# May 2014 Core 2

① i) Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}(8)(10)\sin 65$

$= 36.3\text{cm}^2$ (3sf)

ii) $BD^2 = 8^2 + 10^2 - 2(8)(10)\cos 65$

$BD^2 = 18.99\ldots$

$BD = 4.36\text{cm}$ (3sf)

iii) $\frac{BC}{\sin 65} = \frac{8}{\sin 30}$

$BC = \frac{8\sin 65}{\sin 30}$

$= 14.5\text{cm}$ (3sf)

② $u_n = 3n - 1$

i) $u_1 = 2$

$u_2 = 5$

$u_3 = 8$

ii) $\sum_{n=1}^{40} u_n = \frac{40}{2}(2(2) + 39(3))$

$= 2420$

③ i) arc length $= r\theta$

$= 8\pi$

ii) Area of Shaded = Area of Segment − Area of triangle

$= \frac{1}{2}r^2\theta - \frac{1}{2}ab\sin C$

$\frac{2}{3}\pi \equiv 120°$

$= \frac{1}{2}(12)^2\left(\frac{2}{3}\pi\right) - \frac{1}{2}(12)^2\sin\left(\frac{2}{3}\pi\right)$

$= 48\pi - 36\sqrt{3}$

~~$= 12(12\pi - 9\sqrt{3})$~~

$= 12(\pi - 3\sqrt{3})$

④ i) $\sin x - \cos x = \frac{6\cos x}{\tan x}$

$\sin x \tan x - \cos x \tan x = 6\cos x$

$\frac{\sin x}{\cos x}\tan x - \tan x = 6$

$\tan^2 x - \tan x - 6 = 0$

ii) $(\tan x - 3)(\tan x + 2) = 0$

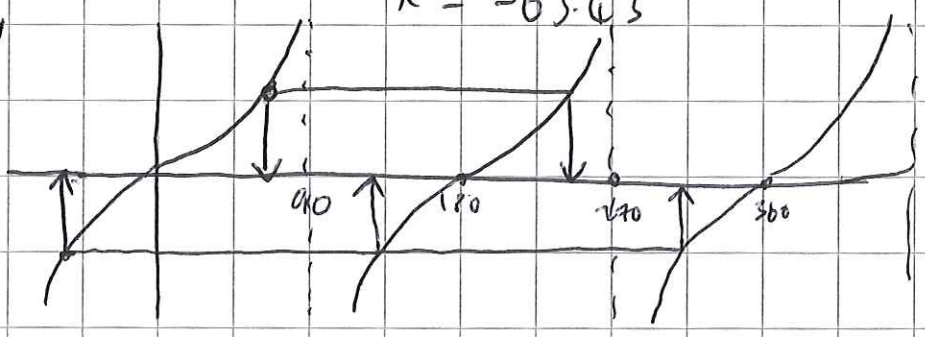
$\tan x = 3$

$x = 71.6,$

$\tan x = -2$

$x = -63.43$

$x = 116.6°, 296.6°, 71.6°, 251.6°$



(5) $2^{4x-1} = 3^{5-2x}$ (2)

$(4x-1)\log_{10}2 = (5-2x)\log_{10}3$

$4x\log_{10}2 - \log_{10}2 = 5\log_{10}3 - 2x\log_{10}3$

$4x\log_{10}2 + 2x\log_{10}3 = 5\log_{10}3 + \log_{10}2$

$x(4\log_{10}2 + 2\log_{10}3) = 5\log_{10}3 + \log_{10}2$

$$x = \frac{\log_{10}(3^5 \times 2)}{\log_{10}(2^4 \times 3^2)}$$

$$x = \frac{\log_{10}486}{\log_{10}144}$$

(6) $\left(x^3 + 2x^{-2}\right)^4$

$= {}^4C_0\left(x^3\right)^4 + {}^4C_1\left(x^3\right)^3\left(2x^{-2}\right) + {}^4C_2\left(x^3\right)^2\left(2x^{-2}\right)^2$

$+ {}^4C_3\left(x^3\right)\left(2x^{-2}\right)^3 + {}^4C_4\left(2x^{-2}\right)^4$

$= x^{12} + 8x^7 + 24x^2 + 32x^{-3} + 16x^{-8}$

ii) $\int\left(x^3 + \frac{2}{x^2}\right)^2 dx = \frac{1}{13}x^{13} + x^8 + 8x^3 - 16x^{-2} - \frac{16}{7}x^{-7} + C$

(7) $f(x) = 12 - 22x + 9x^2 - x^3$

i) $f(-2) = 12 - 22(-2) + 9(-2)^2 - (-2)^3$

$= 12 + 44 + 36 + 8$

$= 100$

ii) if $(3-x)$ is a factor then $f(x3) = 0$

$f(3) = 12 - 22(3) + 9(3)^2 - 3^3$

$= 12 - 66 + 81 - 27$

$= 0$

iii) $12 - 22x + 9x^2 - x^3$

$= (3 - x)(x^2 + bx + 4)$

$$\begin{array}{r} 3x^2 \\ -bx^2 \\ \hline 3 - b = 9 \\ b = -6 \end{array}$$

$\cancel{-4x}$

$\cancel{3bx}$

$\cancel{3b - 4} \; 1 = 1 \; 2 \; 2$

$= (3 - x)(x^2 - 6x + 4)$

<u>check</u>

$$\begin{array}{r} 3x^2 - 18x + 12 \\ -x^3 + 6x^2 - 4x \\ \hline -x^3 + 9x^2 - 22x + 12 \end{array}$$

✓✓

iv) $f(x) = 0$ $\qquad$ $\underline{x = 3}$

$(x^2 - 6x + 4)$

$$x = \frac{6 \pm \sqrt{36 - 4(1)(4)}}{2}$$

$$x = \frac{6 \pm \sqrt{20}}{2} \qquad \underline{x = 3 \pm \sqrt{20}}$$

⑧ $u_1 = 50 \qquad r = 0.8$

a) $u_k = ar^{k-1}$

$50 \times 0.8^{k-1} < 0.15$

~~log 50~~

$0.8^{k-1} < 0.003$

~~$(k-1)\log 0.8$~~

$(k-1)\log 0.8 < \log 0.003$

$k\log 0.8 - \log 0.8 < \log 0.003$

$k\log 0.8 < \log(0.8 \times 0.003)$

$k > \dfrac{\log 0.0024}{\log 0.8}$

$k > 27.03321\ldots$

smallest value of $k = \underline{\underline{27}}$

$-1 < r < 1$

b) $u_2 = ar$ $\qquad S_\infty = \frac{a}{1-r}$

$-3 = ar$ $\qquad 4 = \frac{a}{1-r}$

$a = \frac{-3}{r}$

$4 = \frac{-3}{r} \div (1-r)$

$4 = \frac{-3}{r} \times \frac{1}{1-r}$

$4 = \frac{-3}{r-r^2}$

$4(r-r^2) = -3$

$4r - 4r^2 = -3$

$0 = 4r^2 - 4r - 3$

$r = \frac{4 \pm \sqrt{16 - 4(4)(-3)}}{8}$

$r = \frac{4 \pm 8}{8}$

$r \neq \frac{3}{2}$, $r = -\frac{1}{2}$

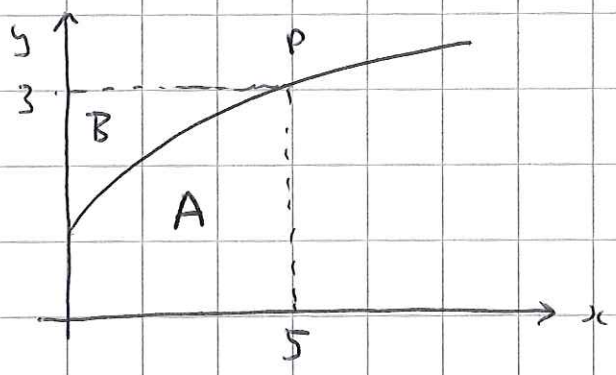
$\therefore u_2 = a \times -\frac{1}{2}$

$-3 = a \times -\frac{1}{2}$

$a = 6$

As sequence converges, $r = -\frac{1}{2}$

(9)




$y = -3 + 2\sqrt{x+4}$

i) Area $A \approx \frac{1}{2}(2.5)(y_0 + y_2 + 2(y_1))$

$= \frac{1}{2}(2.5)(1 + 3 + 2\sqrt{6} - 6 + 2\sqrt{6})$

$= 10.2$ units$^2$ (3 s.f.)

ii) Area B $\approx (3 \times 5)$ − Area of A

$= 15 - 10.2 \ldots$

$= 4.75$ units$^2$ (3 s.f.)

iii) $y = -3 + 2\sqrt{x+4}$

$\left(\frac{y+3}{2}\right)^2 - 4 = x$

$x = \frac{1}{4}(y^2 + 6y + 9) - 4$

$x = \frac{1}{4}y^2 + \frac{6}{4}y + \frac{9}{4} - 4$

$x = \frac{1}{4}y^2 + \frac{6}{4}y - \frac{7}{4}$

$\int_1^3 f(y)\,dy = \left[\frac{1}{12}y^3 + \frac{3}{4}y^2 - \frac{7}{4}y\right]_1^3$

$= \left(\frac{9}{4} + \frac{27}{4} - \frac{21}{4}\right) - \left(\frac{1}{12} + \frac{3}{4} - \frac{7}{4}\right)$

$= \frac{15}{4} - -\frac{11}{12}$

$= \frac{14}{3}$ ✓✓