Jan 2009 C2

1 i) $\int x^3 + 8x - 5 \, dx = \frac{1}{4}x^4 + 4x^2 - 5x + c$

ii) $\int 12\sqrt{x} \, dx = \int 12x^{1/2} \, dx = 8x^{3/2} + c$

2 i) $140^\circ = \frac{2\pi}{360} \times 140 = \frac{7\pi}{9}^c$

$2\pi^c = 360^\circ$

ii) Arc length $= \frac{\pi d}{360} \times 140$

$= \frac{14\pi}{360} \times 140 = \frac{49\pi}{9}$

~~chord AB =~~ ~~Cosine Rule~~

$\cancel{c^2 = a^2 + b^2 - 2ab\sin c}$

$\cancel{AB^2 = 2 \times 7^2 - 2(7)(7)\sin 140}$

$\cancel{= 98 - 62.993\ldots}$

Sine Rule

$\frac{AB}{\sin 140} = \frac{7}{\sin 20}$

$AB = \sin 140 \times \frac{7}{\sin 20}$

$= 13.15569669\ldots$

Perimeter $= \frac{49\pi}{9} + \frac{7\sin 140}{\sin 20}$

$= 30.25992336$ c

$= 30.3\text{cm}$

(3) $U_n = 24 - \frac{2}{3}n$

i) $U_1 = 24 - \frac{2}{3}(1)$ $\quad U_2 = 24 - \frac{4}{3}$ $\quad U_3 = 24 - \frac{6}{3}$

$= \underline{23\tfrac{1}{3}}$ $\quad = \underline{22\tfrac{2}{3}}$ $\quad = \underline{22}$

ii) $U_k = 0$ $\quad \therefore \quad 0 = 24 - \frac{2}{3}(k)$

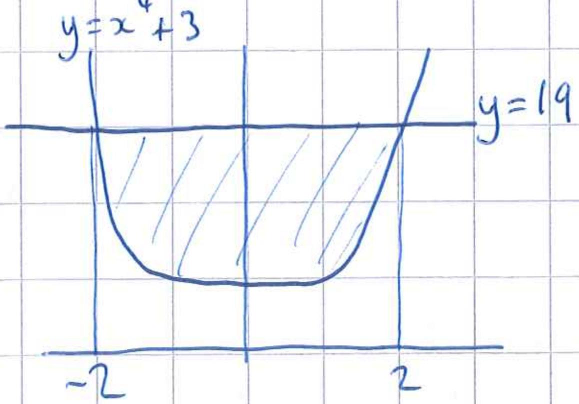
$24 = \frac{2k}{3}$

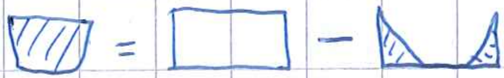
$\underline{k = 36}$

iii) $\sum_{n=1}^{20} U_n$ $\quad S_{20} = \frac{20}{2}\left(46\tfrac{2}{3} + 19\left(-\tfrac{2}{3}\right)\right)$

$= \underline{340}$

(4)





$\int_{-2}^{2} x^4 + 3 \, dx = \left[\frac{1}{5}x^5 + 3x\right]_{-2}^{2}$

$= (6.4 + 6) - (-6.4 + -6)$

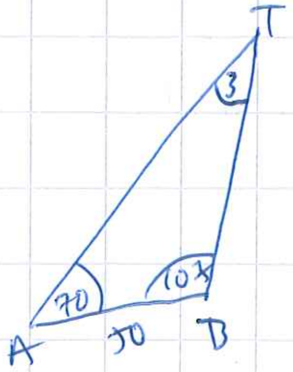
$= 12.4 + 12.4$

$= 24.8$

Shaded area $= (4 \times 19) - 24.8$

$= 51.2$

5



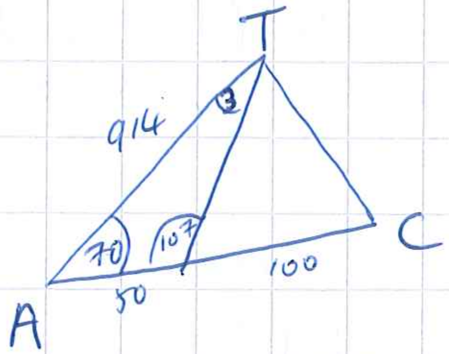
i) $\dfrac{AT}{\sin 107} = \dfrac{50}{\sin 3}$

$AT = \dfrac{50 \sin 107}{\sin 3}$

$= 913.6211742$

$= 914\,m$

ii)



cosine rule.

$TC^2 = 914^2 + 150^2 - 2(914)(150)\cos 70$

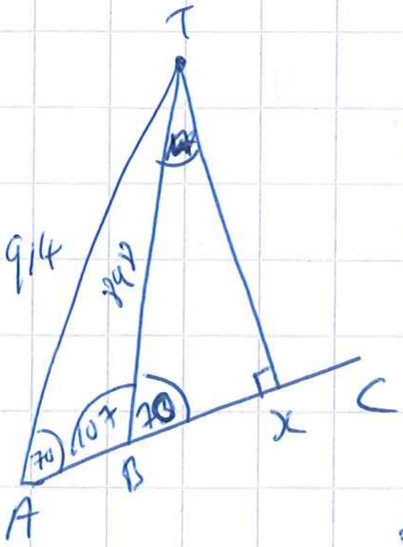
$= \sqrt{914^2 + 150^2 - 2(914)(150)\cos 70}$

$TC = \cancel{8527846}$

$= 874.1361889$

$= 874\,m$

iii)



$BT = \sin 70 \times \dfrac{914}{\sin 107}$

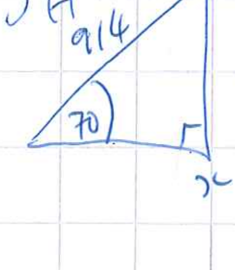
$= 898.12\ldots$

$= 898\,m$

$xT = \sin 70 \times 914$

$= 858.879\ldots$

$= 859\,m$ (nrm).



⑥ G.P. $U_1 = 20 = a \qquad r = 0.9$

i) $S_\infty = \dfrac{a}{1-r} = \dfrac{20}{0.1}$

$= 200$

ii) ~~$S_{30} = 30(20(1-0.9)$~~

$S_n = \dfrac{n(1-r^n)}{1-r} = \dfrac{20(1-0.9^{30})}{1-0.9}$

$= 191.5217683$

$= \underline{192}$

iii) $U_n = ar^{n-1}$

$20 \times 0.9^{p-1} < 0.4$

~~$\log 20 + (p-1)\log 0.9 < \log 0.4$~~

$0.9^{p-1} < 0.02$

$(p-1)\log 0.9 < \log 0.02$

$p-1 \overset{*}{>} \dfrac{\log 0.02}{\log 0.9}$

$p > \dfrac{\log 0.02}{\log 0.9} + 1$

$p > 38.1$

$p = 39$

* sign change as $\div$ by $\log 0.9$, $\log 0.9$ is a negative number!

7 $(k+ax)^4$ coefficient f $x^2 = 24$

coeff of $x^2 = {}^4C_2 (k)^2 (ax)^2$

$$24 = {}^4C_2 k^2 a^2$$

$$4 = (ak)^2$$

$ak = 2$ as a and k are +ve.

ii) coeff of x: ${}^4C_3 (ax)k^3 = 4k^3 a x$

x^4	x^3	x^2	x
0	1	2	3

$$4k^3 a = 128$$
$$k^3 a = 32$$

$ak^3 = 32$
$ak = 2$

$$\frac{ak^3}{ak} = \frac{32}{2}$$
$$k^2 = 16$$
$$k = 4$$

$$a = \frac{2}{4}$$
$$a = 1/2$$

iii) ${}^4C_1 \left(\tfrac{1}{2}x\right)^3 \cancel{4} = 4 \times 4 \times \frac{x^3}{8}$

$$= \frac{16x^3}{8} = 2x^3$$

Hence 2 is coefficient of x^3

8 $\log_a x = p \qquad \log_a y = q$

i) $\log_a(xy) = \log_a x + \log_a y$

$= p + q$

ii) $\log_a\left(\frac{a^2x^3}{y}\right) = 2 + 3p - q$

b) i) $\log_{10}(x^2 - 10) - \log_{10} x$

$= \log_{10}\left(\frac{x^2-10}{x}\right)$

ii) $\log_{10}\left(\frac{x^2-10}{x}\right) = \log_{10} 9$

Hence $\frac{x^2 - 10}{x} = 9$

$\cancel{x^2 - 10 = 18}$

$\cancel{x^2 + 8 = 0}$

$\cancel{x^2 = -8}$

$x^2 - 10 = 9x$

$x^2 - 9x - 10 = 0$

$(x-10)(x+1) = 0$

$\underline{x = 10} \qquad x = -1$

9) $f(x) = x^3 - x^2 - 3x + 3$

$f(1) = 1^3 - 1^2 - 3(1) + 3$
$= 1 - 1 - 3 + 3$
$= 0$

Hence $(x-1)$ is a factor.

$(x-1)(x^2 + bx - 3)$

$$\begin{array}{l} x^3 + bx^2 - 3x \\ \quad - x^2 - bx + 3 \\ \hline x^3 + (b-1)x^2 + (-3-b)x + 3 \end{array}$$

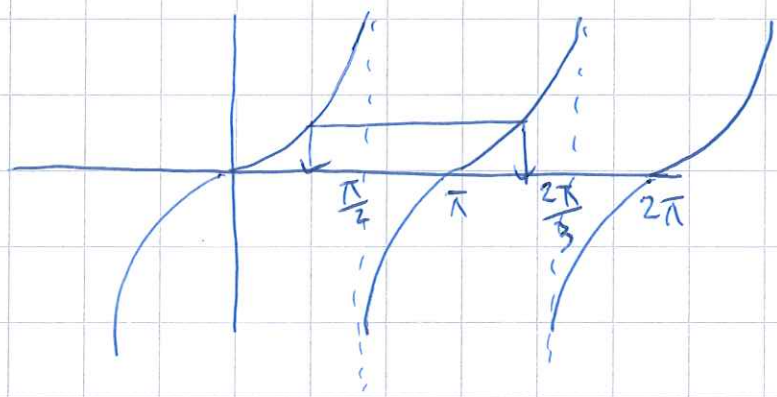
$(b-1) = -1$
$b = 0$

So $(x-1)(x^2-3) = 0$

$x = 1$, $x = \pm\sqrt{3}$

ii) $\tan^3 x - \tan^2 x - 3\tan x + 3 = 0$

$\tan x = 1$ $\quad \tan^{-1}(1) = 45°$ $\quad 45° = \frac{\pi}{4}$



solutions for $\tan x = 1$

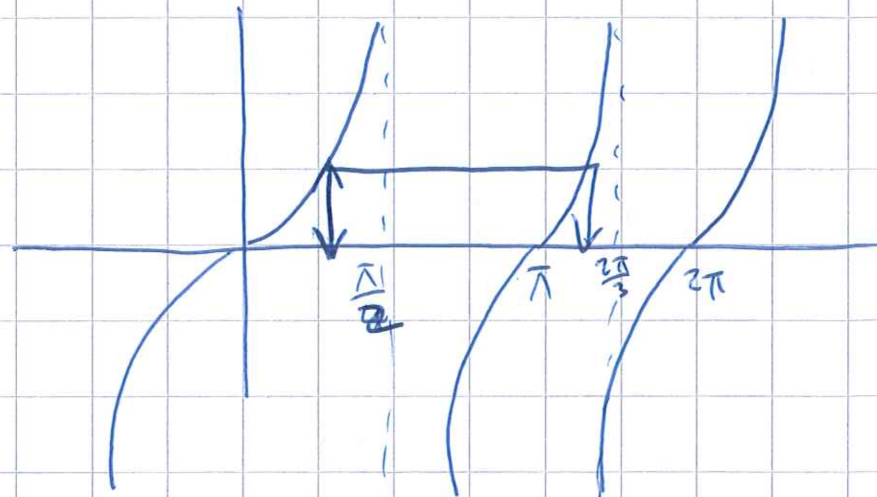
$x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$
$= \frac{\pi}{4}, \frac{5\pi}{4}$

$\tan x = \sqrt{3}$

$x = \tan^{-1}\sqrt{3}$

$= 60^\circ$

$x = \frac{\pi}{3}, \frac{4\pi}{3}$



$\tan x = -\sqrt{3}$

$x = -60^\circ$

$x = \frac{2\pi}{3}, \frac{5\pi}{3}$