

Mechanics - Jan 2011 Miss Watson's solutions

1

$$\begin{array}{ccc} 0.5\text{kg} & 2.4\text{ms}^{-1} & 0.8\text{kg} & 1.5\text{ms}^{-1} \\ \textcircled{P} \longrightarrow & & \longleftarrow \textcircled{Q} & \\ & & -0.2\text{ms}^{-1} & \end{array}$$

(i) $0.5 \times 2.4 + 0.5 \times 0.2 = 1.3$ ✓ [2]

(ii) $0.5 \times 2.4 + 0.8 \times -1.5 = 0.5 \times -0.2 + 0.8 \times V$
 $0.1 = 0.8 \times V$
 $V = 0.125$ ✓ [4]

2 (i) $8 = 10 \times \cos \alpha$
 $\alpha = \cos^{-1}\left(\frac{8}{10}\right) = 36.86989765$
 36.9° ✓ [3]

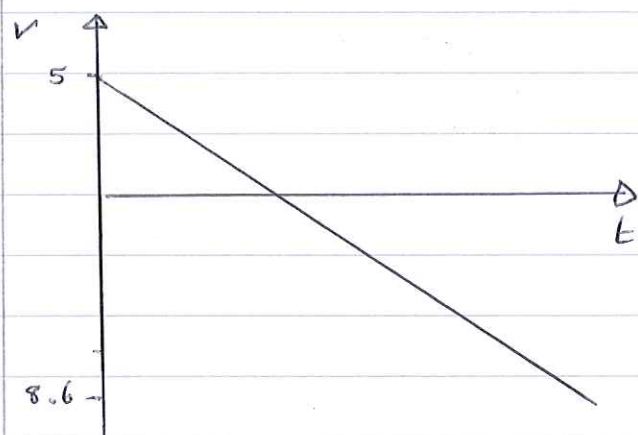
(ii) $F = 10 \times \cos(90 - 36.9)$
 $F = 6.0042$
 $= 6\text{N}$ ✓ [3]

3 (i) $u = 5$ $a = -9.8$ $v = 0$ $s = ?$
 $v^2 = u^2 + 2as$
 $0 = 5^2 + 2 \times -9.8 \times s$
 $s = 1.2755$
 $u = 0$ $a = 9.8$ $s = 2.5 + 1.2775 = 3.7755$ $v = ?$
 $v^2 = u^2 + 2as$
 $v^2 = 0 + 2 \times 9.8 \times 3.7755$
 $v = 8.6023$ ✓ [3]

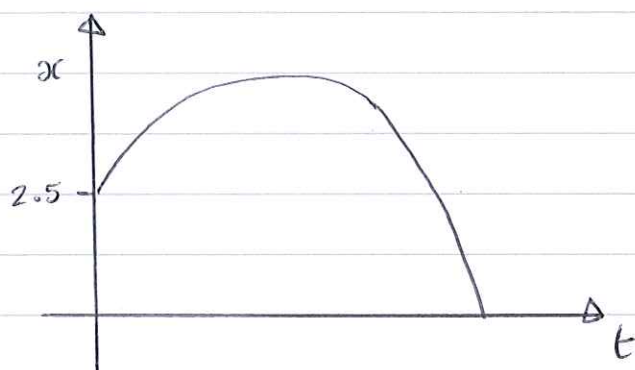
(ii) $u = 5$ $a = -9.8$ $v = -8.6023$ $t = ?$
 $v = u + at$
 $-8.6023 = 5 + -9.8 \times t$
 $t = 1.387989$
 $t = 1.39$ ✓ [3]

continued.

3 (iii)
(a)



(b)



4

4

(i) $2 - T_1 = 0.8 \times 0.2$

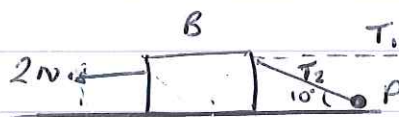
$T_1 = 1.84$

$T_2 \times \cos 10 = T_1$

$T_2 \times \cos 10 = 1.84$

$T_2 = 1.86838$

$= 1.87$



3

(ii) $0.3 \times 9.8 = 2.94$

$R = 2.94 - 1.87 \times \cos 80 = 2.615$

$F_{lim} = \mu \times R$

$F - F_{lim} = ma$

$1.87 \times \cos 10 - F_{lim} = 0.3 \times 0.2$

$1.842 - F_{lim} = 0.06$

$F_{lim} = 1.782$

$1.782 = \mu \times 2.615$

$\mu = 0.68145$

0.681

7

5 (i) P \rightarrow $u = 4.9$ $a = 4.9$ $t = T$ $s = ?$
 Q \rightarrow $u = 0$ $a = 9.8$ $t = T$ $s = ?$

(a)

$$s = ut + \frac{1}{2} at^2$$

P \rightarrow $S_p = 4.9T + \frac{1}{2} \times 4.9 \times T^2$ \checkmark $S_p = 4.9T + 2.45T^2$

Q \rightarrow $S_q = 0 + \frac{1}{2} \times 9.8 \times T^2$ \checkmark $S_q = 4.9T^2$

3

(b) $F = ma$

$$F = m \times 4.9$$

$$4.9m = 9.8m \times \cos \alpha$$

$$4.9 = 9.8 \times \cos \alpha$$

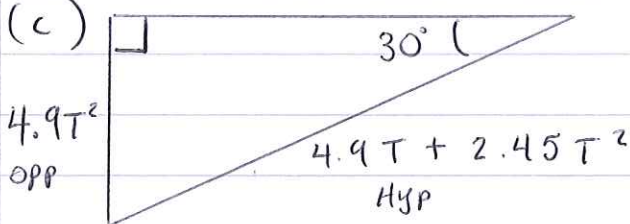
$$\alpha = \cos^{-1} \left(\frac{4.9}{9.8} \right)$$

$$\alpha = 60$$

$$\theta = 90 - 60 = \underline{\underline{30}} \checkmark$$

2

(c)



$$\sin \theta = \frac{\text{OPP}}{\text{HYP}}$$

$$\sin 30 = \frac{4.9T^2}{4.9T + 2.45T^2}$$

$$0.5 \times (4.9T + 2.45T^2) = 4.9T^2$$

$$2.45T + 1.225T^2 = 4.9T^2$$

$$\div T \rightarrow 2.45 + 1.225T = 4.9T$$

$$2.45 = 3.675T$$

$$T = \frac{2.45}{3.675} = \frac{2}{3} \checkmark$$

3

(ii) P \rightarrow $u = 4.9$ $a = 4.9$ $t = \frac{2}{3}$ $v = ?$

$$v = u + at$$

$$v = 4.9 + 4.9 \times \frac{2}{3}$$

$$v = 8.17 \checkmark$$

Q \rightarrow $u = 0$ $a = 9.8$ $t = \frac{2}{3}$ $v = ?$

$$v = u + at$$

$$v = 0 + 9.8 \times \frac{2}{3}$$

$$v = 6.53 \checkmark$$

3

6 (i)

$$v = t^2 - 9$$

$$\int t^2 - 9 \, dt = \frac{1}{3}t^3 - 9t + C \quad \leftarrow \begin{array}{l} x=0 \\ \text{when } t=0 \text{ so } C=0 \end{array}$$

$$x = \frac{1}{3}t^3 - 9t$$

$$x = \frac{1}{3} \times 2^3 - 9 \times 2$$

$$x = -15\frac{1}{3}$$

Displacement is $15\frac{1}{3}$ metres ✓

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(ii)

$$v = t^2 - 9$$

$$v = (t-3)(t+3)$$

$$0 = (t-3)(t+3)$$

$$t=3 \quad t=-3$$

$$x = \frac{1}{3} \times 3^3 - 9 \times 3$$

$$x = -18$$

18 metres ✓

6

(iii)

$$\frac{dv}{dt} = a = 2t$$

$$10 = 2t$$

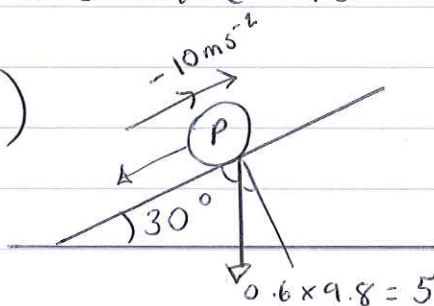
$$t = 5$$

$$x = \frac{1}{3} \times 5^3 - 9 \times 5 = -3\frac{1}{3}$$

Distance is $15\frac{1}{3} - 3\frac{1}{3} = 12$ m ✓

5

7 (i)



$$m = 0.6$$

$$F = ma$$

$$F = -6$$

$$5.88 \times \cos 60 = 2.94$$

$$F_{\text{lim}} = \mu \times R$$

$$R = 5.88 \times \cos 30 = 5.0922$$

$$2.94 + F_{\text{lim}} = 6$$

$$F_{\text{lim}} = 3.06$$

$$3.06 = \mu \times 5.0922$$

$$\mu = 0.600919$$

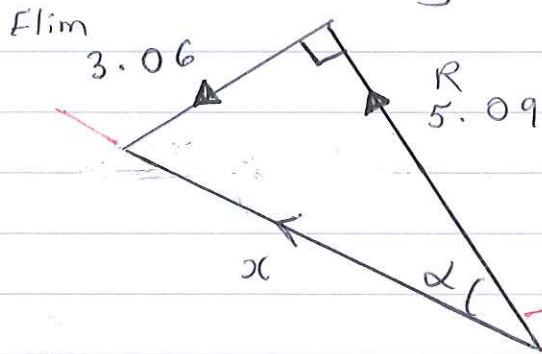
$$= 0.601 \quad \checkmark$$

7

Magnitude of the 'contact force' is the resultant of the normal and friction

↳ both types of contact force

7 (ii) (a) When in motion friction is limiting and acting down the slope.



$$x^2 = 3.06^2 + 5.09^2$$
$$x = 5.938998232$$
$$5.94 \checkmark$$

$$\tan \alpha = \frac{O}{A}$$

$$\alpha = \tan^{-1} \left(\frac{3.06}{5.09} \right) = 31.0133919$$

$$\text{Angle is } 31 + 90 = 121^\circ \checkmark$$

5

(b) When at rest friction is acting up the slope and is equal to force acting down.