

June 2009

(C1)

$$\textcircled{1} \quad y = x^5 + \frac{1}{x^2}$$

$$\text{i) } \frac{dy}{dx} = 5x^4 - 2x^{-3}$$

$$\text{ii) } \frac{d^2y}{dx^2} = 20x^3 + 6x^{-4}$$

$$\textcircled{2} \quad \frac{8+\sqrt{7}}{2+\sqrt{7}} \times \frac{2-\sqrt{7}}{2-\sqrt{7}} = \frac{16-6\sqrt{7}-7}{4-7}$$

$$= \frac{9-6\sqrt{7}}{3}$$

$$= 3-2\sqrt{7}$$

$$a=3 \quad b=-2$$

$$\textcircled{3} \quad \text{i) } \frac{1}{9} = 3^{-2}$$

$$\text{ii) } \sqrt[3]{3} = 3^{\frac{1}{3}}$$

$$\text{iii) } 3^{10} \times 9^{15} = 3^{10} \times (3^2)^{15} = 3^{10} \times 3^{30} = 3^{40}$$

$$④ \quad 4x^2 + y^2 = 10$$

$$2x - y = 4$$

$$y = 2x - 4$$

$$4x^2 + (2x - 4)^2 = 10$$

$$4x^2 + 4x^2 - 16x + 16 = 10$$

$$8x^2 - 16x + 6 = 0$$

$$4x^2 - 8x + 3 = 0$$

$$(2x - 3)(2x - 1) = 0$$

$$2x - 3 = 0$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$$2x - 1 = 0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$y = 2x - 4$$

$$y = 2\left(\frac{1}{2}\right) - 4$$

$$= 1 - 4$$

$$y = \underline{-3}$$

$$x = \underline{\frac{1}{2}}$$

$$y = 2x - 4$$

$$y = 2\left(\frac{3}{2}\right) - 4$$

$$y = 3 - 4$$

$$y = \underline{-1} \quad x = \underline{\frac{3}{2}}$$

$$⑤ \text{ i) } (2x+1)(x-3)(x+4) = (2x+1)(x^2+x-12)$$

$$= 2x^3 + 2x^2 - 24x$$

$$+ x^2 + x - 12$$

$$\hline 2x^3 + 3x^2 - 23x - 12$$

$$\text{ii) } x^4 \Rightarrow$$

$$\cancel{x^2} \cancel{x} \cancel{x^2} = \cancel{x^4}$$

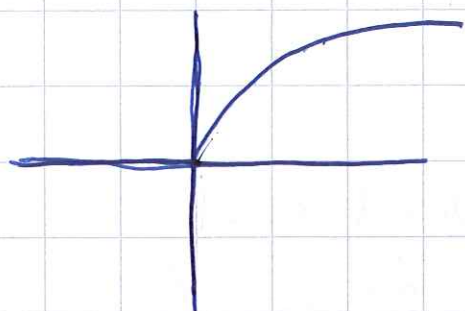
$$x(x^2 + 2x + 3)(x^2 + 7x - 2) = (x^3 + 2x^2 + 3x)(x^2 + 7x - 2)$$

$$\text{for } x^4 = 7x^4 + 2x^4$$

$$= 9x^4$$

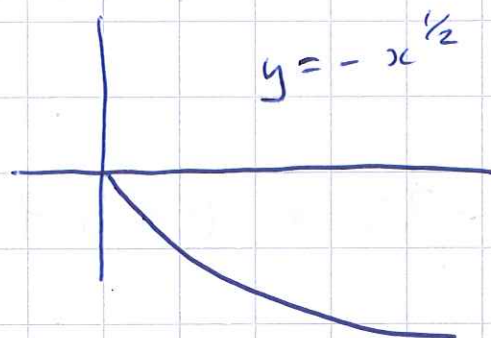
$$\underline{\underline{9}}$$

⑥ i) $y = -\sqrt{x} = -x^{1/2}$



$y = x^{1/2}$

\Rightarrow



* reflection in y-axis

ii) $y = -\sqrt{x} \rightarrow y = 5 - \sqrt{x}$

translation by vector $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$

iii) Stretch S.F 2 parallel to x-axis

$y = -\sqrt{x} \rightarrow y = -\sqrt{\frac{1}{2}x}$

⑦ i) $x^2 - 5x + \frac{1}{4} = \left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + \frac{1}{4}$

$= \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + \frac{1}{4}$

$= \left(x - \frac{5}{2}\right)^2 - \frac{24}{4}$

$= \left(x - \frac{5}{2}\right)^2 - 6$

ii) Centre : $\left(\frac{5}{2}, 0\right)$

radius : $\sqrt{6}$

8) i)

$$-35 < 6x + 7 < 1$$

$$-35 < 6x + 7$$

$$-42 < 6x$$

$$-7 < x$$

$$6x + 7 < 1$$

$$6x < -6$$

$$x < -1$$

$$-7 < x < -1$$

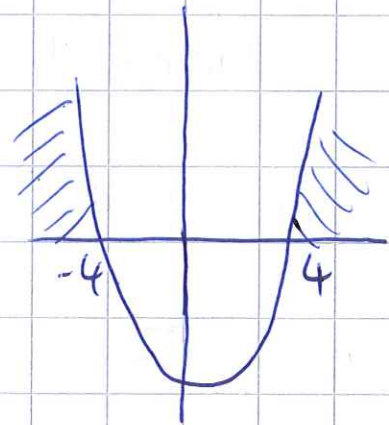
ii)

$$3x^2 > 48$$

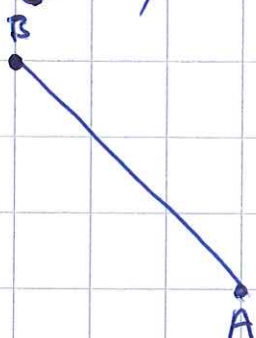
$$x^2 > 16$$

$$x > 4$$

$$\text{or } x < -4$$



9) i) A(4, -3) B(-1, 9)



$$\begin{aligned} \text{i) } AB &= \sqrt{(4 - (-1))^2 + (-3 - 9)^2} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} = \underline{\underline{13}} \end{aligned}$$

$$\begin{aligned} \text{ii) Midpoint} &: \left(\frac{4 + (-1)}{2}, \frac{-3 + 9}{2} \right) \\ &= \left(\frac{3}{2}, 3 \right) \end{aligned}$$

$$m_{AB} = \frac{\Delta y}{\Delta x} = \frac{12}{-5} = -2.4$$

$$y = mx + c \quad (1, 3)$$

$$3 = -2.4(1) + c$$

$$c = 5.4$$

$$y = -2.4x + 5.4$$

$$12x + 5y - 27 = 0$$

$$\textcircled{a} \text{ i) } 9x^2 + 18x - 7 = 0$$

$$(3x+7)(3x-1) = 0$$

$$3x+7=0$$

$$3x = -7$$

$$x = -\frac{7}{3}$$

$$3x-1=0$$

$$3x = 1$$

$$x = \frac{1}{3}$$

$$\text{ii) } \textcircled{a} \text{ stationary point } \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 18x + 18$$

$$18x + 18 = 0$$

$$x = -1$$

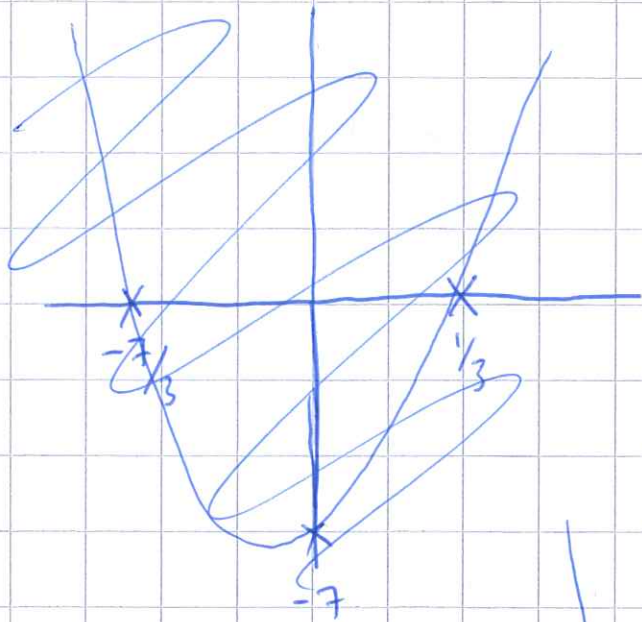
sub $x = -1$ into eqⁿ

$$y = 9(-1)^2 + 18(-1) - 7$$

$$= 9 - 18 - 7$$

$$y = -16$$

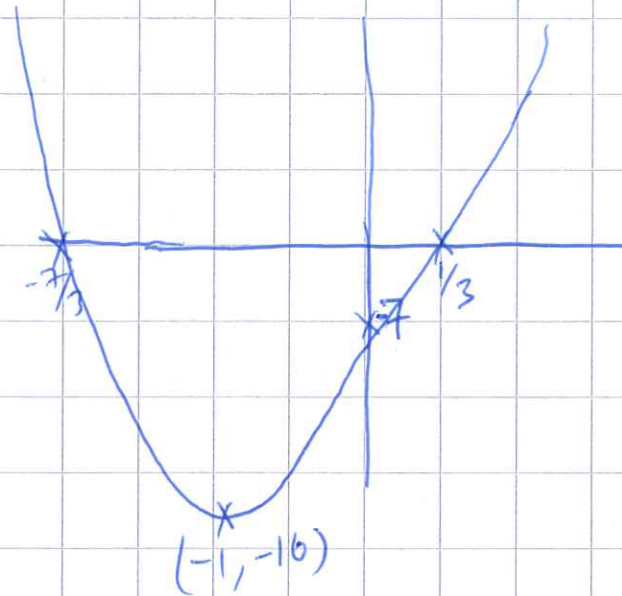
stationary point $\Rightarrow (-1, -16)$



when $x=0$, $y=-7$

when $y=0$, $x=1/3$ and $x=-7/3$

and stationary point $(-1, -16)$



iv) $x > -1$

ii) $y = k\sqrt{x}$ $x=4$, $y=2k$

normal is parallel to $2x+3y=0$; gradient = $-2/3$

\therefore gradient of tangent = $3/2$

$$y = kx^{1/2} \quad \frac{dy}{dx} = \frac{1}{2} k x^{-1/2}$$

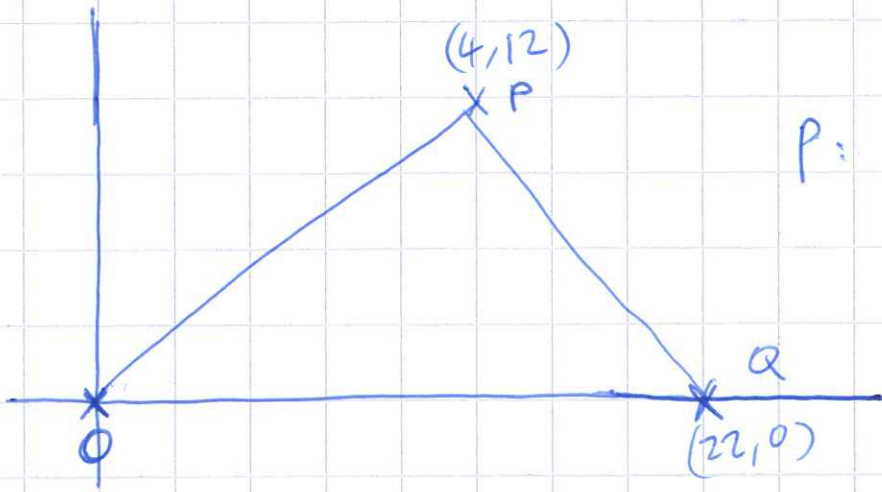
$$x=4 \quad \frac{dy}{dx} = \frac{1}{2} \cdot k \cdot \frac{1}{2}$$

$$= \frac{k}{4}$$

$$\frac{k}{4} = \frac{3}{2}$$

$$\underline{k = 6}$$

ii)



$$P: x=4 \quad y=6\sqrt{4} \\ = 12$$

gradient of normal = $-\frac{2}{3}$
 Passes through $P \Rightarrow$

~~$$x =$$~~

$$y = mx + c$$

$$12 = -\frac{2}{3}(4) + c$$

$$12 = -\frac{8}{3} + c$$

$$c = 14\frac{2}{3}$$

eqn of normal:

$$y = -\frac{2}{3}x + 14\frac{2}{3}$$

when ~~normal~~ $y=0$

$$\frac{2}{3}x = 14\frac{2}{3}$$

$$2x = 44$$

$$x = 22$$

$$Q(22, 0)$$

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 22 \times 12 \\ &= 132 \text{ sq. units.} \end{aligned}$$

