Core 3 · June 13.

i) $\int (4-3x)^7 \, dx$

$= -\dfrac{(4-3x)^8}{24} + C$

ii) $\int \dfrac{1}{4-3x} \, dx$

$= -\dfrac{1}{3}\ln|4-3x| + C$

2 i) $4\cos 2\alpha = \sin^2\alpha$ $\qquad \cos 2\alpha = 1 - 2\sin^2\alpha$

$4(1-2\sin^2\alpha) = \sin^2\alpha$

$4 - 8\sin^2\alpha = \sin^2\alpha$

$4 - 9\sin^2\alpha =$

$\dfrac{4}{9} = \sin^2\alpha$

$\sin\alpha = \pm\dfrac{2}{3}$

ii) $2\tan^2\beta = 3 + 9\sec\beta$

$2(\sec^2\beta - 1) = 3 + 9\sec\beta$

$2\sec^2\beta - 2 = 3 + 9\sec\beta$

$2\sec^2\beta - 9\sec\beta - 5 = 0$

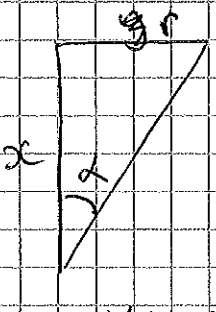
$(2\sec\beta + 1)(\sec\beta - 5) = 0$.

~~$\sec\beta = -\dfrac{1}{2}$~~ or $\sec\beta = 5$.

3) $\tan^{-1}\left(\frac{1}{2}\right) = \alpha$

$\frac{dV}{dt} = 14\ cm^3$ per minute

radius of cone $r =$



$\tan\alpha = \frac{r}{x} = \frac{1}{2}$ $\quad r = \frac{x}{2}$

Vol of cone $= \frac{1}{3}\pi r^2 x$

$= \frac{1}{3}\pi \times \left(\frac{x}{2}\right)^2 x$

$V = \frac{\pi x^3}{12}$

ii) $\frac{dV}{dx} = \frac{1}{4}\pi x^2$ $\quad \frac{dx}{dV} = \frac{4}{\pi x^2}$

$\frac{dx}{dt} = \frac{dV}{dt} \times \frac{dx}{dV}$

$\frac{dx}{dt} = 14 \times \frac{4}{\pi x^2}$

$\frac{dx}{dt} = \frac{56}{\pi x^2}$

when $x = 8$

$\frac{dx}{dt} = \frac{56}{\pi \times 64} = 0.28$ cm per minute

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4) $y = (4x-7)^{1/2} + 4x(2x+1)^{-1}$

$\frac{dy}{dx} = \frac{4}{2}(4x-7)^{-1/2} + \frac{4}{2x+1} + -4x(2x+1)^{-2} \times 2$

$= \frac{2}{\sqrt{4x-7}} + \frac{4}{2x+1} - \frac{8x}{(2x+1)^2}$

At $x = 4$

$\frac{dy}{dx} = \frac{2}{\sqrt{9}} + \frac{4}{9} - \frac{32}{81}$

$= \frac{2}{3} + \frac{4}{9} - \frac{32}{81}$

$= \frac{58}{81}$

or $y = (4x-7)^{1/2} + \frac{4x}{2x+1}$

$\frac{dy}{dx} = \frac{2}{\sqrt{4x-7}} + \frac{4(2x+1) - 8x}{(2x+1)^2}$

$= \frac{2}{\sqrt{4x-7}} + \frac{4}{(2x+1)^2}$

$= \frac{2}{\sqrt{9}} + \frac{4}{81}$

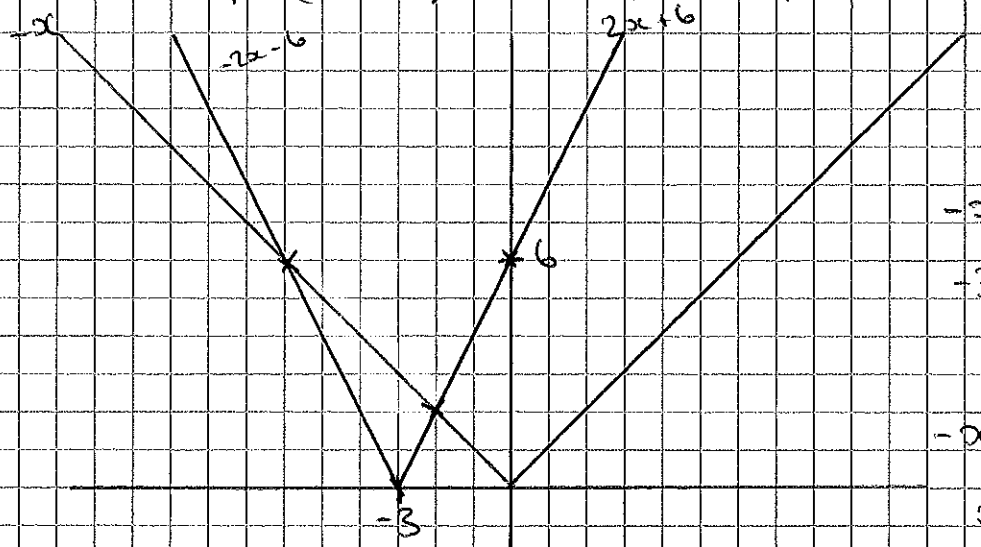
$= \frac{58}{81}$

5i) $y = |x|$ $\quad y = |2(x+3)|$

Stretch scale factor $\frac{1}{2}$ in the x direction parallel to the x axis.

Translation 3 units in the negative x direction.

ii) $|x| > |2(x+3)|$ $\quad |x| > |2x+6|$



$-x = 2x + 6$

$-3x = 6$

$x = -2$

$-x = -2x - 6$

$x = -6$

$-6 < x < -2$

6i) $\int_0^8 \ln(3+x^2)\,dx$

x	0	2	4	6	8
y	$\ln 3$	$\ln 7$	$\ln 19$	$\ln 39$	$\ln 67$

$$A = \frac{2}{3}\left(\ln 3 + 4\ln 7 + 2\ln 19 + 4\ln 39 + \ln 67\right)$$

$$A = 22.4 \text{ (3sf)}$$

ii) $2A =$

$$2\ln(3+x^2)$$
$$= \ln(3+x^2)^2$$
$$= \ln\left((3+x)(3+x)\right)$$
$$= \ln(9+6x+x^2)$$

iii) $\ln(3e+ex^2)$

$$= \int_0^8 \ln e + \int \ln(3+x^2)\,dx$$
$$= \int_0^8 dx + \int \ln(3+x^2)\,dx$$
$$= (8-0) + A$$
$$= 8 + A$$

7)

7) $f(x) = 3 + 4e^{-x}$

i) range is $f(x) > 3$

ii) $y = 3 + \frac{4}{e^x}$

$e^x(y-3) = 4$

$e^x = \frac{4}{y-3}$

$x = \ln \frac{4}{y-3}$

$f^{-1}(x) = \ln\left(\frac{4}{x-3}\right)$

domain: $x > 3$

range: $f(x) \in \mathbb{R}$

iii) $x_{n+1} = 3 + 4e^{-x_n}$

$x_1 = 3$

$x_2 = 3.199$

$x_3 = 3.16319$

$x_4 = 3.16916$

$x_5 = 3.168155$

$x_6 = 3.168325$

$x_7 = 3.168296$

$x = 3.168$

when $\underline{x = 3.168}$

$\underline{y = 3.168}$

iv) reflection in the line $y = x$.

8i) $4\cos\theta - 2\sin\theta = R\cos(\theta+\alpha)$

$R\cos(\theta+\alpha) = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

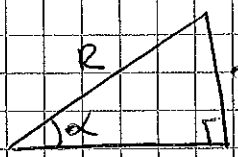
$4\cos\theta - 2\sin\theta = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

when $\theta = 0$

$4 = R\cos\alpha$ $\quad\quad \cos\alpha = \frac{4}{R}$

when $\theta = 90°$

$-2 = -R\sin\alpha$ $\quad\quad \sin\alpha = \frac{2}{R}$



$R^2 = 2^2 + 4^2$

$R = \sqrt{20}$

$\alpha = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$

$\sqrt{20}\cos(\theta+26.6)$

ii)a) $4\cos\theta - 2\sin\theta = 3$

$\sqrt{20}\cos(\theta+26.6) = 3$

$\cos(\theta+26.6) = \frac{3}{\sqrt{20}}$

$\theta + 26.6 = 47.896, 312.130$

$\theta = 21.3°, 286°$ (3sf)

b) $25 - \left(\sqrt{20}\cos(\theta+26.6)\right)^2$

$= 25 - 20\cos^2(\theta+26.6)$

Max value 25 when $\theta + 26.6 = 90°$, so $\theta = 63.4°$

Min value 5 when $\theta + 26.6 = 360$, so $\theta = 333°$

9) $y = e^{2x} - 18x + 15$

$\frac{dy}{dx} = 2e^{2x} - 18$

at min point $\frac{dy}{dx} = 0$

$2e^{2x} - 18 = 0$

$e^{2x} = 9$

$2x = \ln 9$

$x = \frac{\ln 9}{2} = \frac{1}{2}\ln 9$

$= \ln \sqrt{9}$

$= \underline{\ln 3}$.

ii) ~~Area of trapezium~~

Co-ords of P

when $x = 0$

$y = 1 + 15 = 16$.

P $(0, 16)$

~~Q $(\ln 3, \ln n$~~

when $x = \ln 3$

$y = 9 - 18\ln 3 + 15$

$= 24 - 18\ln 3$

Q $(\ln 3, 24 - 18\ln 3)$

Area of trapezium $= \frac{24 - 18\ln 3 + 16}{2} \times \ln 3$

$= \frac{40 - 18\ln 3}{2} \times \ln 3$

$= (20 - 9\ln 3)\ln 3 = 20\ln 3 - 9(\ln 3)^2$

$$\int_0^{\ln 3} \left(e^{2x} + 18x + 15\right) dx$$

$$= \left[\frac{e^{2x}}{2} + \frac{\cancel{18}^{9}x^2}{\cancel{2}} + 15x\right]_0^{\ln 3}$$

$$= \left(\frac{e^{\ln 9}}{2} + 9(\ln 3)^2 + 15\ln 3\right) - \left(\frac{1}{2}\right)$$

$$= \frac{9}{2} + 9(\ln 3)^2 + 15\ln 3 - \frac{1}{2}$$

$$= 4 + 9(\ln 3)^2 + 15\ln 3 .$$

So Shaded area = trapezium $- \int_0^{\ln 3} y$

$$= 20\ln 3 - 9(\ln 3)^2 - \left(4 + 9(\ln 3)^2 + 15\ln 3\right)$$

$$= 20\ln 3 - 9(\ln 3)^2 - 4 - 9(\ln 3)^2 - 15\ln 3$$

$$= 5\ln 3 - 4$$