Binomial Expansion Questions involving Partial Fractions SOLUTIONS

1)a) $9x - 14 \equiv A(x-1)^2 + B(x+4)(x-1) + C(x+4)$

$x = -4$	$\Rightarrow$	$-50 = 25A$	$\Rightarrow$	$A = -2$
$x = 1$	$\Rightarrow$	$-5 = 5C$	$\Rightarrow$	$C = -1$
coeffs of x^2	$\Rightarrow$	$0 = A + B$	$\Rightarrow$	$B = 2$

b)

$f(x) = \frac{-2}{x+4} + \frac{2}{x-1} - \frac{1}{(x-1)^2}$

$\frac{-2}{4+x} = \frac{-\frac{2}{4}}{\frac{4}{4} + \frac{x}{4}}$

$= \frac{-\frac{1}{2}}{1 + \frac{x}{4}}$

$= -\frac{1}{2}\left(1 + \frac{x}{4}\right)^{-1}$

$-\frac{1}{2}\left(1 + \frac{x}{4}\right)^{-1} = -\frac{1}{2}\left[1 + (-1)\left(\frac{x}{4}\right) + \frac{(-1)(-2)}{2}\left(\frac{x}{4}\right)^2\right]$

$= -\frac{1}{2}\left[1 - \frac{x}{4} + \frac{x^2}{16}\right]$

$\frac{-2}{x+4} = -\frac{1}{2} + \frac{x}{8} - \frac{x^2}{32}$

$\frac{2}{x-1} = \frac{-2}{1-x}$

$= -2(1-x)^{-1}$

$-2(1-x)^{-1} = -2\left(1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2\right)$

$= -2\left(1 + x + x^2\right)$

$\frac{-2}{1-x} = -2 - 2x - 2x^2$

$\frac{-1}{(x-1)^2}$

$= \frac{-1}{(-1+x)^2}$

$= \frac{-1}{(-1)^2(1-x)^2}$

$= \frac{-1}{(1-x)^2} = -1(1-x)^{-2}$

$-(1-x)^{-2} = -\left(1 + (-2)(-x) + \frac{(-2)(-3)}{2}(-x)^2\right)$

$= -\left(1 + 2x + 3x^2\right)$

$\frac{-1}{(x-1)^2} = -1 - 2x - 3x^2$

So $f(x) = \left(-\frac{1}{2} + \frac{x}{8} - \frac{x^2}{32}\right) + \left(-2 - 2x - 2x^2\right) + \left(-1 - 2x - 3x^2\right)$

$= -\frac{7}{2} - \frac{31x}{8} - \frac{161x^2}{32}$

c) Valid for $|x| < 1$

2).a) A = 0, B = 4, C = 3

b) $4 + \frac{39x^2}{4}$

3).

a $\dfrac{2-11x}{1-5x+4x^2} \equiv \dfrac{A}{1-x} + \dfrac{B}{1-4x}$

$2 - 11x \equiv A(1 - 4x) + B(1 - x)$

$x = 1 \quad \Rightarrow \quad -9 = -3A \quad \Rightarrow \quad A = 3$

$x = \frac{1}{4} \quad \Rightarrow \quad -\frac{3}{4} = \frac{3}{4}B \quad \Rightarrow \quad B = -1$

b $\dfrac{2-11x}{1-5x+4x^2} \equiv \dfrac{3}{1-x} - \dfrac{1}{1-4x}$

$\dfrac{3}{1-x} = 3(1-x)^{-1} = 3[1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(-x)^3 + \ldots]$

$= 3 + 3x + 3x^2 + 3x^3 + \ldots, \ |-x| < 1 \ \ \therefore |x| < 1$

$\dfrac{1}{1-4x} = (1-4x)^{-1} = 1 + (-1)(-4x) + \frac{(-1)(-2)}{2}(-4x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(-4x)^3 + \ldots$

$= 1 + 4x + 16x^2 + 64x^3 + \ldots, \ |-4x| < 1 \ \ \therefore |x| < \frac{1}{4}$

$\therefore \ \dfrac{2-11x}{1-5x+4x^2} = (3 + 3x + 3x^2 + 3x^3 + \ldots) - (1 + 4x + 16x^2 + 64x^3 + \ldots)$

$= 2 - x - 13x^2 - 61x^3 + \ldots, \ \text{valid for } |x| < \frac{1}{4}$

4).

a $\dfrac{4-17x}{(1+2x)(1-3x)^2} \equiv \dfrac{A}{1+2x} + \dfrac{B}{1-3x} + \dfrac{C}{(1-3x)^2}$

$4 - 17x \equiv A(1 - 3x)^2 + B(1 + 2x)(1 - 3x) + C(1 + 2x)$

$x = -\frac{1}{2} \quad \Rightarrow \quad \frac{25}{2} = \frac{25}{4}A \quad \Rightarrow \quad A = 2$

$x = \frac{1}{3} \quad \Rightarrow \quad -\frac{5}{3} = \frac{5}{3}C \quad \Rightarrow \quad C = -1$

coeffs of $x^2 \ \Rightarrow \quad 0 = 9A - 6B \quad \Rightarrow \quad B = 3$

$\therefore \ f(x) \equiv \dfrac{2}{1+2x} + \dfrac{3}{1-3x} - \dfrac{1}{(1-3x)^2}$

b $\dfrac{2}{1+2x} = 2(1+2x)^{-1} = 2[1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(2x)^3 + \ldots]$

$= 2 - 4x + 8x^2 - 16x^3 + \ldots$

$\dfrac{3}{1-3x} = 3(1-3x)^{-1} = 3[1 + (-1)(-3x) + \frac{(-1)(-2)}{2}(-3x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(-3x)^3 + \ldots]$

$= 3 + 9x + 27x^2 + 81x^3 + \ldots$

$\dfrac{1}{(1-3x)^2} = (1-3x)^{-2} = 1 + (-2)(-3x) + \frac{(-2)(-3)}{2}(-3x)^2 + \frac{(-2)(-3)(-4)}{3\times2}(-3x)^3 + \ldots$

$= 1 + 6x + 27x^2 + 108x^3 + \ldots$

$f(x) = (2 - 4x + 8x^2 - 16x^3 + \ldots) + (3 + 9x + 27x^2 + 81x^3 + \ldots) - (1 + 6x + 27x^2 + 108x^3 + \ldots)$

$= 4 - x + 8x^2 - 43x^3 + \ldots$