Jan 2013 Core 2

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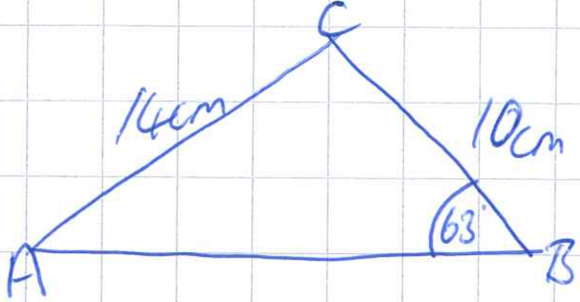


i) $$\frac{\sin CAB}{10} = \frac{\sin 63}{14}$$

$$CAB = \sin^{-1}\left(\frac{10\sin 63}{14}\right)$$
$$= 39.52636\ldots$$
$$= \underline{39.5°}$$

ii) $$\frac{AB}{\sin(180-(63+39.5))} = \frac{14}{\sin 63}$$

$$AB = 15.34011674$$
$$= \underline{15.3\text{cm}}$$

2 i) $U_1 = 7$ $\quad a=7 \quad d=4$

$U_{n+1} = U_n + 4$

Hence $U_{17} = 7 + (16\times 4)$
$= 71$

ii) $$\sum_{n=1}^{35} U_n = \sum_{n=36}^{50} U_n$$

$$S_{35} = \tfrac{1}{2}35(7+143) = \underline{2625}$$

$$S_{(36-50)} = \tfrac{1}{2}15(147+203) = \underline{2625}$$

$U_{35} = 7 + (34\times 4)$
$= 143$

$U_{36} = 7 + (35\times 4)$
$= 147$

$U_{50} = 7 + (49\times 4)$
$= 203$

(3) $\frac{dy}{dx} = kx(2x-1)$

$\frac{dy}{dx} = 2kx^2 - kx$

$y = \frac{2}{3}kx^3 - \frac{k}{2}x^2$ when $x=2$ $y=7$

$7 = \frac{2}{3} \times 8 \times k - \frac{1}{2}k \times 4$

$7 = \frac{16}{3}k - 2k$

$7 = 3\frac{1}{3}k$

$k =$

i) $\frac{dy}{dx} = 9$ $\quad 9 = kx(2x-1)$

$x = 2 \quad \therefore \quad 9 = 6k$

$k = 3/2$

ii) $\frac{dy}{dx} = \frac{3}{2}x(2x-1)$

$= 3x^2 - \frac{3}{2}x$

$y = x^3 - \frac{3}{4}x^2 + c$ @ $(2,7)$

$7 = 8 - 3 + c$

$c = 2$

$\therefore \quad y = x^3 - \frac{3}{4}x^2 + 2$

④ i) $(2+x)^5 = {}^5C_0 x^5 + {}^5C_1 x^4(2) + {}^5C_2 x^3(2)^2 + {}^5C_3 x^2(2)^3 + {}^5C_4 x(2)^4 + {}^5C_5 2^5$ ②

$= x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$

ii) $(2 + 3y + y^2)$

$40(3y+y^2)^3 = 1080y^3 + \ldots$

$80(3y+y^2)^2 = 80(9y^2 + 6y^3 + y^4)$

$= \ldots + 480y^3 + \ldots$

Hence
$$\begin{array}{r} 1080 + \\ 480 \\ \hline 1560 \\ \hline \end{array}$$
is y^3 coefficient.

⑤ i) $2\sin x = \dfrac{4\cos x - 1}{\tan x}$ $\qquad \tan x = \dfrac{\sin x}{\cos x}$

$$2\sin x = \frac{4\cos x - 1}{\frac{\sin x}{\cos x}}$$

$2\sin^2 x = 4\cos^2 x - \cos x$

$2(1-\cos^2 x) = 4\cos^2 x - \cos x$

$0 = 6\cos^2 x - \cos x - 2$

ii)

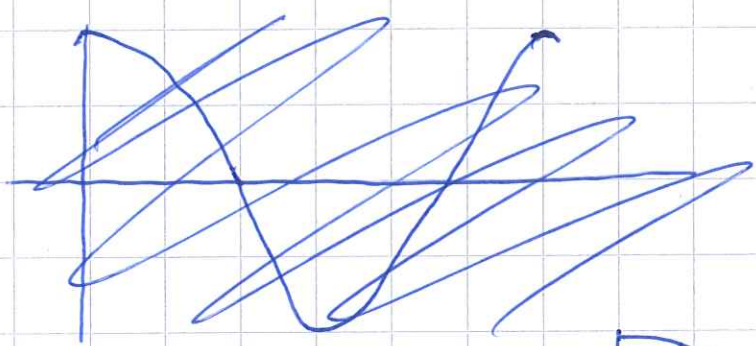
$$6\cos^2 x - \cos x - 2 = 0$$

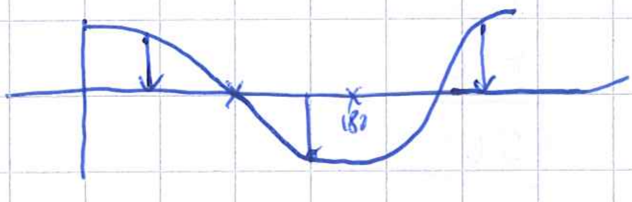
$$(3\cos x - 2)(2\cos x + 1) = 0$$

$3\cos x = 2$ $\qquad$ $2\cos x = -1$

$\cos x = 2/3$ $\qquad$ $\cos x = -1/2$

$x = 48.189\ldots,\ 311.8°$ $\qquad$ $x = 120,\ 240°$





⑥ i) $2x,\ x+4,\ 2x-7$

$$2x-7-(x+4) = x+4-2x$$
$$x - 11 = -x + 4$$
$$2x = 15$$
$$x = 7.5$$

ii) a) $x = 8$

$$2x = 16$$
$$x+4 = 12$$
$$2x-7 = 9$$

$r = \frac{3}{4}$ $\qquad$ $S_\infty = \frac{9}{1-3/4}$

$$= \frac{16}{1/4}$$

$$= \underline{\underline{64}}$$

b)
$$\frac{2x-7}{x+4} = \frac{x+4}{2x}$$

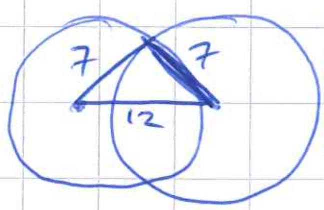
$$\frac{2x(2x-7)}{2x(x+4)} = \frac{(x+4)(x+4)}{2x(x+4)}$$

$$4x^2 - 14x = x^2 + 8x + 16$$
$$3x^2 - 22x - 16 = 0$$

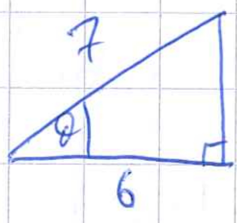
$$x = \frac{22 \pm \sqrt{(-22)^2 - 4(3)(-16)}}{6}$$

$$= \frac{22 \pm \sqrt{676}}{6} = \frac{22 \pm 26}{6} = 8 \text{ or } -\tfrac{2}{3}$$

7

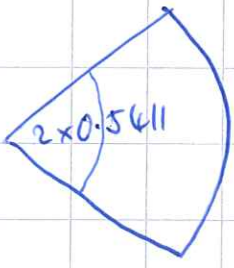


i)



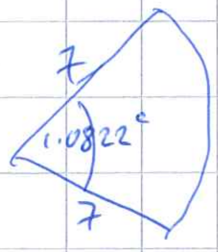
$\theta = \cos^{-1}\left(\frac{6}{7}\right)$
$\theta = 0.54109\ldots$
$\theta = 0.5411^c$ (3sf)

ii) 2x Arc length $= r\theta \times 2$
$= 7 \times 2 \times 0.5411 \times 2$
$= 15.1507..$
$= 15.2$ cm



iii) Area of sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times (2 \times 0.5411) \times 7^2$



Area of triangle $= \frac{1}{2} ab \sin C$

Shaded region $= 2 \times \left(\frac{1}{2} \times 2 \times 0.5411 \times 7^2 - \frac{1}{2}(7)(7)\sin(2 \times 0.5411) \right)$

$\approx \cancel{26.75439}$

$= 9.761162880$

$= \underline{\underline{9.76 \text{ cm}^2}}$

⑧ i) $y = \log_2 x \longrightarrow y = \log_2 (x-3)$

translation by vector $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$.

ii) $y = \log_2 x$ $\quad \log_2 x = 3$

$\underline{x = 8}$

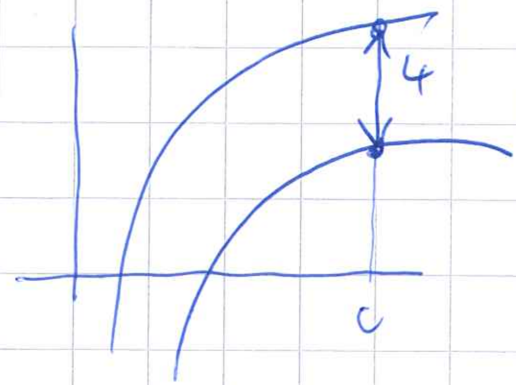
iii) $\log_2 (x-3) = 1.8$

$2^{1.8} = x - 3$

$x = 2^{1.8} + 3$

$x = 6.4822\ldots$

$x = 6.48$



$4 = \cancel{\log_2(c-3)}$ $\log_2 c - \log_2(c-3)$

$4 = \log_2\left(\frac{c}{c-3}\right)$

$4 = \log_2 16$ $\quad \log_2 16 = \log_2 \frac{c}{c-3}$

$\frac{c}{c-3} = 16$

$c = 16c - 48$

$48 = 15c$

$c = 48/15 = 3.2$

9) $\int_a^{2a} \frac{2x^3 - 5x^2 + 4}{x^2}\, dx = \int_a^{2a} 2x - 5 + 4x^{-2}\, dx$

$= \left[x^2 - 5x - 4x^{-1}\right]_a^{2a}$

$= \left((2a)^2 - 5(2a) - 4(2a)^{-1}\right) - \left(a^2 - 5a - 4(a)^{-1}\right)$

$= \left(4a^2 - 10a - \frac{2}{a}\right) - \left(a^2 - 5a - \frac{4}{a}\right)$

$0 = 3a^2 - 5a + 2/a$

(x by a)

$0 = 3a^3 - 5a^2 + 2$

ii) if $a = 1$ is a root the $(a-1)$ will have no remainder and $f(1) = 0$

$$f(1) = 3(1)^3 - 5(1)^2 + 2$$
$$= 3 - 5 + 2$$
$$= 0$$ Hence $a = 1$ is a root.

$$(a-1)(3a^2 + ba - 2)$$

ba^2
$-3a^2$

$(b-3) = -5$

$\underline{b = -2}$

$\Rightarrow (a-1)(3a^2 - 2a - 2)$

$$a = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-2)}}{6}$$

$$= \frac{2 \pm \sqrt{28}}{6} = \frac{2 \pm 2\sqrt{7}}{6}$$

$= \cancel{\frac{1}{3}\ 2\ 2\sqrt{7}}$

$$= \frac{1 \pm \sqrt{7}}{3}$$

$\frac{\sqrt{28}}{\cancel{6}} = 2\sqrt{7}$