GCSE Homework Booklet 5

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Composite and Inverse Functions

Q1.

f(x) $\frac{x}{x+2}$ g(x) = $x^2 - 2$

Work out fg(x)

Give your answer in the form $a + bx^n$ where a, b and n are integers.

Answer _____

(Total 3 marks)

Q2.

$f(x) = \frac{2x+3}{x-4}$

Work out $f^{-1}(x)$

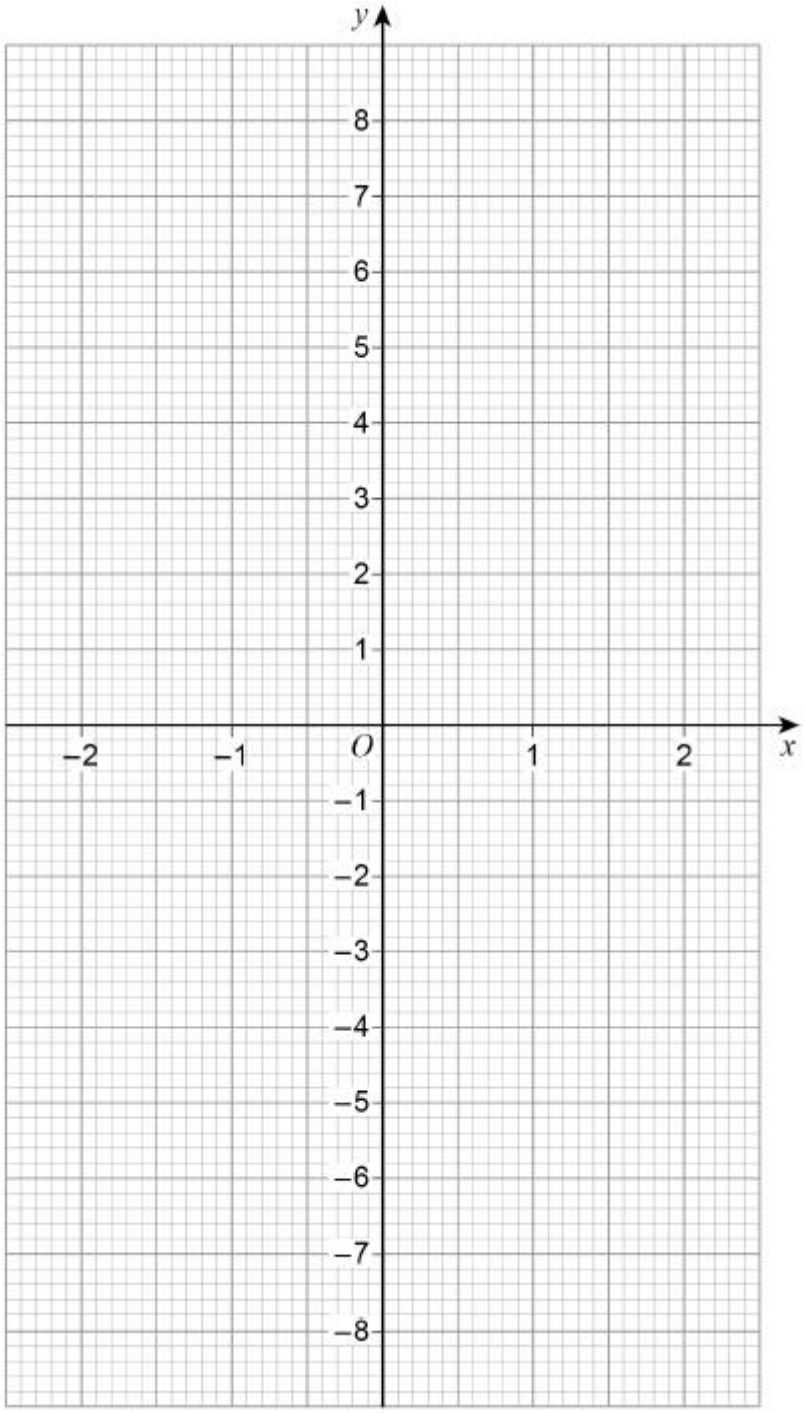
Answer _____

(Total 4 marks)

Q3.

(a) $h(x) = \sqrt[3]{x}$ for all values of x

On the grid, draw the graph of the inverse function $y = h^{-1}(x)$ for $-2 \leq x \leq 2$



(2)

Q4.

For all values of x, $f(x) = x^2 + 1$ $g(x) = x - 5$

(a) Show that $fg(x) = x^2 - 10x + 26$

(2)

(b) Solve $fg(x) = gf(x)$

$x =$ _____

(4)

(Total 6 marks)

Q5.

$f(x) = \frac{x}{3} + 4$ for all values of x.

$g(x) = 6x^2 + 3$ for all values of x.

Work out $fg(x)$.

Give your answer in the form $ax^2 + b$ where a and b are integers.

Answer _____

(Total 2 marks)

Q6.

$f(x) = 3^{2x}$ and $g(x) = x^3$ for all values of x.

(a) Work out the value of $f(1) + g(4)$

Answer _____

(2)

(b) Work out the value of $g^{-1}(-27)$

Answer _____

(2)

(c) Work out an expression for gf(X)

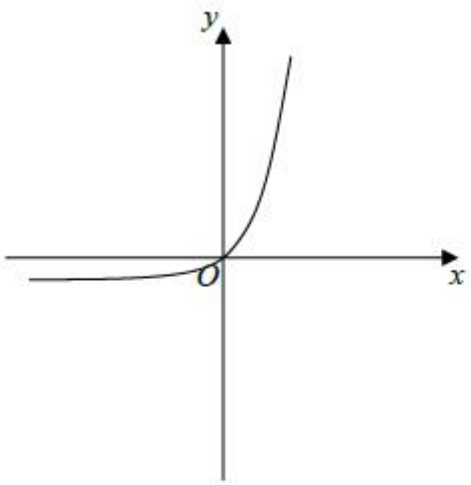
Give your answer as a power of 3 in its simplest form.

Answer _____

(2)

(d) One of these graphs is a sketch of $y = 3^{2X}$
Which one?
Circle the correct letter.

A



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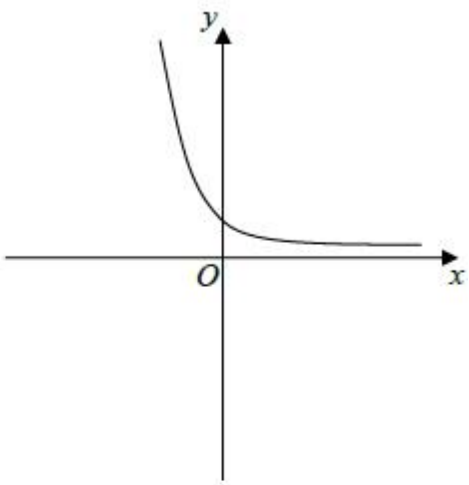

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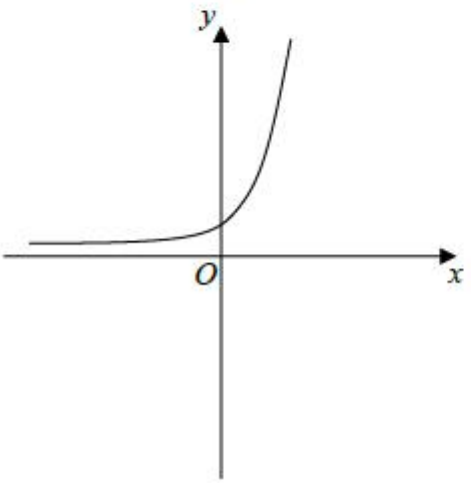
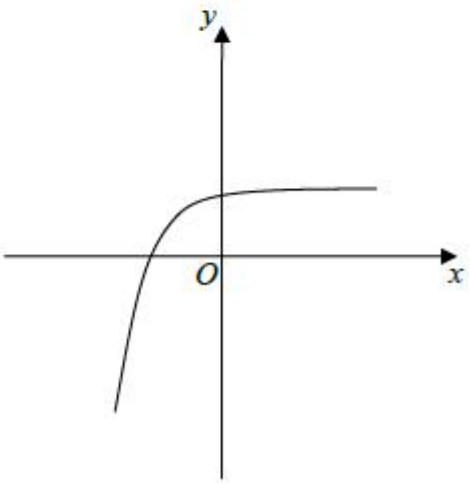


D



(1)

(Total 7 marks)

Q7.

$f(x) = 2x^2$
$g(x) = x + 5$

Circle the composite function $fg(x)$

$2x^2 + 5$ $2(x + 5)^2$ $2(x^2 + 5)$ $4(x + 5)^2$

(Total 1 mark)

Q8.

$f(x) = 2x + c$
$g(x) = cx + 5$
$fg(x) = 6x + d$
c and d are constants.

Work out the value of d.

Answer _____

(Total 3 marks)

Equations of Circles

Q1.

Work out the diameter of the circle $x^2 + y^2 = 64$

Circle your answer.

8 16 32 128

(Total 1 mark)

Q2.

(a) What is the equation of a circle with centre (0, 0) and diameter 6 units?
Circle your answer.

$x^2 + y^2 = 3$ $x^2 + y^2 = 6$ $x^2 + y^2 = 9$ $x^2 + y^2 = 36$

(1)

(b) Which of these points lie on the circumference of the circle $x^2 + y^2 = 25$?
Circle your answer.

(−3, 4) (6.25, 6.25) (9, 16) (−1, 12)

(1)

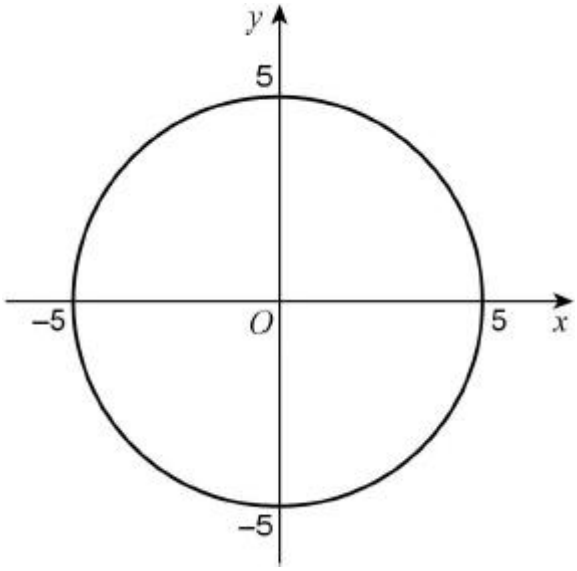
(c) Circle True (T) or False (F) for each statement.

The centre of the circle $x^2 + y^2 = 25$ is (0, 0)	T	F
The equation of the tangent to the circle $x^2 + y^2 = 25$ at the point (5, 0) is $y = 5$	T	F
The equation of a circle and the equation of a straight line can have 0, 1 or 2 solutions if solved simultaneously	T	F

(2)
(Total 4 marks)

Q3.

A circle, centre O, passes through (5, 0).



What is the equation of the circle?

Circle your answer.

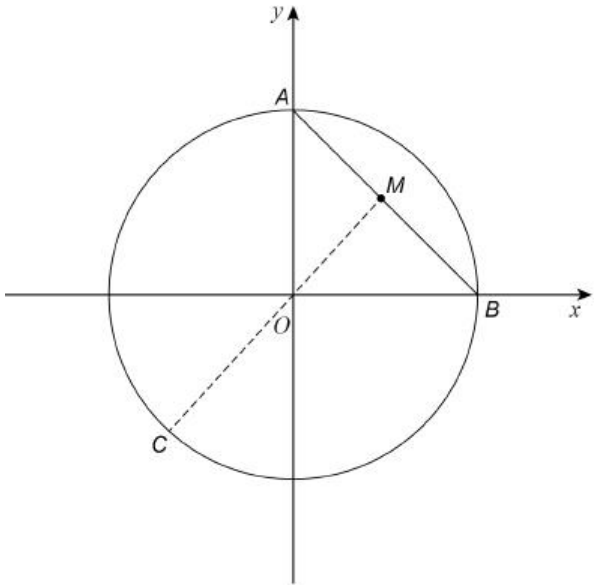
$x^2 + y^2 = 25$ $x^2 + y^2 = 5$ $x^2 + y^2 = 10$ $x^2 + y^2 = 100$

(Total 1 mark)

Q4.

A, *B* and *C* are points on the circle $x^2 + y^2 = 36$ as shown.

A is on the *y*-axis.
B is on the *x*-axis.
M is the midpoint of *AB*.
COM is a straight line.



(a) Show that the coordinates of *A* are (0, 6)

_____

_____

(1)

(b) Work out the coordinates of *B*.

_____

_____

Answer (_____ , _____)

(1)

(c) Show that the equation of the straight line passing through *C*, *O* and *M* is $y = x$

_____

_____

_____

_____

(2)

(d) Work out the coordinates of C.
Give your answers in surd form.

_____

_____

Answer (_____ , _____)

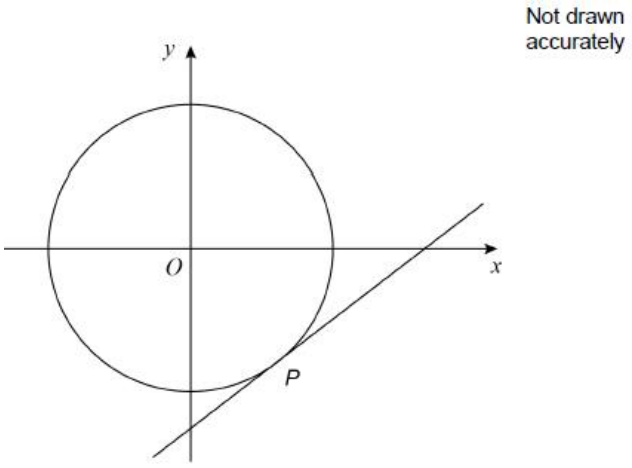
(3)

(Total 7 marks)

Q5.

P is a point on the circle with equation $x^2 + y^2 = 80$

P has x-coordinate 4 and is below the x-axis.



Work out the equation of the tangent to the circle at P.

_____

_____

_____

_____

_____

_____

_____

_____

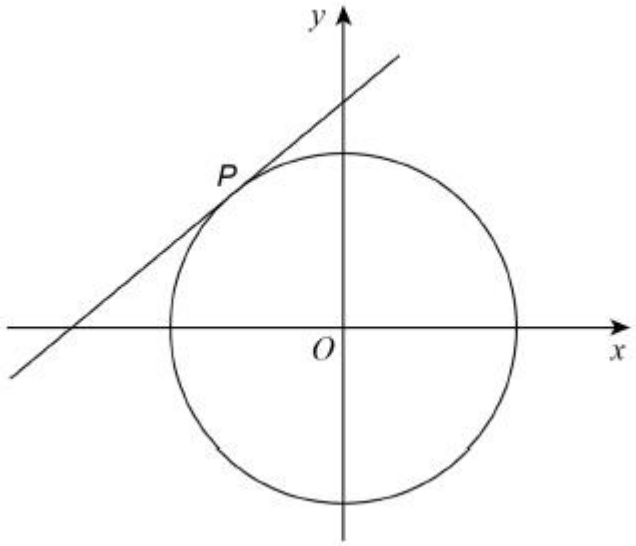
_____

Answer _____

(Total 5 marks)

Q6.

P (−1, 4) is a point on a circle, centre *O*



Not drawn accurately

Work out the equation of the tangent to the circle at *P*.

Give your answer in the form $y = mx + c$

Answer _____

(Total 4 marks)

Q7.

The line $y = 3x + p$ and the circle $x^2 + y^2 = 53$ intersect at points A and B.

p is a positive integer.

(a) Show that the x-coordinates of points A and B satisfy the equation

$10x^2 + 6px + p^2 - 53 = 0$

(3)

(b) The coordinates of A are (2, 7)

Work out the coordinates of B.

You **must** show your working.

Answer (_____ , _____) **[5]**

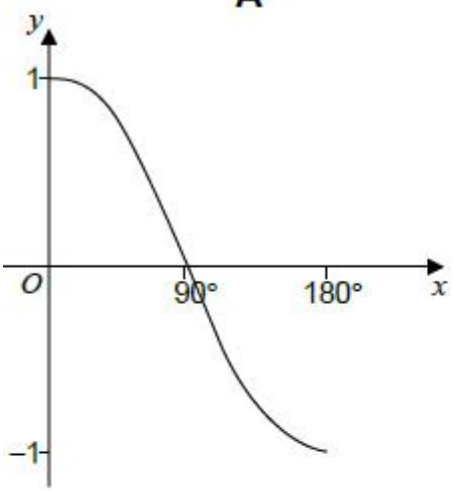
Graphs of Trigonometric Functions

Q1.

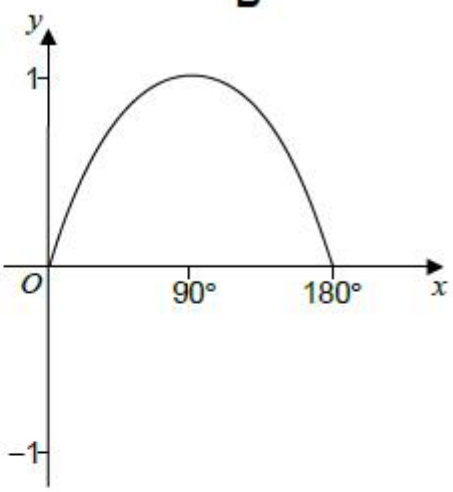
One of these is a sketch of $y = \cos x$ for $0° \leq x \leq 180°$

Which one?
Circle the correct letter.

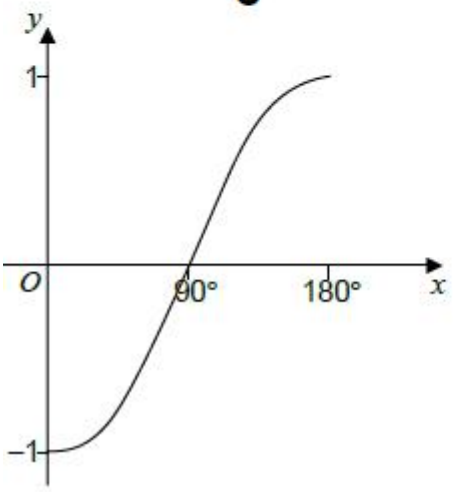
A



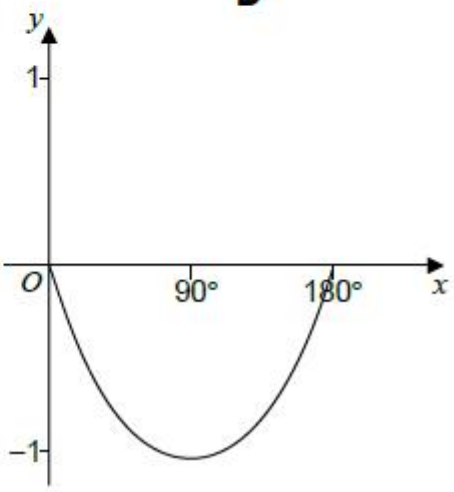
B



C



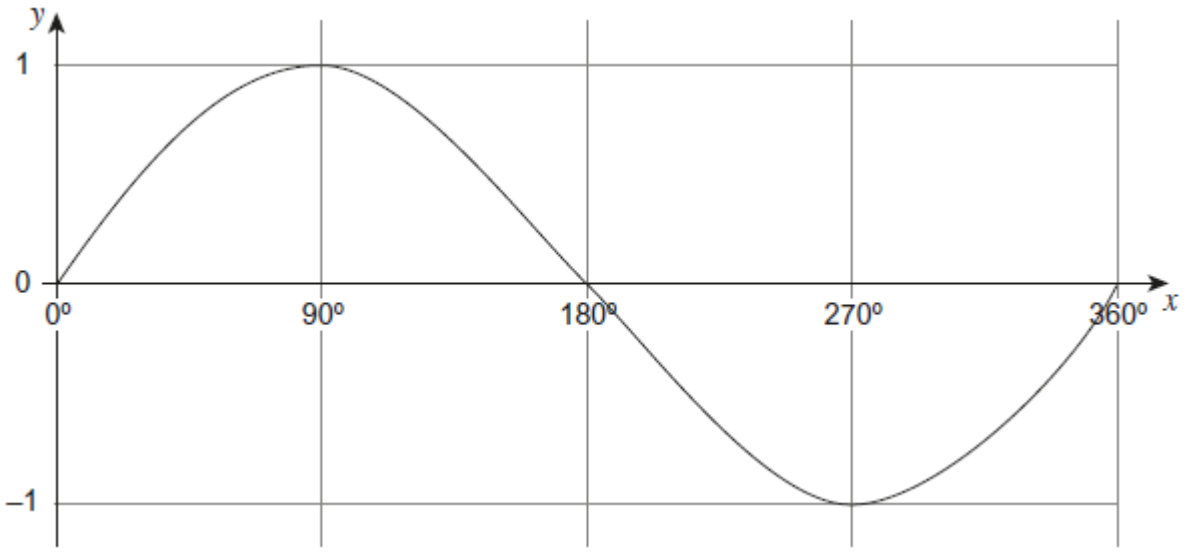
D



(Total 1 mark)

Q2.

This is a sketch graph of $y = \sin x$ for $0° \leq x \leq 360°$



(a) Write down the number of solutions for $\sin x = 0.5$ for $0° \leq x \leq 360°$

Answer _____

(1)

(b) $\sin x = \sin 10$

Write down the value of x for $90° \leq x \leq 180°$

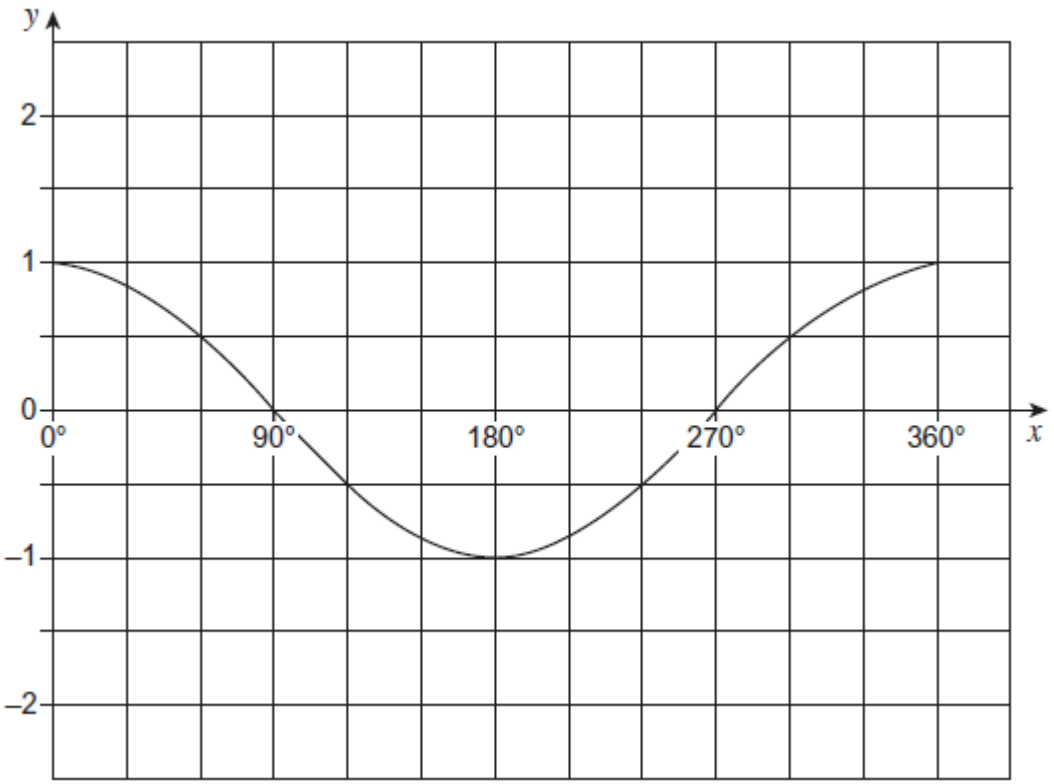
Answer _____

(1)

(Total 2 marks)

Q3.

The graph $y = \cos x$ for $0° \leq x \leq 360°$ is shown.



Write down the **two** solutions to the equation $\cos x = 0.5$ for $0° \leq x \leq 360°$

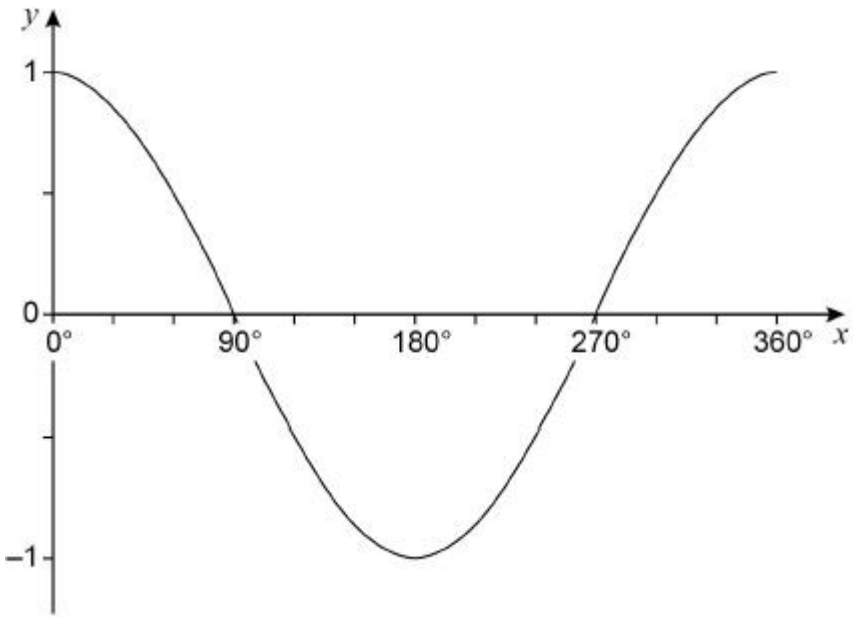
Answer _____ degrees

and _____ degrees

(Total 1 mark)

Q4.

Here is a sketch of the graph of $y = \cos x$ for values of x from 0° to 360°



(a) $\cos x = \cos 60°$

Work out the value of x when $90° \leq x \leq 360°$

Answer _____ degrees

(1)

(b) $\cos x = -\cos 60°$

Work out the value of x when $180° \leq x \leq 360°$

Answer _____ degrees

(1)

(Total 2 marks)

Q5.

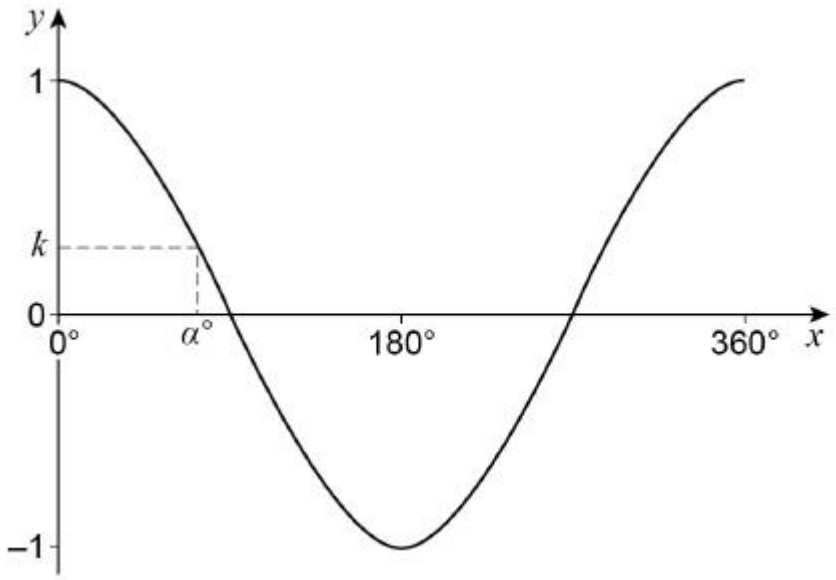
Which of these values **cannot** be the cosine of an angle?
Circle your answer.

−0.5 0 0.5 1.5

(Total 1 mark)

Q6.

Here is a sketch of $y = \cos x$ for values of x from 0° to 360°



α° is an acute angle.

$\cos \alpha° = k$

(a) Circle the value of $\cos (180° - \alpha°)$

$1 - k$ k $-k$ $-1 - k$

(1)

(b) Circle the value of $\cos (360° + \alpha°)$

$k - 1$ $k + 1$ $-k$ k

(1)

(Total 2 marks)

Q7.

For which acute angle do sin x and cos x have the same value?
Circle your answer.

0° 30° 45° 60°

(Total 1 mark)

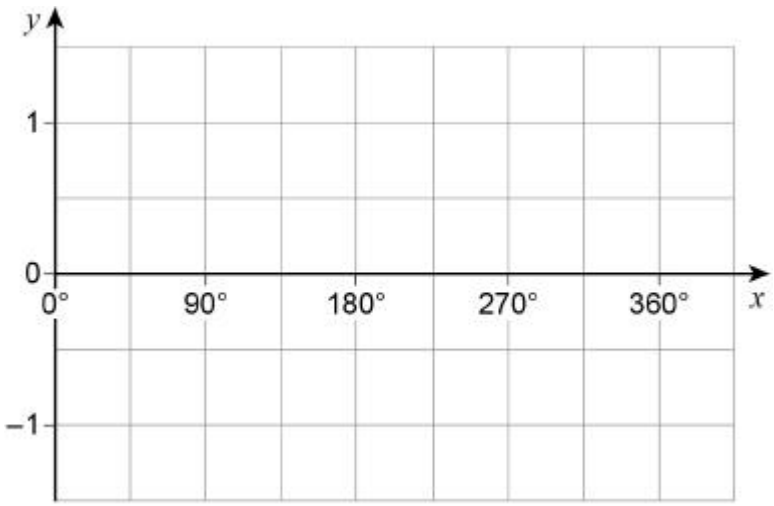
Q8.

For all values of x

$f(x) = \sin x$

$g(x) = x + 90$

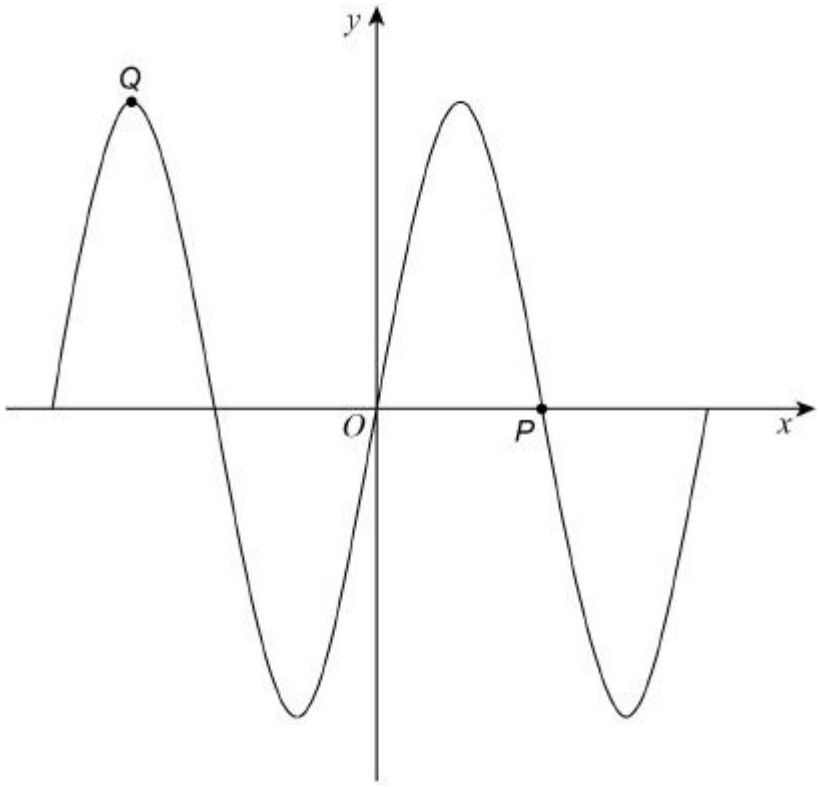
On the grid, draw the graph of the composite function $y = fg(x)$ for $0° \leq x \leq 360°$



(2)

Q9.

Here is a sketch of $y = \sin x°$ for $-360 \leq x \leq 360$



(a) Write down the coordinates of *P*.

Answer (_____ , _____)

(1)

(b) Write down the coordinates of *Q*.

Answer (_____ , _____)

(1)

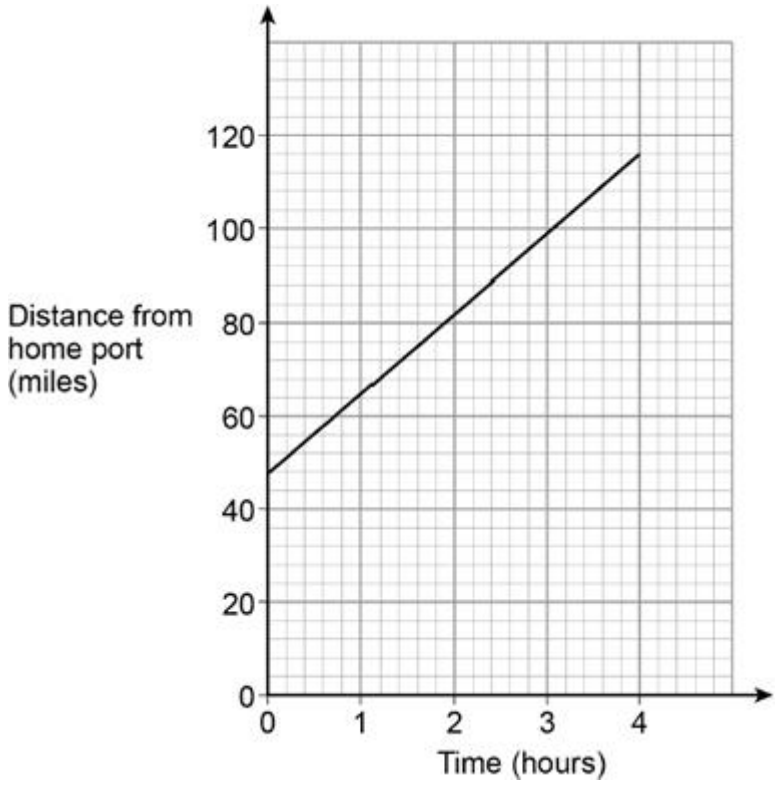
(Total 2 marks)

Displacement time and Velocity time Graphs

Q1.

A ship is sailing in a straight line from its home port.

The distance-time graph shows 4 hours of the journey.



Work out the speed of the ship during these 4 hours.

_____

_____

_____

_____

_____

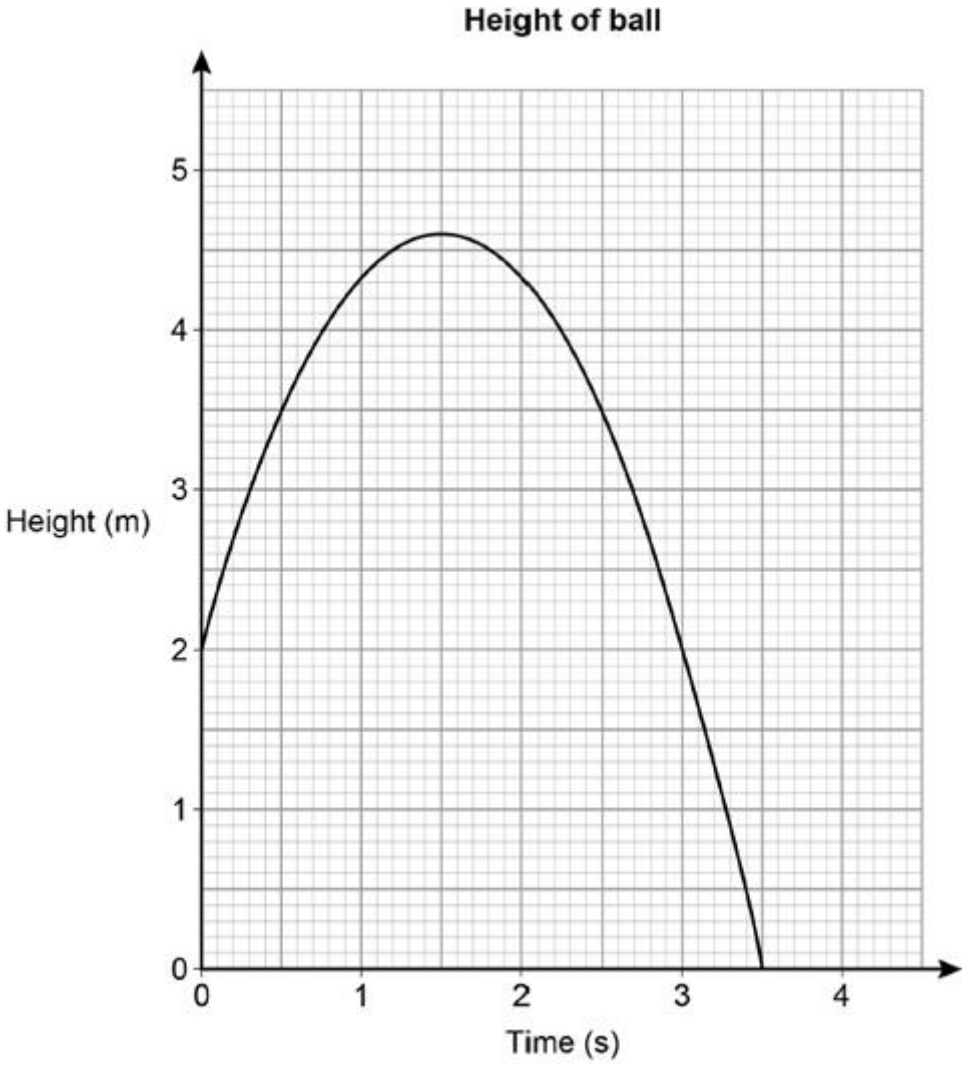
Answer _____ mph

(Total 3 marks)

Q2.

A ball is thrown vertically upwards.

The graph shows the height of the ball above the ground after it is thrown.



(a) For how many seconds is the ball at a height of **more than** 2 metres?

Answer _____ s

(1)

(b) After how many seconds is the ball at instantaneous rest when it is in the air?

Answer _____ s

(1)

(c) Work out the average speed of the ball when it is moving downwards.

_____

_____

Answer _____ m/s **(2)**

Q3.

Lily goes on a car journey.

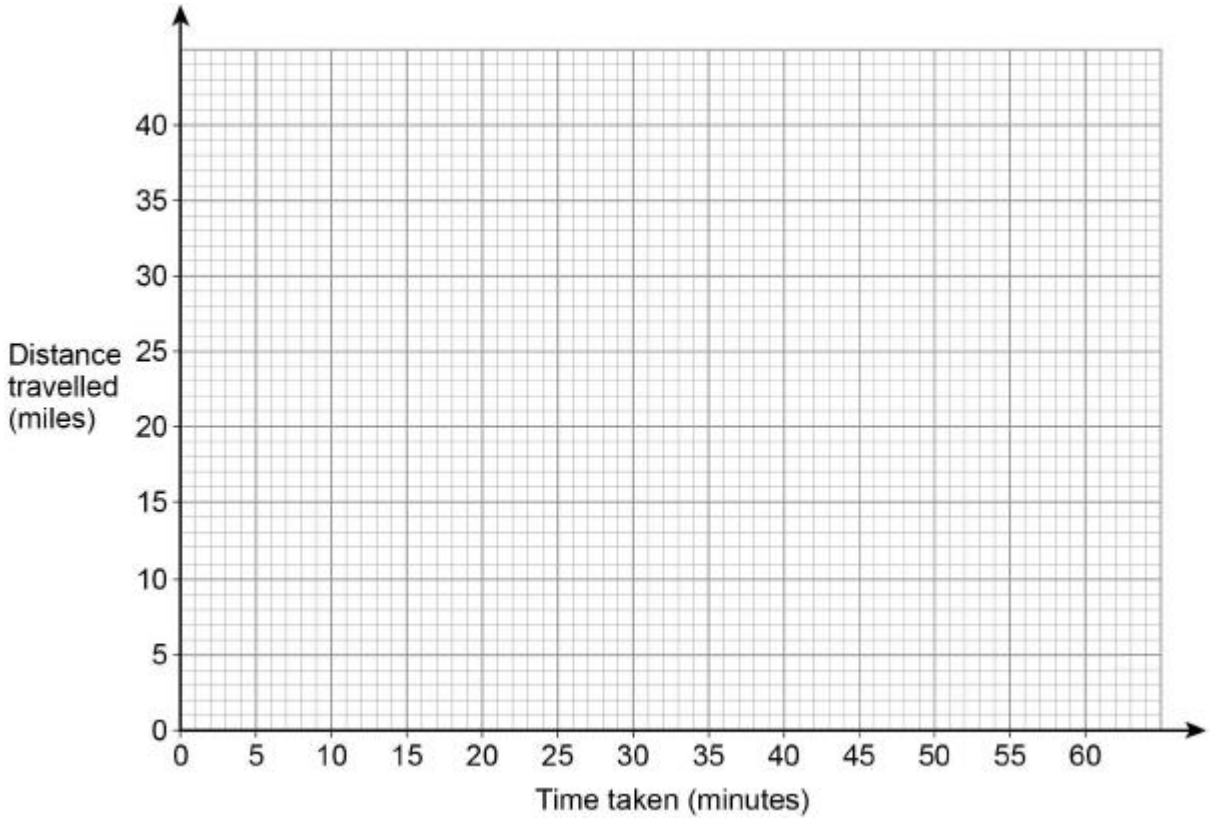
For the first 30 minutes her average speed is 40 miles per hour.

She then stops for 15 minutes.

She then completes the journey at an average speed of 60 miles per hour.

The total journey time is 1 hour.

(a) Draw a distance-time graph for her journey.



(3)

(b) Write down the average speed for the total journey.

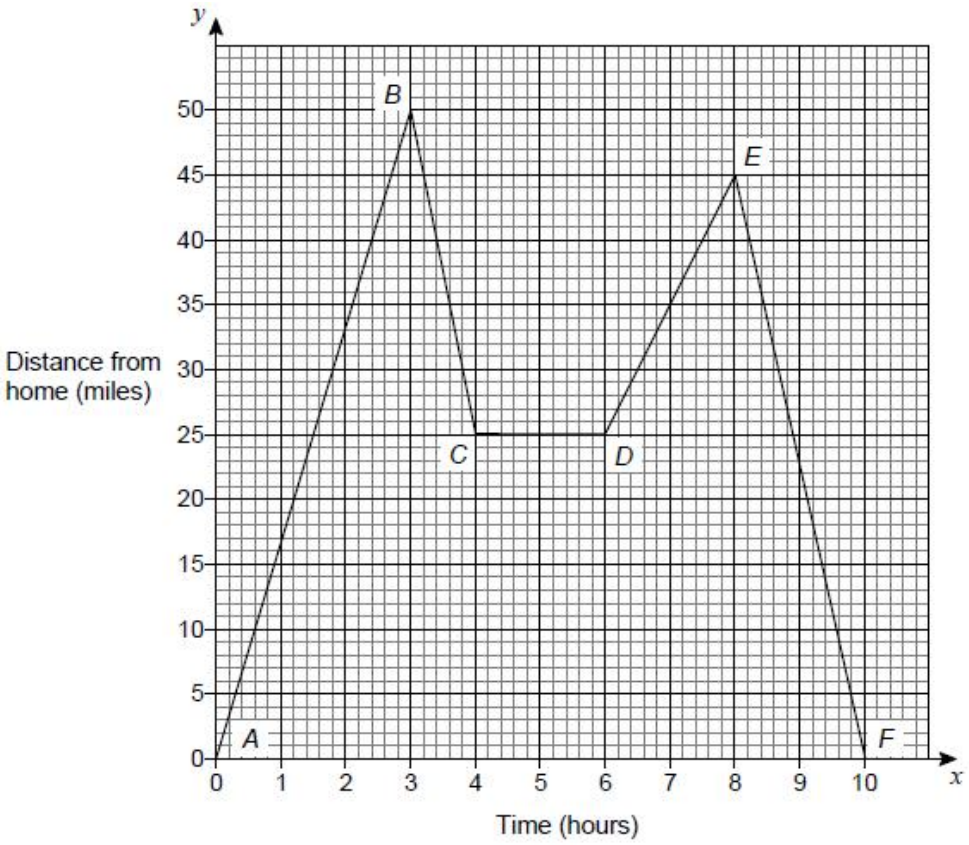
Answer _____ mph

(1)

(Total 4 marks)

Q4.

Here is a distance-time graph.



(a) Match each statement to one section of the journey.

Average speed = 25 miles per hour ..... to .....

Average speed = 10 miles per hour ..... to .....

Stationary ..... to .....

Fastest part of the journey ..... to .....

(4)

(b) How far is the whole journey?

_____

_____

Answer _____ miles

(2)

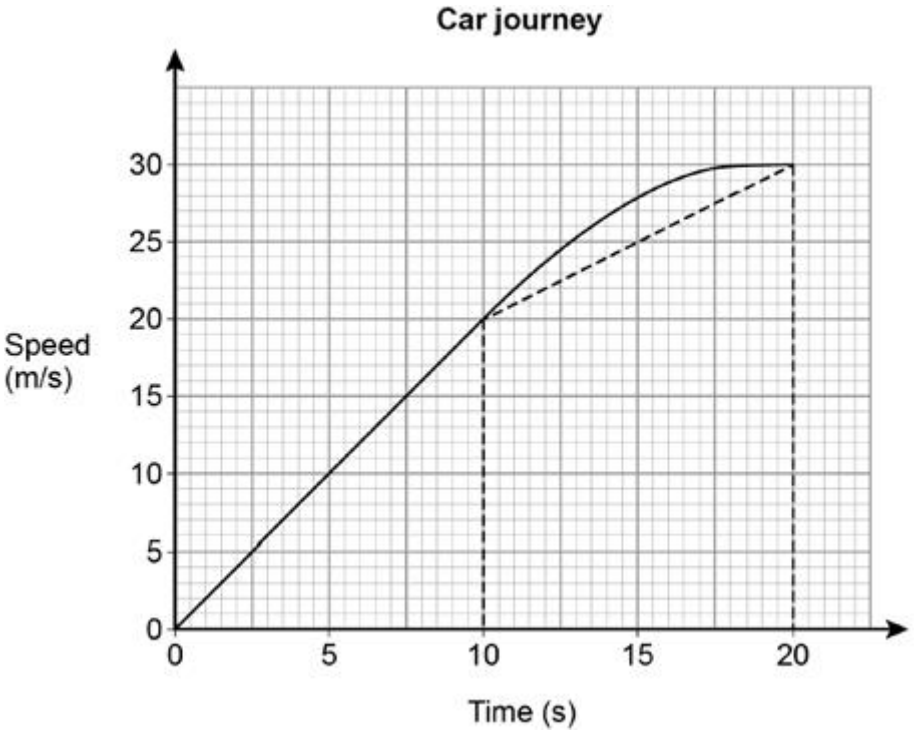
(Total 6 marks)

Q5.

The speed-time graph shows 20 seconds of a car journey.

Harry wants to estimate the distance the car travels in this time.

He uses a triangle and a trapezium, as shown, to estimate the area under the graph.



(a) Complete Harry's method to estimate the distance the car travels.

Answer = _____ m

(3)

(b) For this journey, which of these is true for Harry's method?

Tick **one** box.

☐ It works out an overestimate of the distance

☐ It works out an underestimate of the distance

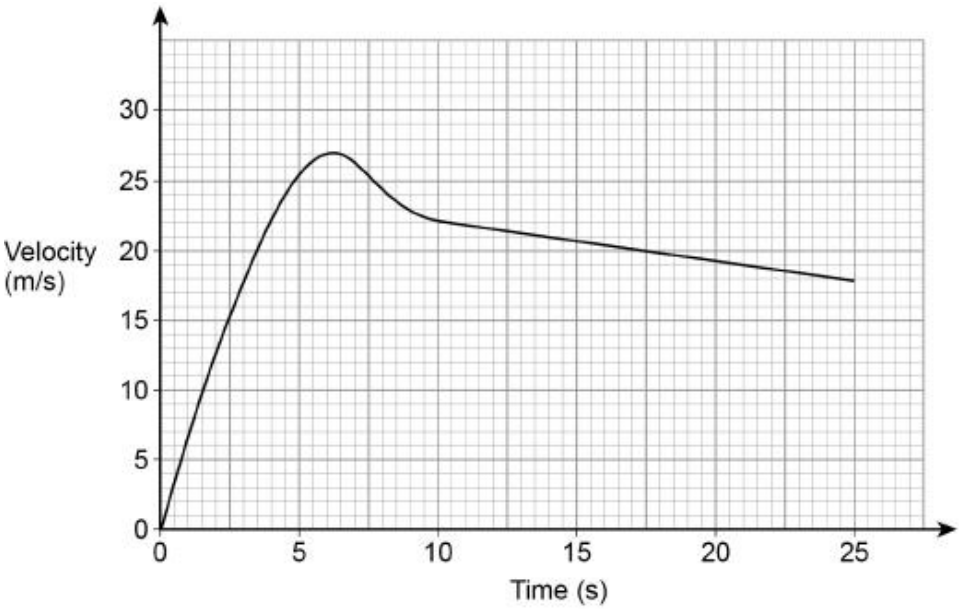
☐ It could work out an overestimate or an underestimate of the distance

(1)

(Total 4 marks)

Q6.

Here is a velocity-time graph of a motorbike for 25 seconds.



(a) After how many seconds was the acceleration zero?

Answer _____ seconds

(1)

(b) Work out the distance travelled in the last 15 seconds.

_____

_____

_____

_____

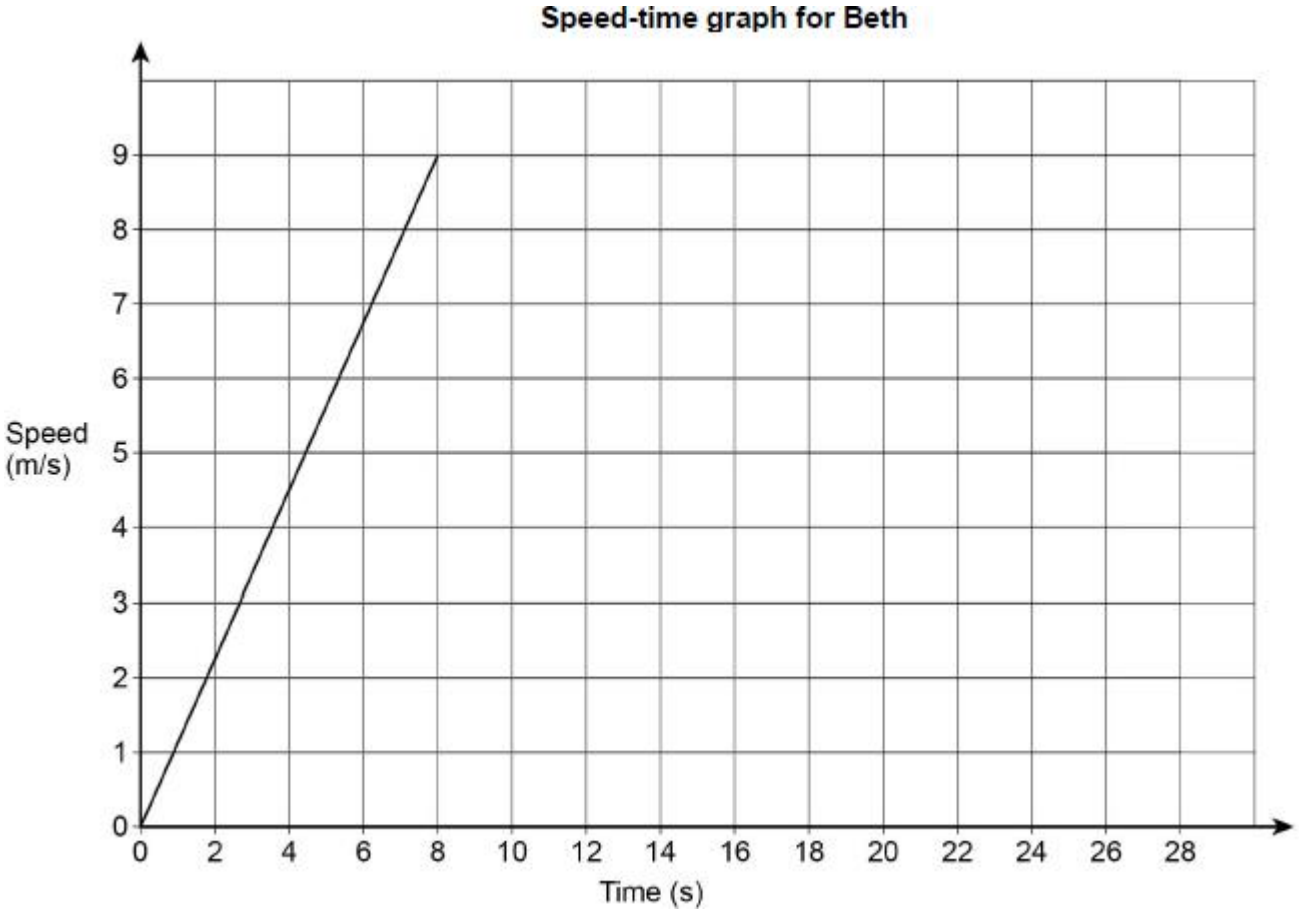
Answer _____ metres **(2)**

Q7.

Beth ran a 200 metre race.

Here is a graph of the first 8 seconds of her race.

She completed the race at a constant speed of 9 m/s



Amy completed the race in 27 seconds.

Did Beth finish before Amy?

You **must** show your working.

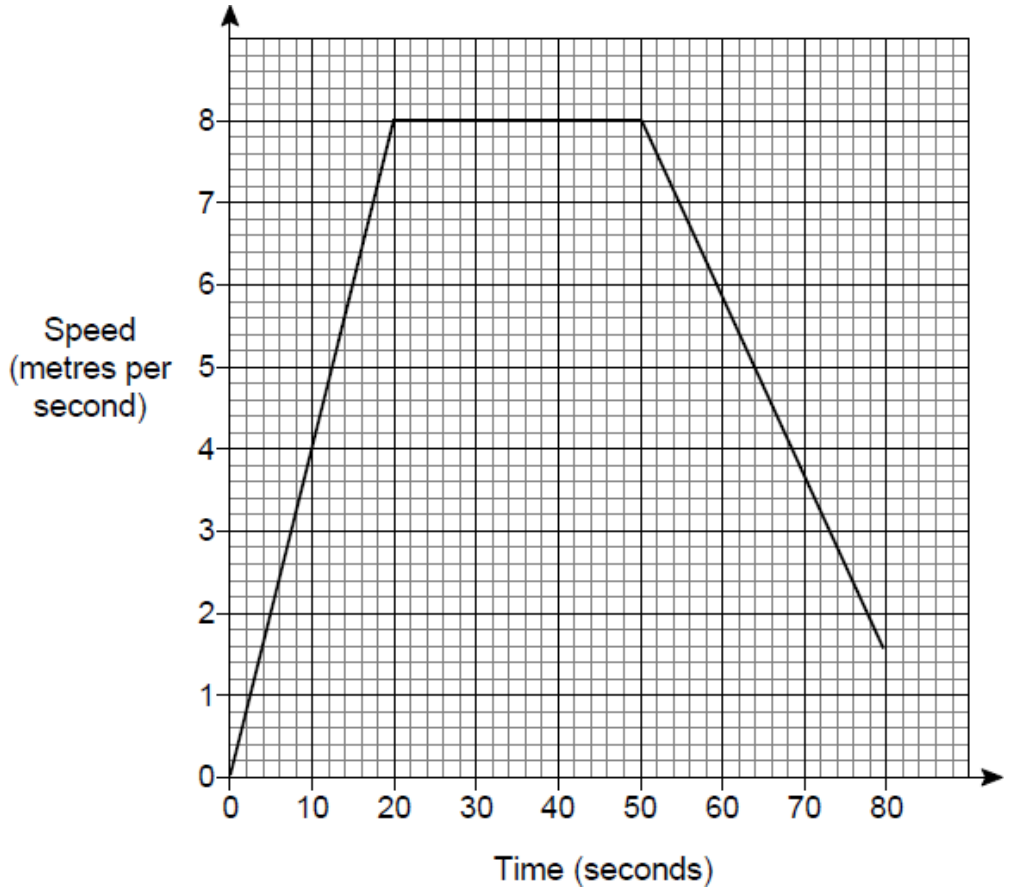
Answer _____

(Total 3 marks)

Q8.

Amina and Ben had a cycle race.

Here is Amina's speed-time graph from the start of the race.



The distance of the race was 400 metres.

Ben cycled the 400 metres in 64 seconds.

Who won the race?

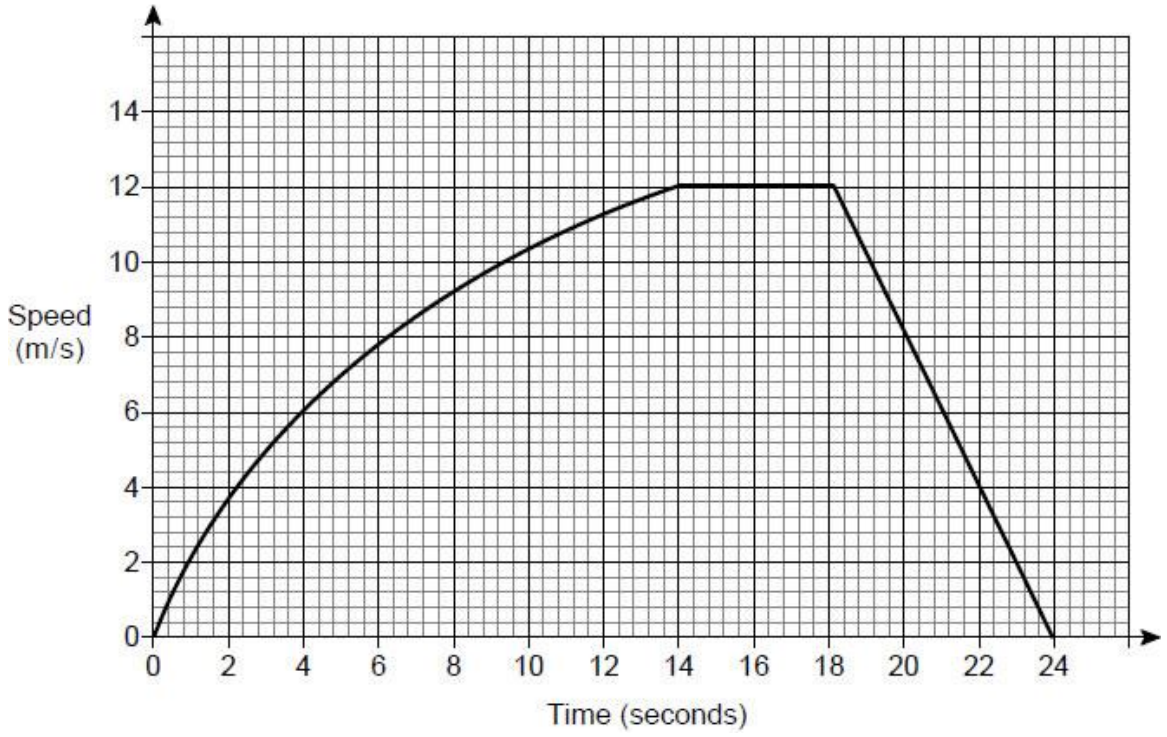
You **must** show your working.

Answer _____

(Total 4 marks)

Q9.

The speed-time graph for a car's journey is shown.



(a) Estimate the acceleration at 6 seconds.
You **must** show your working.

Answer _____ m/s^2

(3)

(b) Estimate the average speed of the car for the journey.
You **must** show your working.

Answer _____ m/s

(4)

(c) Evaluate your answer to part (b).

Tick a box.

☐ underestimate ☐ exact ☐ overestimate

Comment _____

_____

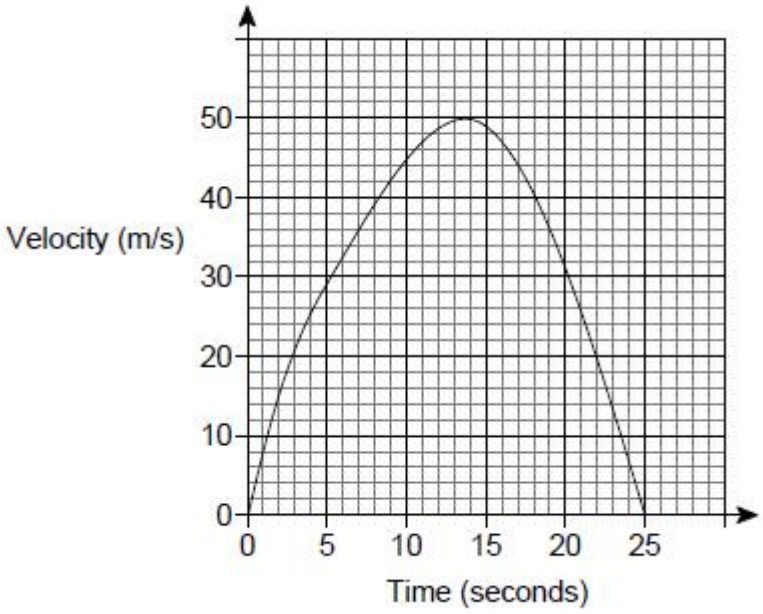
_____

(1)

(Total 8 marks)

Q10.

Here is a velocity-time graph of a car.



Estimate the distance travelled in the first 10 seconds.

_____

_____

_____

_____

Answer ..... metres

(Total 2 marks)

Q11.

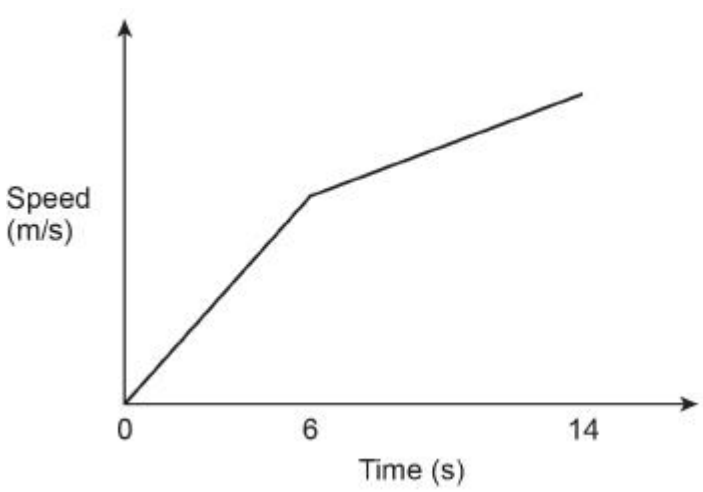
Izzy runs an 80-metre race in 14 seconds.

During the first 6 seconds her speed increases at a constant rate.

During the last 8 seconds her speed increases at a different constant rate.

Her speed at 14 seconds is 2 m/s more than her speed at 6 seconds.

Here is a sketch of her speed-time graph.



(a) Work out her acceleration during the last 8 seconds.
State the units of your answer.

Answer _____

(2)

(b) When Izzy finishes the 80-metre race, her speed is v m/s

Work out the value of v.

Answer _____ **(4)**

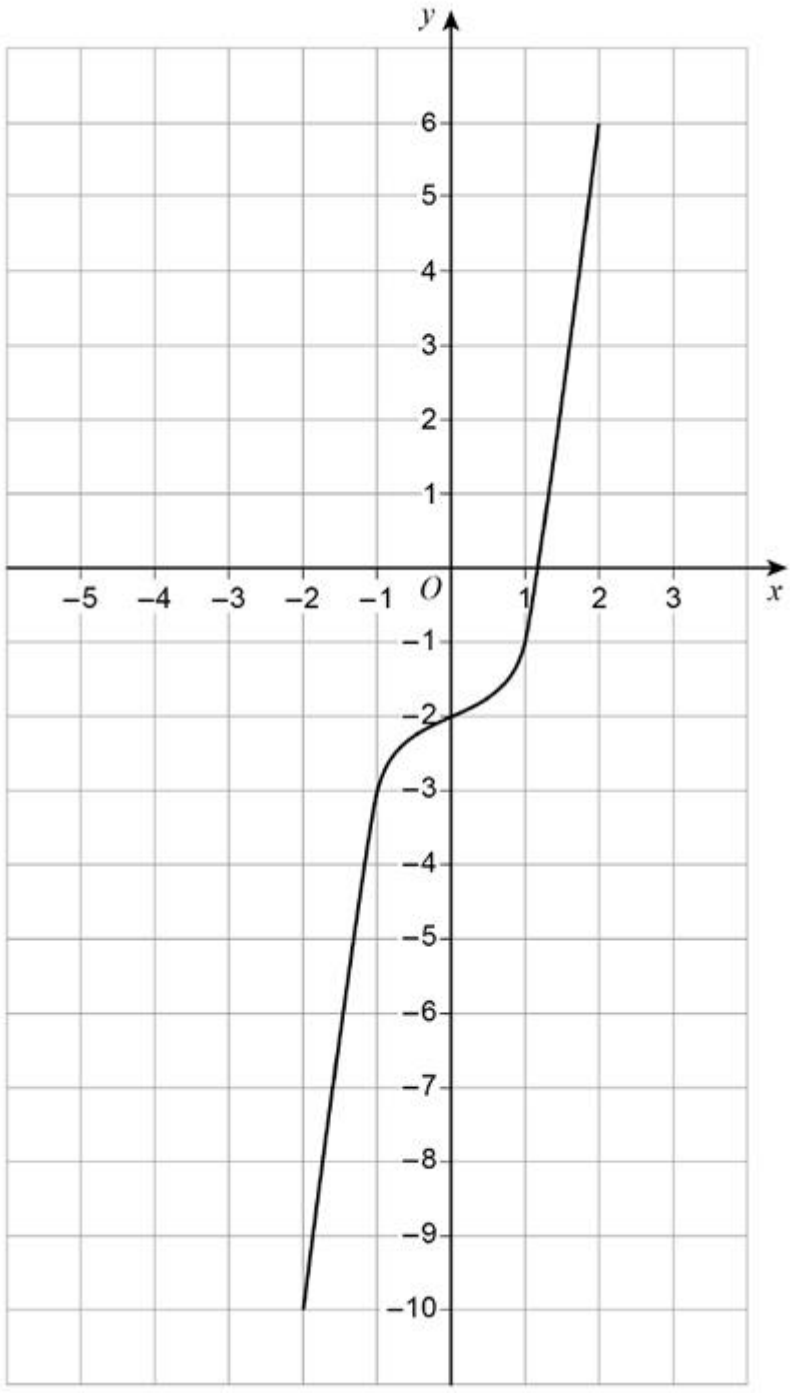
Graph Transformations

Q1.

Here is a sketch of $y = f(x)$

The curve passes through the points

(–2, –10) (–1, –3) (0, –2) (1, –1) (2, 6)



On the grid, sketch the curve $y = f(x + 2)$

(Total 2 marks)

Q2.

The curve with equation $y = x^2 - 5x + 2$ is reflected in the x-axis.

Circle the equation of the reflected curve.

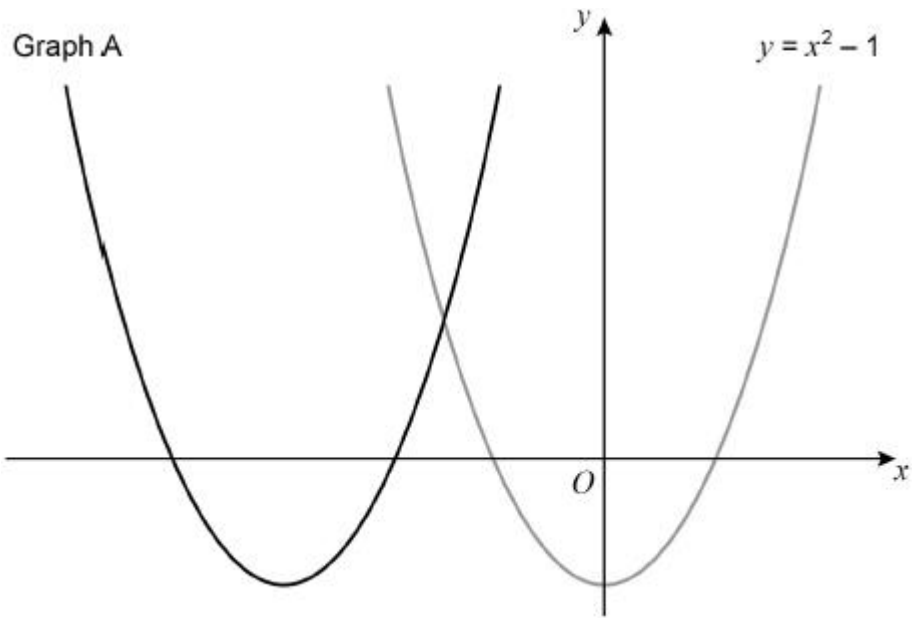
$y = x^2 - 5x - 2$ $\quad$ $y = -x^2 + 5x + 2$

$y = -x^2 + 5x - 2$ $\quad$ $y = x^2 + 5x + 2$

(Total 1 mark)

Q3.

Here are sketches of two graphs.



The graph of $y = x^2 - 1$ is translated 3 units to the left to give graph A.

(a) The equation of graph A can be written in the form $y = x^2 + bx + c$

Work out the values of b and c.

$b =$ _____

$c =$ _____

(3)

(b) The graph of $y = x^2 - 1$ is reflected in the x-axis to give graph B.

Work out the equation of graph B.

_____

_____

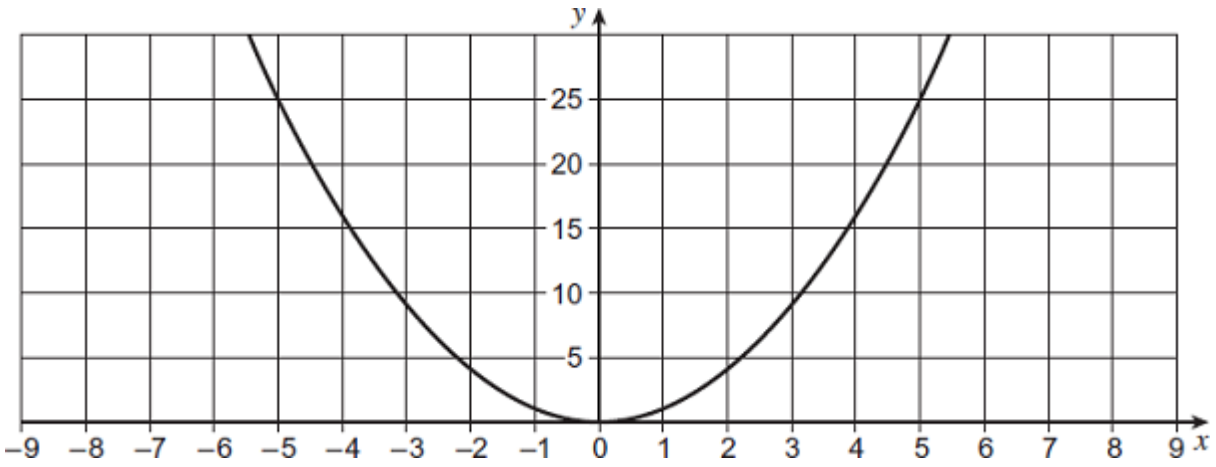
Answer _____

(1)

(Total 4 marks)

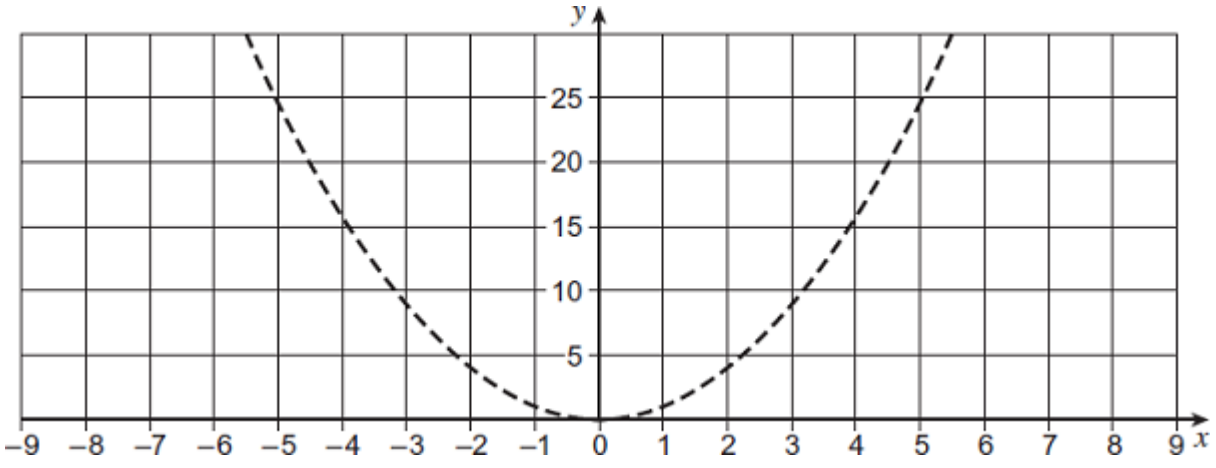
Q4.

This graph is a sketch of $y = x^2$



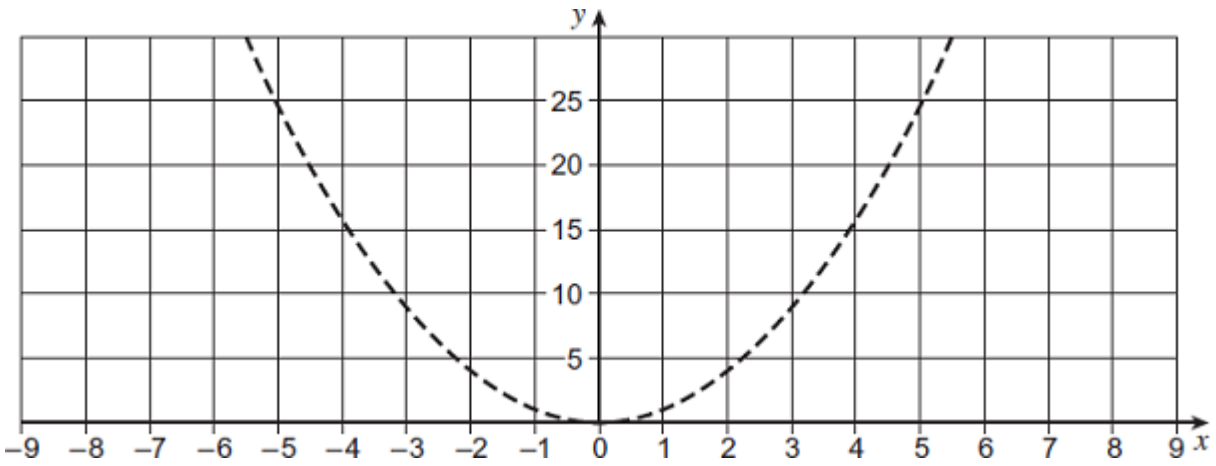
On each grid, the graph of $y = x^2$ is shown dashed to help you.

(a) Sketch the graph of $y = x^2 + 5$ on the grid.



(1)

(b) Sketch the graph of $y = (x - 3)^2$ on the grid.

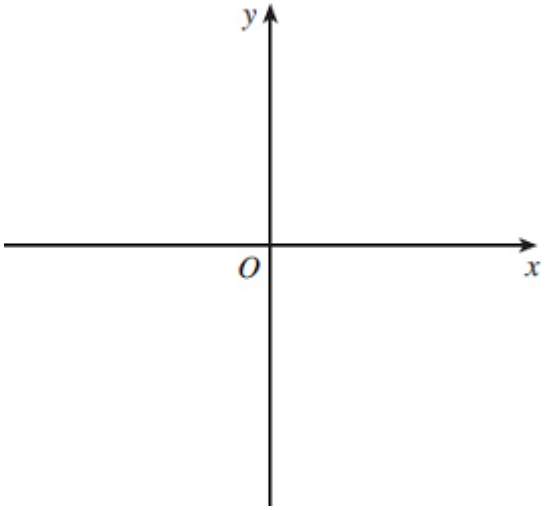


(1)

(Total 2 marks)

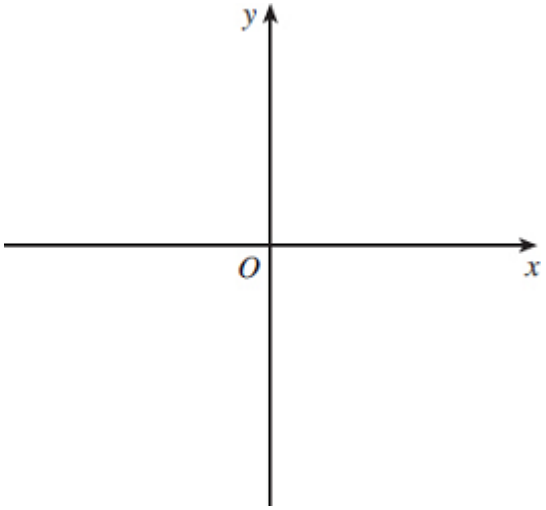
Q5.

(a) On the axes below sketch the graph of $y = x^3$



(1)

(b) On the axes below sketch the graph of $y = x^3 + 8$



(1)

(Total 2 marks)

Q6.

Curve P has equation $y = 2(x - 1)^2 - 5$

Curve Q is a reflection in the y-axis of curve P.

Work out the equation of curve Q.

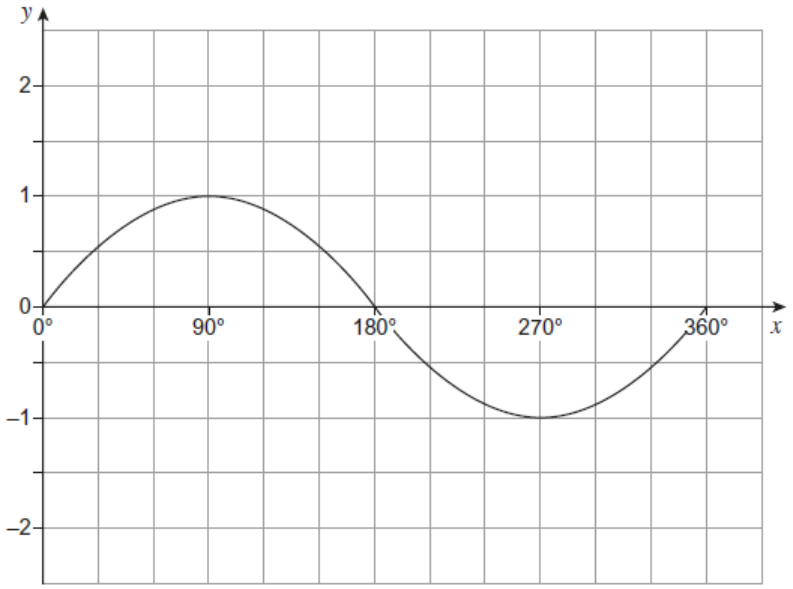
Give your answer in the form $y = ax^2 + bx + c$ where a, b and c are integers.

Answer _____

(Total 3 marks)

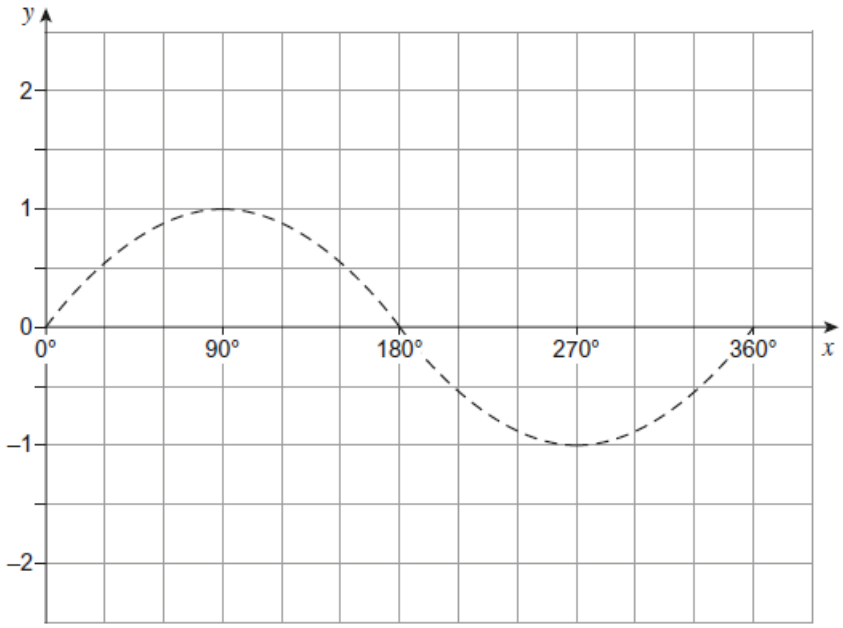
Q7.

The graph of $y = \sin x$ for $0° \leq x \leq 360°$ is shown.



(a) On the grid below, draw the graph of $y = 1 + \sin x$ for $0° \leq x \leq 360°$

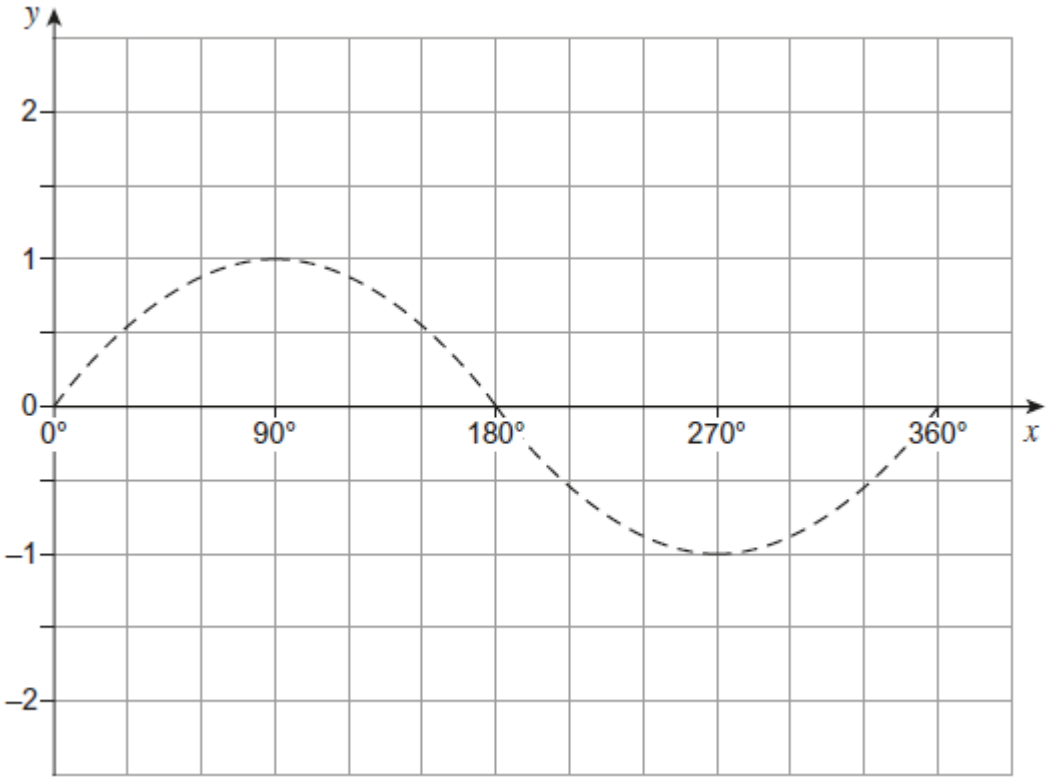
The graph of $y = \sin x$ is shown to help you.



(1)

(b) On the grid below, draw the graph of $y = \sin(x + 90°)$ for $0° \leq x \leq 360°$

The graph of $y = \sin x$ is shown to help you.



(1)

(Total 2 marks)

Q8.

The graph with equation $y = x^2$ is translated by vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

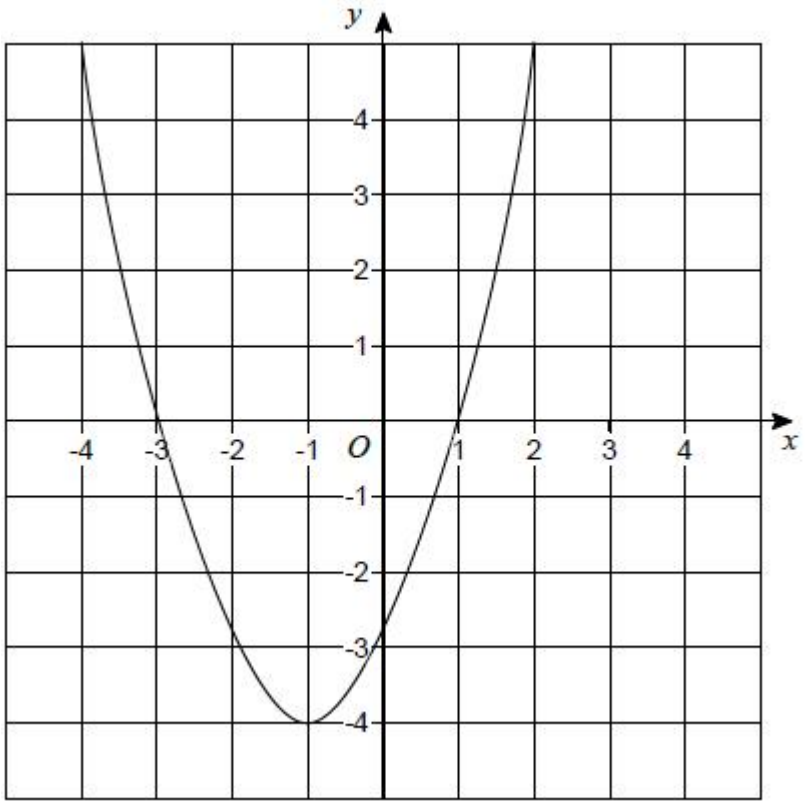
Circle the equation of the translated graph.

$y = (x - 2)^2$ $y = (x + 2)^2$ $y = x^2 + 4$ $y = x^2 + 2$

(Total 1 mark)

Q9.

(a) Here is the graph of $y = f(x)$
The graph has a turning point at $(-1, -4)$



On the grid, draw the graph of $y = f(x - 2)$

(1)

(b) The graph of $y = -3x^2 + 4x - 5$ is reflected in the y-axis.

Work out the equation of the reflected graph.
Give your answer in its simplest form.

_____

_____

_____

Answer _____

(2)

(Total 3 marks)

Iteration

Q1.

Use trial and improvement to find a solution to $2^x - 30 = 0$
Give your answer to 1 decimal place.

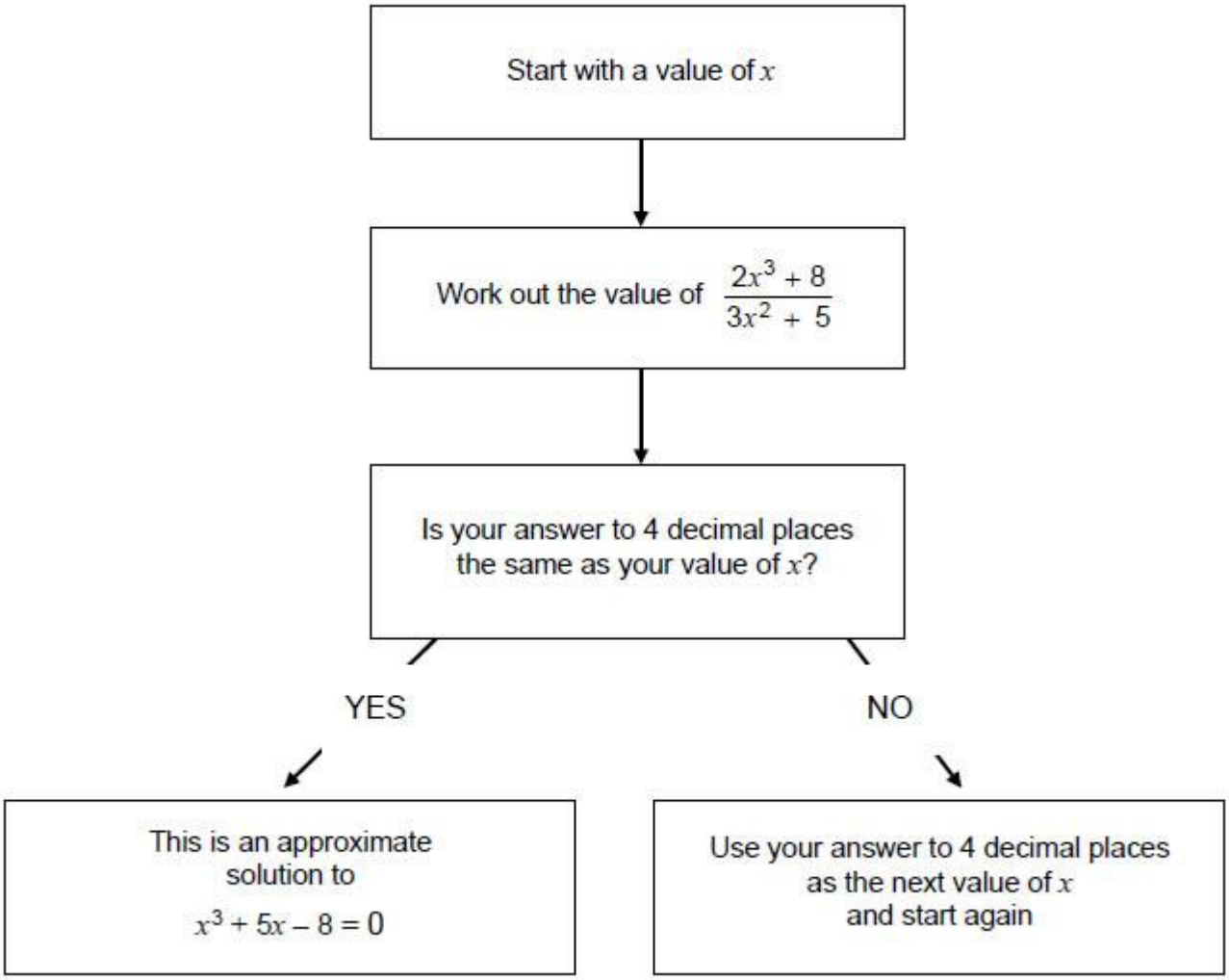
x	$2^x - 30$	Comment
4	– 14	Too small

x = _____

(Total 4 marks)

Q2.

This iterative process can be used to find approximate solutions to $x^3 + 5x - 8 = 0$



(a) Use this iterative process to find a solution to 4 decimal places of $x^3 + 5x - 8 = 0$

Start with the value $x = 1$

Answer _____

(3)

(b) By substituting your answer to part (a) into $x^3 + 5x - 8$

Comment on the accuracy of your solution to $x^3 + 5x - 8 = 0$

(2)
(Total 5 marks)

Q3.

An approximate solution to an equation is found using the iterative formula

$$x_{n+1} = \frac{(x_n)^3 - 2}{10}$$ with $x_1 = -1$

(a) Work out the values of x_2 and x_3

$x_2 =$ _____

$x_3 =$ _____

(2)

(b) Work out the solution to 5 decimal places.

$x =$ _____

(1)
(Total 3 marks)

Q4.

An approximate solution to an equation is found using this iterative process.

$$x_{n+1} = \frac{(x_n)^3 - 3}{8} \text{ and } x_1 = -1$$

(a) Work out the values of x_2 and x_3

x_2 = _____

x_3 = _____

(2)

(b) Work out the solution to 6 decimal places.

x = _____

(1)

(Total 3 marks)

Q5.

Use the iteration $x_{n+1} = 4 - \frac{2}{x_n^2}$

to work out an approximate solution to $x = 4 - \frac{2}{x^2}$

Start with $x_1 = 1$
Give your answer to 2 decimal places.

Answer _____

(Total 3 marks)

Combinations

Q1.

A music app has a shuffle play function.

This means that songs are played in a random order **without repeat**.

(a) Ruth puts 10 songs on shuffle play.

One of them is her favourite song.

Write down the probability that her favourite song plays first.

Answer _____

(1)

(b) Ted puts songs A, B and C on shuffle play.

List all the possible orders of songs A, B and C.

One has been done for you.

A B C

(2)

(Total 3 marks)

Q2.

A code has 4 digits.

Each digit is a number from 0 to 9

Digits may be repeated.

The code starts 5 4 1

5	4	1	

(a) Joe chooses a number at random for the last digit.

Write down the probability that he chooses the correct number.

Answer _____

(1)

(b) Amy knows the last digit is odd but **not** 7

She chooses a different odd number at random.

What is the probability that she chooses the correct number?

Answer _____

(1)

(Total 2 marks)

Q3.

Meal Deal
Choose one sandwich, one drink and one snack

There are

7 different sandwiches

5 different drinks

and

3 different snacks.

(a) How many different Meal Deal combinations are there?

_____

_____

Answer _____

(2)

(b) Two of the sandwiches have cheese in them.

Three of the drinks are fizzy.

Eva picks a Meal Deal at random.

Work out the probability that the sandwich has cheese in it **and** the drink is fizzy.

Give your answer as a fraction.

Answer _____

(2)

(Total 4 marks)

Q4.

In a college canteen students can choose

a starter and a main course

or

a main course and a pudding.

Starter	Main Course	Pudding
Soup	Curry	Jelly
	Burger	Fruit
	Pasta	

(a) One combination is soup and curry.

How many different combinations are there?

Answer _____

(2)

(b) All of the combinations are equally popular.

A student is chosen at random.

What is the probability that he has jelly?

_____

Answer _____

(1)

(c) The canteen serves 270 students one Monday.

How many jellies do they expect to serve?

_____

_____

_____

Answer _____

(2)

(Total 5 marks)