joined by smooth curve  
not straight lines



- 13 The probability of Heads when a biased coin is thrown is 0.6  
The coin is thrown 500 times.  
Circle the expected number of Tails.

20

200

250

300

[1 mark]

$$500 \times 0.6 = 300$$

- 14 The mean mass of a squad of 19 hockey players is 82 kg  
A player of mass 93 kg joins the squad.  
Work out the mean mass of the squad now.

[3 marks]

$$\frac{19 \times 82 + 93}{20} = 82.55$$

Answer \_\_\_\_\_ kg



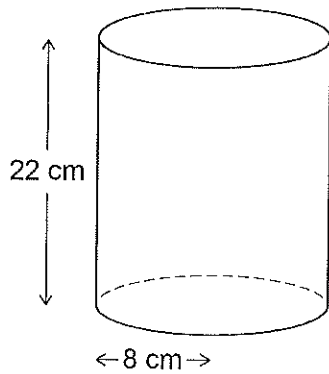
15

A company makes two types of lampshade using fabric on wire frames.

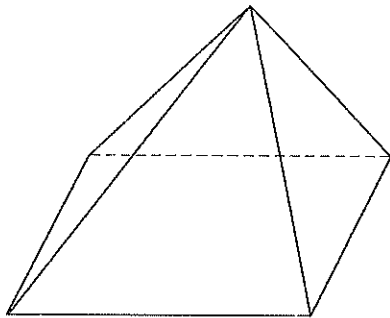
**Lampshade A**

Fabric is used to make the curved surface of a cylinder.

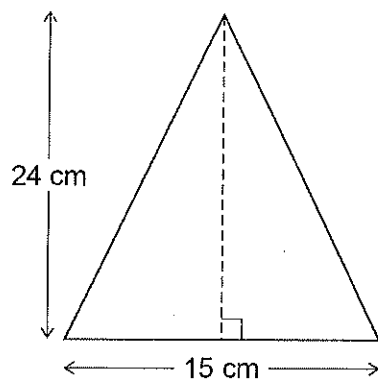
The cylinder has radius 8 cm and height 22 cm

**Lampshade B**

Fabric is used to make the four triangular faces of a pyramid.



Each triangular face has base 15 cm and perpendicular height 24 cm



Not drawn  
accurately



Cost of fabric	£400 per square metre
Other costs for A	£3.50 per lampshade
Other costs for B	£7.50 per lampshade

Work out the ratio cost of one lampshade A : cost of one lampshade B

Give your answer in the form  $n : 1$

[5 marks]

$$\text{Area A } \pi d h = \pi \times 16 \times 22 = 352\pi \text{ cm}^2$$

$$\text{B } 4 \times \frac{1}{2} \times 15 \times 24 = 720 \text{ cm}^2$$

$$\text{Cost A } \quad \text{note} \quad 1\text{m}^2 = 10,000 \text{ cm}^2$$

$$\frac{352\pi \times 400}{10,000} + 3.5 = 445.8362456$$

Cost B

$$\frac{720 \times 400}{10,000} + 7.5 = 295.5$$

$$445.8362456 : 295.5$$

$$1.5087521 : 1$$

Answer \_\_\_\_\_ :



16

In a running club there are 50 females and 80 males.

If a female is chosen at random, the probability she has blue eyes is 0.38

If a male is chosen at random, the probability he has blue eyes is 0.6

One person is chosen at random.

Show that the probability the person has blue eyes is **more than** 0.5

[4 marks]

$$P(f) = \frac{50}{130} \quad P(m) = \frac{80}{130}$$

$$P(B) = \frac{50}{130} \times 0.38 + \frac{80}{130} \times 0.6$$

$$= \frac{67}{130}$$

17

$$w = \frac{3}{5\sqrt{x}}$$

Circle the expression for  $w^2$

[1 mark]

$$\frac{6}{10x^2}$$

$$\frac{9}{25x^2}$$

$$\frac{6}{10x}$$

$$\frac{9}{25x}$$

$$\frac{3}{5\sqrt{x}} \times \frac{3}{5\sqrt{x}}$$



18

Here is some information about the ages of people at a concert.

Age, $x$ (years)	Frequency
$10 \leq x < 15$	8
$15 \leq x < 25$	24
$25 \leq x < 40$	30
$40 \leq x < 70$	39

$$FD = F/cw$$

$$8/5 = 1.6$$

$$24/10 = 2.4$$

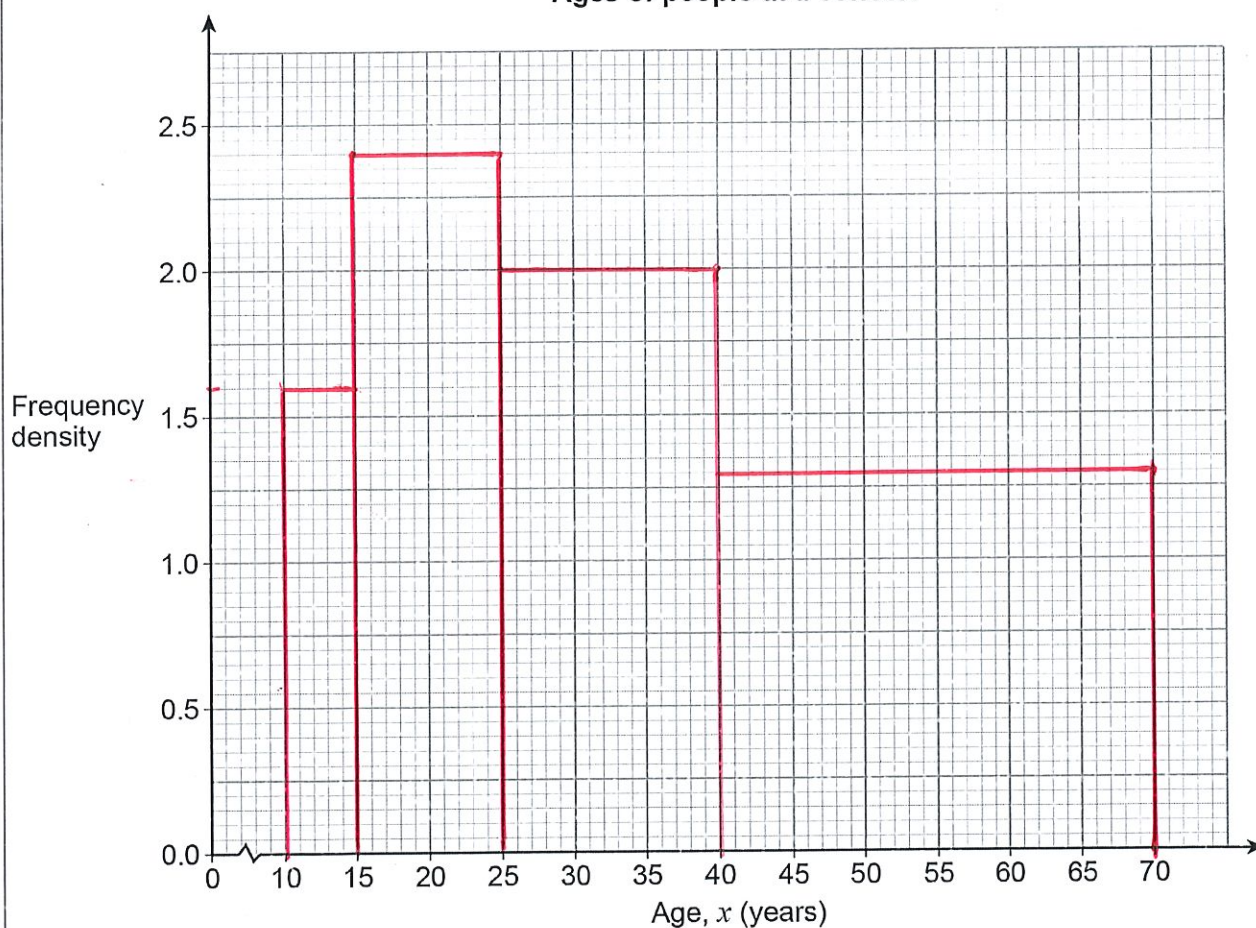
$$30/15 = 2$$

$$39/30 = 1.3$$

Draw a histogram to represent the information.

[3 marks]

Ages of people at a concert



Turn over ►



19

The length of a roll of ribbon is 30 metres, correct to the nearest half-metre.

A piece of length 5.8 metres, correct to the nearest 10 centimetres, is cut from the roll.

Work out the maximum possible length of ribbon left on the roll.

[3 marks]

$$\cancel{29.5} \leq r < 30.25$$

$$29.75$$

$$5.75 \leq l < 5.85$$

$$30.25 - 5.75 = 24.5$$

Answer \_\_\_\_\_ metres





20

Curve P has equation  $y = 2(x - 1)^2 - 5$ Curve Q is a reflection in the  $y$ -axis of curve P.

Work out the equation of curve Q.

Give your answer in the form  $y = ax^2 + bx + c$  where  $a$ ,  $b$  and  $c$  are integers.

[3 marks]

$$y = 2((-x) - 1)^2 - 5$$

$$2(x^2 + 2x + 1) - 5 = 2x^2 + 4x - 3$$

Answer \_\_\_\_\_

Turn over for the next question



21

Priya and Joe travel the same 16.8 km route.

Priya starts at 9.00 am and walks at a constant speed of 6 km/h

Joe starts at 9.30 am and runs at a constant speed.

Joe overtakes Priya at 10.20 am

At what time does Joe finish the route?

[5 marks]

$$10.20 \rightarrow 9.00 = 1\frac{1}{3} \text{ hrs}$$

$$10.20 \rightarrow 9.30 = 50 \text{ min } \frac{5}{6} \text{ hr}$$

$$6 \times 1\frac{1}{3} = 8 \text{ km}$$

$$8 \div \frac{5}{6} = 9.6 \text{ km/hr}$$

$$16.8 \div 9.6 = 1\frac{3}{4} \quad | \text{ hr } 45 \text{ min}$$

$$9.30 + 1 \text{ hr } 45 \text{ min} = 11:15 \text{ am}$$

Answer 11:15 am



- 22 An approximate solution to an equation is found using the iterative formula

$$x_{n+1} = \frac{(x_n)^3 - 2}{10} \quad \text{with } x_1 = -1$$

- 22 (a) Work out the values of  $x_2$  and  $x_3$

[2 marks]

$$\frac{(-1)^3 - 2}{10} = \frac{-3}{10} \quad x_2 \quad \frac{(-3/10)^3 - 2}{10} =$$

$$x_2 = \frac{-3}{10}$$

$$x_3 = \frac{-2027}{10,000}$$

- 22 (b) Work out the solution to 5 decimal places.

[1 mark]

$$-0.2008528394, -0.2008100357, -0.2008097594,$$

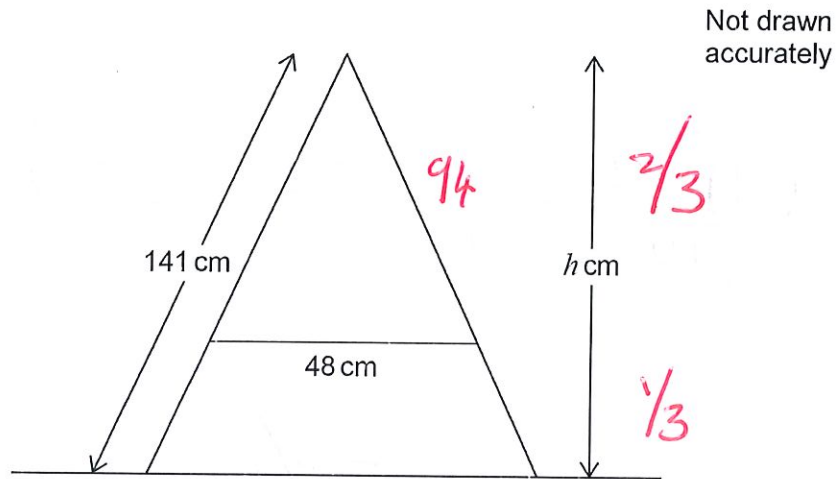
$$-0.2008097565,$$

$$x = -0.20081$$



23

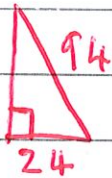
The diagram shows the side view of a step ladder with a horizontal strut of length 48 cm. The strut is one third of the way up the ladder. The symmetrical cross section of the ladder shows two similar triangles.



Work out the vertical height,  $h$  cm, of the ladder.

[5 marks]

$$141 \times \frac{2}{3} = 94$$



$$a^2 + b^2 = c^2$$

$$c^2 = 8260$$

$$\sqrt{8260} \times \frac{1}{2/3} = \sqrt{8260} \times \frac{3}{2}$$

$$= 136.3268132$$

Answer 136.3 cm



24

Volume of a sphere =  $\frac{4}{3}\pi r^3$  where  $r$  is the radius

Volume of a cone =  $\frac{1}{3}\pi r^2 h$  where  $r$  is the radius and  $h$  is the perpendicular height

A sphere has radius  $2x$  cm

A cone has

radius  $3x$  cm

perpendicular height  $h$  cm

The sphere and the cone have the same volume.

Work out radius of cone : perpendicular height of cone

Give your answer in the form  $a : b$  where  $a$  and  $b$  are integers.

[4 marks]

$$\begin{array}{l} \text{Sphere} \\ \frac{4}{3} \pi (2x)^3 = \frac{1}{3} \pi (3x)^2 h \\ \text{Cone} \end{array}$$

$$32x^3 = 9x^2 h$$

$$\frac{32}{9} x = h$$

$$\text{so } 32x = 9h$$

$$x \quad h$$

$$\text{so } 9 : 32$$

$$3x : h$$

$$27 : 32$$

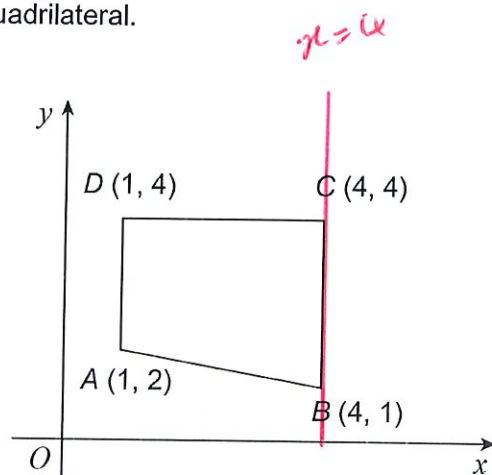
Answer 27 : 32

9

Turn over ►



25

 $ABCD$  is a quadrilateral.Not drawn  
accuratelyThe quadrilateral is reflected in the line  $x = 4$ 

Which vertices are invariant?

Circle your answer.

[1 mark]

A and D

C and D

B and C

B and D



26

$$f(x) = \frac{2x+3}{x-4}$$

Work out  $f^{-1}(x)$ 

[4 marks]

$$y = \frac{2x+3}{x-4}$$

$$y(x-4) = 2x+3$$

$$yx - 4y = 2x+3$$

$$yx - 2x = 3+4y$$

$$x(y-2) = 3+4y$$

$$x = \frac{3+4y}{y-2}$$

Answer  $f^{-1}(x) = \frac{3+4x}{x-2}$

Turn over for the next question

Turn over ►



- 27 The line  $y = 3x + p$  and the circle  $x^2 + y^2 = 53$  intersect at points A and B.  
 $p$  is a positive integer.

- 27 (a) Show that the  $x$ -coordinates of points A and B satisfy the equation

$$10x^2 + 6px + p^2 - 53 = 0$$

[3 marks]

$$\begin{aligned}x^2 + (3x + p)^2 &= 53 \\x^2 + 9x^2 + 6xp + p^2 &= 53 \\10x^2 + 6xp + p^2 - 53 &= 0\end{aligned}$$





27 (b) The coordinates of A are (2, 7)

Work out the coordinates of B.

You **must** show your working.

[5 marks]

$$y = 3x + p$$

$$7 = 3 \times 2 + p \Rightarrow p = 1$$

$$\text{So } 10x^2 + 6x - 52 = 0$$

$$5x^2 + 3x - 26 = 0$$

$$(5x + 13)(x - 2) = 0$$

$$5x + 13 = 0$$

$$\text{or } x - 2 = 0$$

$$x = -13/5$$

$$x = 2$$

already seen (2, 7)

$$y = 3x - 13/5 + 1$$

$$y = -34/5 = -6\frac{4}{5}$$

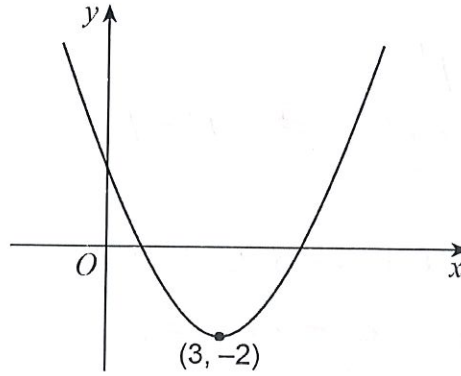
Answer  $(-1\frac{3}{5}, -6\frac{4}{5})$

Turn over for the next question



28

Here is a sketch of a quadratic curve.

The turning point is  $(3, -2)$ Not drawn  
accuratelyCircle the correct statement about the gradient of the curve for  $x < 3$ 

[1 mark]

gradient is positive

gradient is negative

gradient is zero

gradient could be any value

END OF QUESTIONS

