Sale Grammar School

Mathematics – Year 8

KNOWLEDGE ORGANISER

To support your revision for the End of Year Assessment

To effectively revise for a Maths assessment, you must finish, check, and correct many Maths questions.

- This document is to support your revision but remember the key with Maths revision is to **finish lots of questions**. Techniques like rewriting revision notes or copying from a revision guide, colour coding, and making posters can be enjoyable, but generally, they aren't the most effective use of revision time.
- Use your progress books and finish outstanding chapters or redo questions you struggled with.
- www.corbettmaths.com has lots of helpful videos and worksheets you can use as well.
- Use notes, and work through examples and questions in your book.
- Don't use your calculator unless the question specifically asks for it, you need to practise non-calculator skills as well. But checking answers with a calculator is very useful.
- If your struggle with anything, come to the support session during Tuesday lunchtime, in M51 or ask your teacher.
- The full-colour version can be found on www.smlmaths.com.

Year 8 Mathematics Knowledge Organiser – Unit 1: Factors and powers

Keywords

Integer	is any of the positive or negative whole numbers and zero.	*Example: ...-2, -1, 0, +1, +2 ...*
Even number	is an integer that is divisible by 2.	*Example: 2, 4, 6, 8, 10, 12, 14, …*
Odd number	is an integer that has a remainder of 1 when divided by 2.	*Example: 1, 3, 5, 7, 9, 11, 13, 15, …*
Product	is the result of multiplying one number by another.	
Index notation	is used as a shortcut for multiplication when a number is being multiplied by itself.	*Example:* $5^4 = 5 \times 5 \times 5 \times 5$

FACTORS

Factors are numbers that divide another number without leaving a remainder.

Example:
Find factors of number 32

1×32
2×16
4×8

Factors of number 32 are: 1, 2, 4, 8, 16 and 32

PRIME NUMBERs

Prime numbers are whole numbers greater than 1 that have **exactly** two factors, themselves and 1.

Composite numbers are integers that are divisible without remainder by at least one positive integer other than themselves and one.

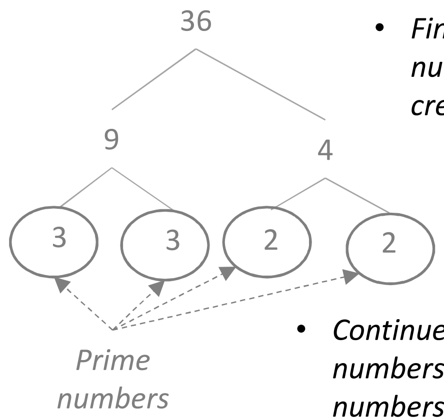
Example:
15 *is divisible by 1, 15, <u>3, 5</u>, therefore it is* ***not*** *a prime number.*
2 *is divisible by 1 and 2 only, therefore it* ***is*** *a prime number (the only even prime number)*

<u>Important note</u>: Number **1** is not prime nor composite number.
Prime numbers that you should remember are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

PRIME FACTORISATION

The **prime numbers** that can be multiplied to give the original number are called prime factors.
Prime factorization is a method of finding which **prime** numbers multiply together to make the original number.

Example: Write number 36 as a product of prime factors



- *Find any two factors of number 36 and start creating the 'factor tree'.*
- *Check whether the factors are prime numbers.*
- *Continue to break any composite numbers into factors until all the numbers at the end of the branches are prime numbers.*

Answer: $36 = 2 \times 2 \times 3 \times 3 = \underline{2^2 \times 3^2}$

LCM OR THE LOWEST / LEAST COMMON MULTIPLE

LCM is smallest positive number that is a multiple of two or more numbers.

Example:
Find the lowest common multiple of 6 and 9.
Multiples of 6: 6, 12, 18, 24, 30, 36…
Multiples of 9: 9, 18, 27, 36, 45, 54, …
Common multiples of 6 and 9: 18, 36 …
The LCM is 18

HCF OR THE HIGHEST COMMON FACTOR

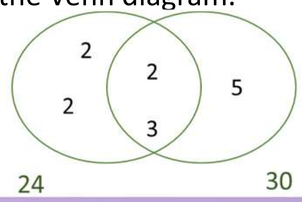
HCF is the greatest number that is a factor of two (or more) other numbers.

Example:
Find the highest common factor of 18 and 24.
Factors of 18: 1, 2, 3, 6, 9, 18
Factors of 2 : 1, 2, 3, 4, 6, 8, 12, 24
Common factors of 18 and 24: 1, 2, 3 and 6
The HCF is 6

LCM AND HCF USING PRIME FACTORISATION

By drawing a Venn diagram to display the prime factors of each number, we can easily see which factors are common to both numbers.
To find the HCF, find any prime factors that are common to both numbers from the cross section of the Venn diagram.
To find LCM, multiply together all of the numbers in the Venn diagram.
Example: Find the HCF and LCM of 24 and 30.

$24 = 2 \times 2 \times 2 \times 3$
$30 = 2 \times 3 \times 5$



HCF = 2 x 3 = 6
LCM = 2 x 2 x 2 x 3 x 5 = 120

Year 8 Mathematics Knowledge Organiser – Unit 1: Factors and powers

Keywords

Powers	The power of a number shows you how many times to use the number (base number) in a multiplication.
The base number	is the number that is being multiplied by Itself.
The power number	tells us how many times the base number is multiplied by itself.
Index notation	The notation in which a product such as $a \times a \times a \times a$ is recorded as a^4. In this example the number 4 is the index (plural indices) *Example:* $8^2 = 8 \times 8 = 64$

NOTABLE POWERS

$$a^1 = p$$
$$a^0 = 1$$

Examples: $7^1 = 47$

$7^0 = 1$

DIVISION INDEX LAW

When expressions with the same base are divided, the indices are subtracted.

$$a^m \div a^n = a^{(m-n)} \quad \text{or} \quad \frac{a^m}{a^n} = a^{m-n}$$

Examples: $15^7 \div 15^4 = 15^3$

$3^9 \div 3^2 = 3^7$

$\frac{5^{11}}{5^3} = 5^8$

$\frac{7^4}{7^{10}} = 7^{-6}$

MULTIPLICATION INDEX LAW

When expressions with the same base are multiplied, the indices are added.

$$a^m \times a^n = a^{(m+n)}$$

Examples: $7^5 \times 7^3 = 7^8$

$5^{12} \times 5 = 5^{13}$

$4^5 \times 4^8 = 4^{13}$

BRACKETS INDEX LAWS

When raising a power to another power, multiply the powers together.

$$(a^m)^n = a^{m \times n}$$

Examples: $(3^2)^5 = 3^{10}$

$(6^3)^4 = 6^{12}$

$(7^6)^3 = 7^{18}$

ROUNDING TO SIGNIFICANT FIGURES (S.F.)

Rules:

- the first significant figure is the first digit that is not a zero

  Example: 1505.6 (the 1st significant figure: 1) 0.0002306 (the 1st significant figure: 2)

- the 2nd, 3rd, 4th … sig.figures follow immediately after
- any non zero digits ARE significant
- zeros in between significant figures ARE significant
- zeros at the end of decimal numbers ARE significant
- zeros at the end of whole numbers are NOT significant

  Example: 150500 0.00023060000

  green zeros ARE significant and

  red zeros are NOT significant

Example:

Round	to 1 s.f.	to 2 s.f.	to 3 s.f.
1582	2000	1600	1580
6.351	6	6.4	6.35
34.026	30	34	34.0
0.005038	0.005	0.0050	0.00504

* green digits are significant

POWERS OF 10

Trillion	1 000 000 000 000	10^{12}	12	Tera (T)
Billion	1 000 000 000	10^9	9	Giga (G)
Million	1 000 000	10^6	6	Mega (M)
Thousand	1 000	10^3	3	kilo (k)
Hundred	100	10^2	2	hecto (h)
Ten	10	10^1	1	Deca (da)
One	1	10^0	0	
One Tenth	0.1	10^{-1}	-1	deci (d)
One Hundredth	0.01	10^{-2}	-2	centi (c)
One Thousandth	0.001	10^{-3}	-3	milli (m)
One Millionth	0.000 001	10^{-6}	-6	micro (μ)
One Billionth	0.000 000 001	10^{-9}	-9	nano (n)
One Trillionth	0.000 000 000 001	10^{-12}	-12	pico (p)

Example:

$1.15 \times 10^2 = 1.15 \times 100 = 115$

$2.18 \times 10^3 = 2.18 \times 1000 = 2180$

$223 \times 10^{-2} = 223 \times 0.01$ or $223 \div 100 = 2.23$

$15 \times 10^{-3} = 15 \times 0.001$ or $15 \div 1000 = 0.015$

Convert 3 kg to g

1 kg	3 kg
1000 g	3000 g

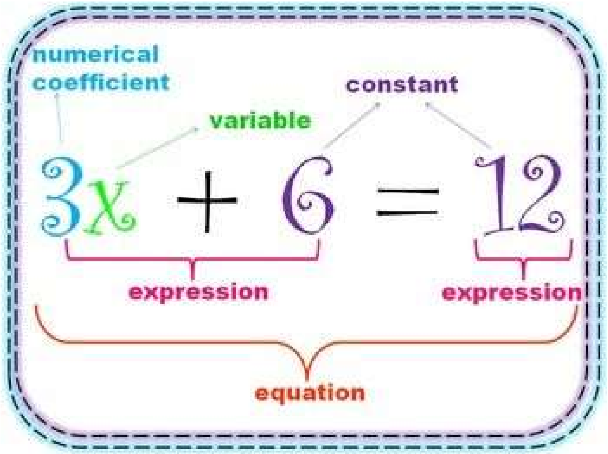
Convert 5 km into cm

1 km	5 km
1000 m	5000 m

	5 km
1 m	5000 m
100 cm	500000 cm

Keywords

A **variable** is a letter or symbol that represents an unknown value.
When variables are used with other numbers, parentheses, or operations, they create an **algebraic expression**.
The equation is an algebraic expression with an equal sign, which can be solved (the value of a variable is found).
A **coefficient** is a number multiplied by the variable in an algebraic expression.
A **term** is the name given to a number, a variable, or a number and a variable combined by multiplication or division, including + or – symbol in front of it.
A **constant** is a number that cannot change its value.
Identity is an equation that is true no matter what values are chosen. (symbol ≡)
A **formula** is an equation linking sets of physical variables.



SIMPLIFYING EXPRESSIONS

Multiplication of a number and variable is written without multiplication symbol, numbers first, letters in alphabetical order:

Example: $3 \times x = 3x$
$y \times 6 \times x = 6xy$

Division is written as a fraction:

Example: $6 \div x = \frac{6}{x}$

Multiplying and dividing variables

Example: $x \times x \times x = x^3$
$2 \times x \times y \times 3 = 2 \times 3 \times x \times y = 6xy$
$6x \div 2 = 3x$

COLLECTING LIKE TERMS

'Like terms' are terms whose **variables** (and their powers) are the same, the **coefficients** can be different.

Example:

$x + x + 2x = 4x$
$x + 4 + 3x - 5 = 4x - 1$
$x + 3y + 2x - 2y + 3 = 3x + y + 3$
$9x^2 - 2x - 5x^2 - 5x = 4x^2 - 7x$

MULTIPLICATION INDEX LAW WITH ALGEBRA

When expressions with the same base are multiplied, the indices are added.

$$a^m \times a^n = a^{(m+n)}$$

Examples: $x^5 \times x^3 = x^8$
$4x^5 \times 2x^8 = 4 \times 2 \times x^5 \times x^8 = 8x^{13}$

DIVISION INDEX LAW WITH ALGEBRA

When expressions with the same base are divided, the indices are subtracted.

$$a^m \div a^n = a^{(m-n)} \quad \text{or} \quad \frac{a^m}{a^n} = a^{m-n}$$

Examples: $x^7 \div x^4 = x^3$
$\frac{z^5}{z^8} = z^{-3}$
$\frac{20\ ^8}{5a^3} = 4a^3$

BRACKETS INDEX LAWS WITH ALGEBRA

When raising a power to another power, multiply the powers together.

$$(a^m)^n = a^{m \times n}$$

Examples: $(x^2)^5 = x^{10}$
$(2x^2)^3 = 2^3 \times x^6 = 8x^6$
$2(x^2)^3 = 2x^6$
$\left(\frac{x^5}{2}\right)^2 = \frac{x^{10}}{2^2} = \frac{x^{10}}{4}$

Please spot the important difference between these two calculations:
- Cubing the whole expression in the brackets $2x^2$
- Cubing only x^2

Year 8 Mathematics Knowledge Organiser – Unit 2: Working with powers

Keywords

Expanding brackets means removing the brackets.
Factorising means putting brackets back into expressions.
Factors of a number are the numbers that divide the original number without a remainder.
Writing a number as a product of factors is called a **factorisation** of the number.
The **Highest Common Factor (HCF)** is the largest common factor (the factor that two or more numbers have in common).

EXPANDING SINGLE BRACKETS

- Multiply everything in the brackets by a number or variable in front of the bracket

Examples: Expand

$4(a + 6) = 4a + 24$

$-2(b - 4) = -2b + 8$

$c(2c - 5) = 2c^2 - 5c$

$2d(3d - e) = 6d^2 - 2de$

$f^3(2f^2 - 4) = 2f^5 - 4f$

Expanding collection of single brackets

- Expand each bracket
- Collect like terms

Example: Expand and simplify

$2(3a^2 + 4a - 1) + 3(4a + 2) =$
$6a^2 + 8a - 2 + 12a + 6 =$
$6a^2 + 20a + 4$

FACTORISING

- Find the HCF of the terms in the brackets (highest numerical factor and the highest power of the variable)
- Put the HCF in front of the brackets
- Check your answers by expanding the brackets

Examples: Factorise

$4x + 12 = 4(x + 3)$

$7x^2 + 3x = x(7x + 3)$

$8x^2 + 16x = 8x(x + 2)$

SUBSTITUTION

If we are told what number a variable represents, we can **substitute** this into expressions to find their value.

Examples:
Find the value of expressions when $x = 5$, $y = 4$

$7x = 7 \times 5 = 35$

$3(x + 1) = 3 \times (5 + 1) = 3 \times 6 = 18$

$\frac{2(x-1)}{4} = \frac{2(5-1)}{4} = \frac{2\times4}{4} = \frac{8}{4} = 2$

$x^2 = 5^2 = 25$

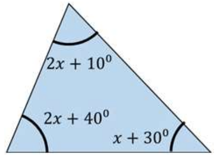
$2x^2 = 2 \times 5^2 = 2 \times 25 = 50$

$(2x)^2 = (2 \times 5)^2 = 10^2 = 100$

$xy = 5 \times 4 = 20$

FORMING AND SOLVING EQUATIONS

Example: Find the angles in this triangle.

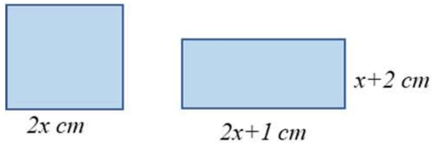


Write an equation:
Angles in the triangle add up to 180°.
$(2x + 10) + (2x + 40) + (x + 30) = 180°$
$5x + 80 = 180°$

Solve the equation:
$5x + 80 = 180°$
$5x = 100$
$x = 20$

Substitute x:
$x = 20$
Sizes of angles are
$2x + 10 = 2\times20 + 10 = 50°$
$2x + 40 = 2\times20 + 40 = 80°$
$x + 30 = 20 + 30 = 50°$

Example: The perimeters of the square and rectangle are the same, find their dimensions.



Write an equation:
$4(2x) = 2(2x + 1) + 2(x + 2)$

Solve the equation:
$8x = 4x + 2 + 2x + 4$
$8x = 6x + 6$
$2x = 6$
$x = 3$

Substitute x:
Square: width = 2 × 3 = 6cm
Rectangle: length = 2×3 + 1 = 7cm
width = 3 + 2 = 5cm
Perimeter of square = 4 × 6 = 24 cm
Perimeter of rectangle = 2×7 + 2×5 = 24 cm

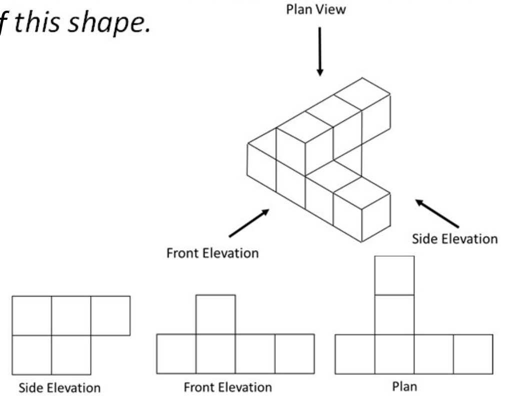
Keywords

Vertex	the point at which two or more lines intersect. Plural: vertices.
Edge	a line segment, joining two vertices of a figure. *Examples: a square has four edges; and a cuboid has twelve edges.*
Face	one of the flat surfaces of a solid shape. *Example: a cube has six faces.*
Area	the amount of space inside the boundary of a flat 2-dimensional shape.
Surface area	the total area of the surface of a three-dimensional object.
Volume	the amount of space a 3D shape takes up.

PLANS AND ELEVATIONS

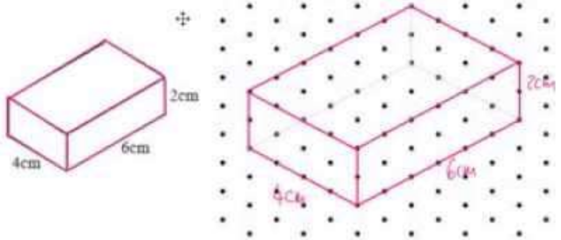
Plan is a scale drawing showing a 3D shape when it is looked at from **above**.
Elevation is the view of a 3D shape when it is looked at from the **side** or from the **front**.

Example: Draw plan, front and side elevations of this shape.



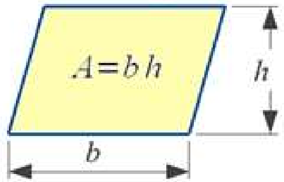
ISOMETRIC DRAWING

A method for visually **representing 3D objects in 2D**.
Example:

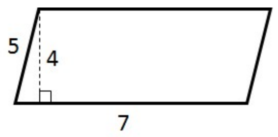


AREAS OF 2D SHAPES

Parallelogram

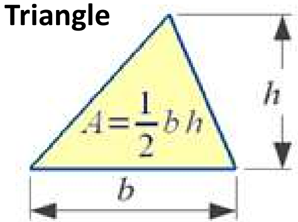


Example: Find the area of this parallelogram.

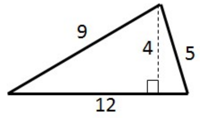


$A = 7 \times 4 = 28$ units^2

Triangle

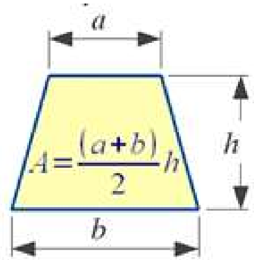


Example: Find the area of this triangle.

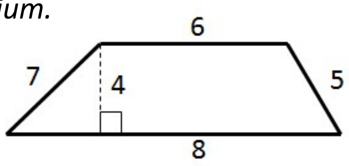


$A = \frac{12 \times 4}{2} = 24$ units^2

Trapezium



Example: Find the area of this trapezium.

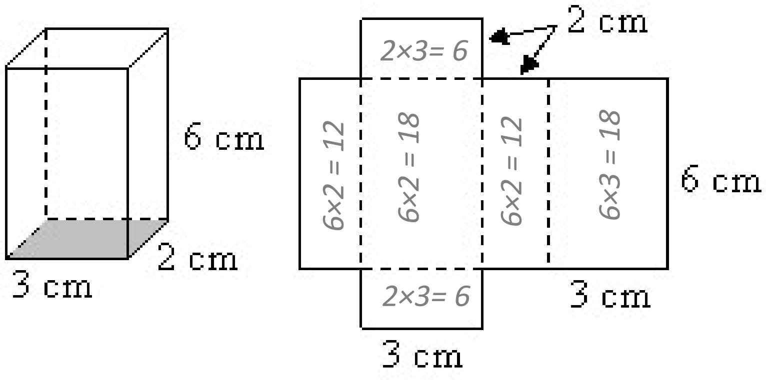


$A = \frac{(8+6)\times 4}{2} = \frac{14 \times 4}{2} = 28$ units^2

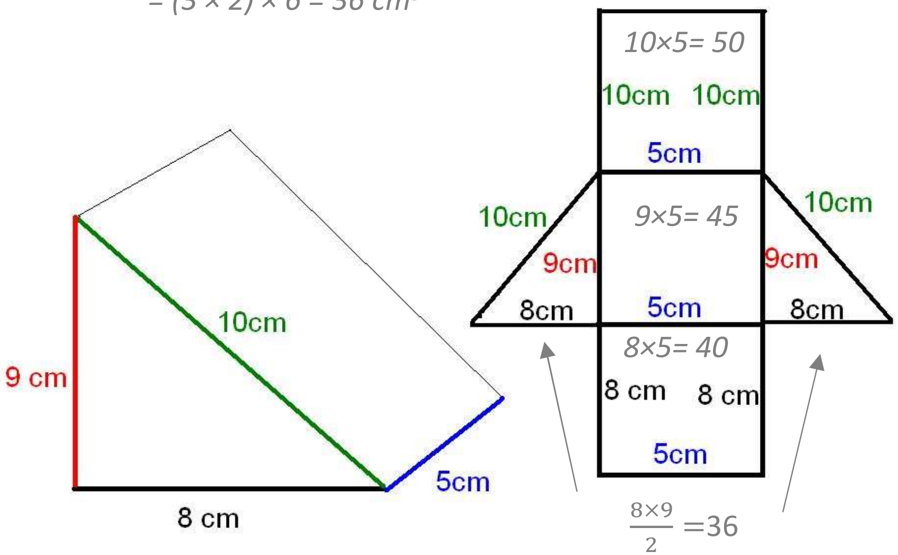
SURFACE AND VOLUME OF PRISMS

To find surface, add areas of all faces together.
To find volume, find the area of the **cross section and multiply it by height/length of the prism.**

Examples:



Surface = 12 + 18 + 12 + 18 + 6 + 6 = 72 cm^2
Volume = Area of cross section (front rectangle) × length of the prism
= (3 × 2) × 6 = 36 cm^3



Surface = 50 + 45 + 40 + 36 + 36 = 207 cm^2
Volume = area of cross section (front triangle) × length of the prism
$= \frac{8 \times 9}{2} \times 5 = 36 \times 5 = 180$ cm^3

Keywords

PARTS OF CIRCLE

Radius the distance from the centre of a circle to the edge.
Diameter the total distance across the width of a circle through the centre.
Circumference the total distance around the outside of a circle.
Arc a part of the circumference of a circle.
Sector the region of a circle enclosed by two radii and their intercepted arc.
Number π ('pi') Pi is the circumference of a circle divided by the diameter. **$\pi \approx 3.14$**

CIRCUMFERENCE OF A CIRCLE

$$C = \pi d$$

which means 'pi × diameter'.

Example: Find circumference of the circle with diameter 10 cm.

$C = \pi \times 10 = 31.4$cm

Example: Find circumference of the circle with radius 10 cm.

Diameter = 10 × 2 = 20 cm

$C = \pi \times 20 = 62.8$cm

Example: Find radius of the circle with circumference 12.6 cm.

diameter $= \frac{C}{\pi} = \frac{12.6}{\pi} = 4$ cm

Radius = half of diameter = 2 cm

ARC LENGTH

The arc length is fraction of the total circumference.

Example: Find length of an arc and perimeter of this shape.

Radius = 5 cm

Diameter = 10 cm

Circumference of the whole circle is

$C = \pi \times 10 = 31.4$ cm

Arc is ¼ of circumference.

Arc $= \frac{1}{4} \times 31.4 = 7.9$ cm

Perimeter of the shape = arc + radius + radius = 7.9 + 5 + 5 = 17.9 cm

AREA OF A CIRCLE

$$A = \pi r^2$$

which means 'pi x radius squared'.

Example: Find area of the circle with diameter 10 cm.

Radius = 10 ÷ 2 = 5cm

$A = \pi \times 5^2 = 78.5$ cm^2

Example: Find circumference of the circle with radius 10 cm.

$A = \pi \times 10^2 = 314$ cm^2

Example: Find diameter of the circle with area 50.3 cm^2.

radius $= \sqrt{\frac{A}{\pi}} = \sqrt{\frac{50.3}{\pi}} = 4$ cm

diameter = 2 × radius = 8 cm

AREA OF A SECTOR

The area of a sector is fraction of the total area.

Example: Find the area of this shape.

Area of the whole shape is

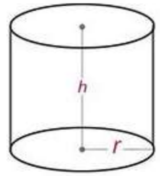
$A = \pi \times 5^2 = 78.5$ cm^2

Shape is the quarter of the whole

area $= \frac{1}{4} \times 78.5 = 19.6$ cm^2

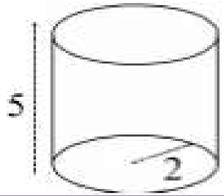
VOLUME OF A CYLINDER

The volume of a cylinder = area of the cross-section (circle) × height of the cylinder.



$$V = (\pi \times r^2) \times h = \pi r^2 h$$

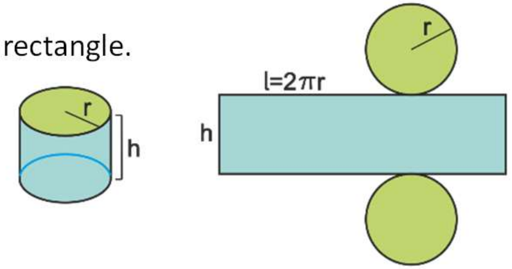
Example: Find the volume of this cylinder:



$V = \pi r^2 h = \pi \times 2^2 \times 5 = 62.8$ cm^3

SURFACE OF A CYLINDER

Net of the cylinder consists of two circles and rectangle.



Area of the circles is $A = \pi r^2$

Dimensions of the curved face (rectangle) are

Length = circumference of the circle $C = \pi d$

Width is the height of the cylinder.

Therefore area of the curved face (rectangle) = length × width = $\pi \times d \times h$

The surface of the cylinder *= 2 × area of circle + area of curved face =*

$$2\pi r^2 + \pi dh$$

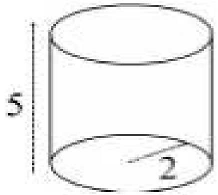
Example: Find surface area of this cylinder

Area of circle $= \pi r^2 = \pi \times 2^2 = 12.6$ cm^2

Diameter = 4 cm

Curved Surface Area $= \pi dh = \pi \times 4 \times 5 = 62.8$ cm^2

Total SA = 2 × 12.6 + 62.8 = 88 cm^2



Keywords

Right angle one quarter of a complete turn. An angle of 90 degrees.

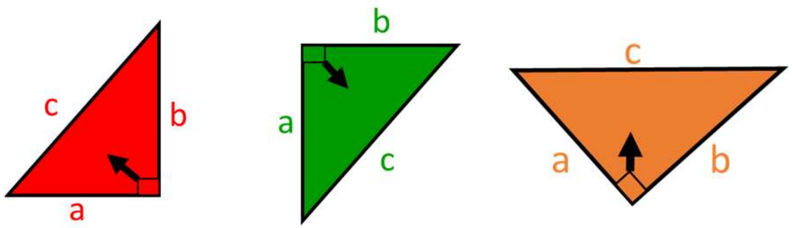
Hypotenuse the longest side in the right-angle triangle, opposite the right angle.

PYTHAGORAS THEOREM

in a right-angled triangle, the square of the length of the **hypotenuse** (labelled **c**) is equal to the sum of the squares of the lengths of the **other sides** i.e. the sides that bound the right angle (labelled **a** and **b**).

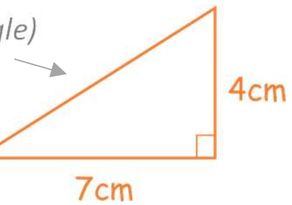
$$c^2 = a^2 + b^2$$

Examples of hypotenuse (labelled **c**) in right angle triangles



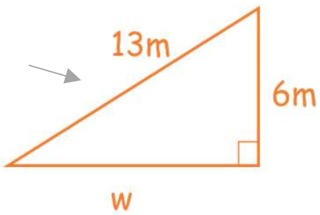
Example: Find the missing side in this right-angle triangle.

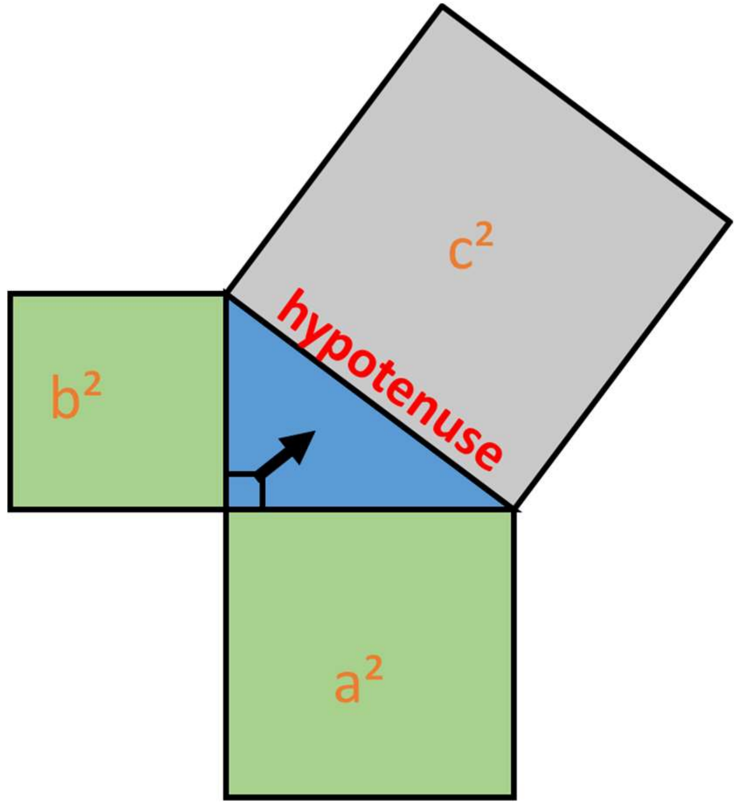
- *Identify hypotenuse (the longest side, opposite to the right angle)*
- *Write the formula* $c^2 = a^2 + b^2$
- *Substitute the known lengths* $c^2 = 7^2 + 4^2$
  $c^2 = 49 + 16 = 65$
- *Square root* $c = \sqrt{65} = 8.1$ cm
- *Don't forget the units*
- *Check if the answer is sensible* *'8.1 cm it is the longest side'*



Example: Find the missing side in this right-angle triangle.

- *Identify hypotenuse (the longest side, opposite to the right angle)*
- *Write the formula* $c^2 = a^2 + b^2$
- *Substitute the known lengths* $13^2 = 6^2 + w^2$
- *Rearrange* $w^2 = 13^2 - 6^2 = 169 - 36 = 133$
- *Don't forget to square root* $w = \sqrt{133} = 11.5$ m
- *Don't forget the units*
- *Check if the answer is sensible* *'11.5 m is less than hypotenuse'*





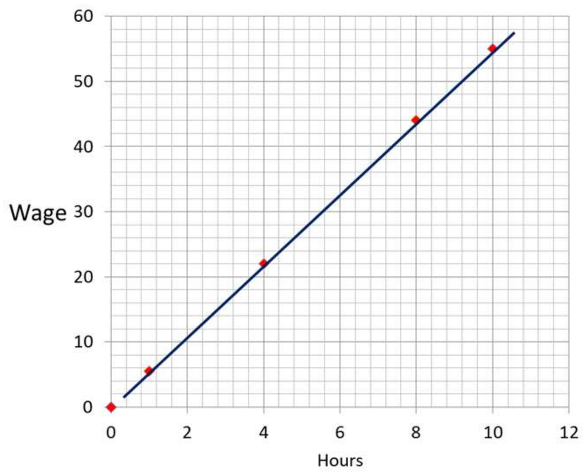
DIRECT PROPORTION

If two quantities are in direct proportion, as one increases, the other increases at the same rate.

A graph of direct proportion is always a **straight line** running through an origin (0, 0).

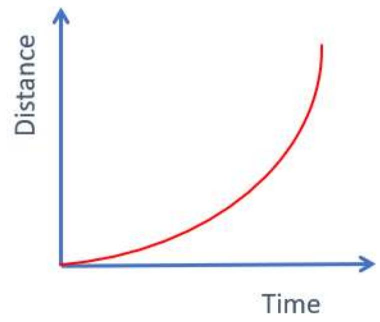
Example: Emily's wage is €5.50 per hour. Draw the graph of hours H against wage W.

H	*0*	*1*	*4*	*8*	*10*
W	*€0*	*€5.50*	*€22.00*	*€44.00*	*€55.00*



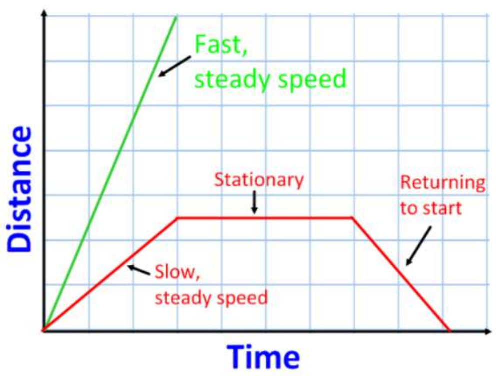
REAL-LIFE GRAPHS

A **curved** line on Distance-time graph shows that object is traveling at **increasing** or **decreasing** speed



Real-life graphs show **changes in time**.

DISTANCE - TIME GRAPH



- A **straight** diagonal line with positive gradient shows the object is moving at a **constant** speed.
- A **steeper** line shows the object is moving **faster.**
- A horizontal line shows that the object has **stopped moving.**
- Diagonal line going back towards the Time axis (negative gradient) shows the object is coming **closer to its starting position** (returning).

$$\text{speed} = \frac{distance}{time}$$

Example: A train travels from Milton to Redville, stops for 30 minutes, and then travels to Leek.

(a) How long did it take the train to travel from Milton to Redville? 2 hours

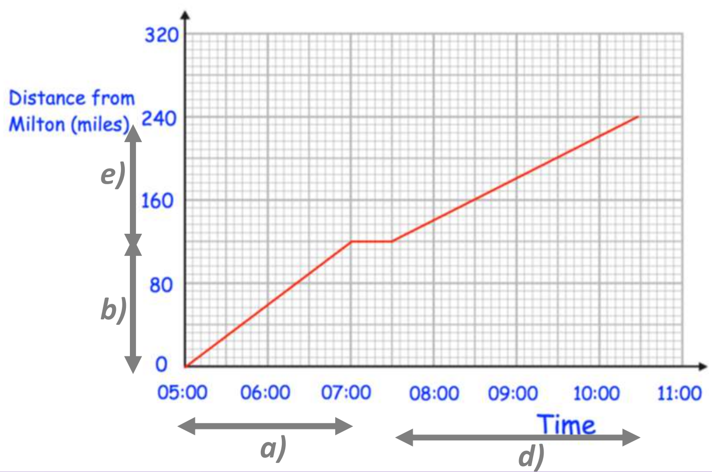
(b) How far is Redville from Milton? 120 miles

(c) Work out the speed of the train for the journey from Milton to Redville. $120 \div 2 = 60$mph

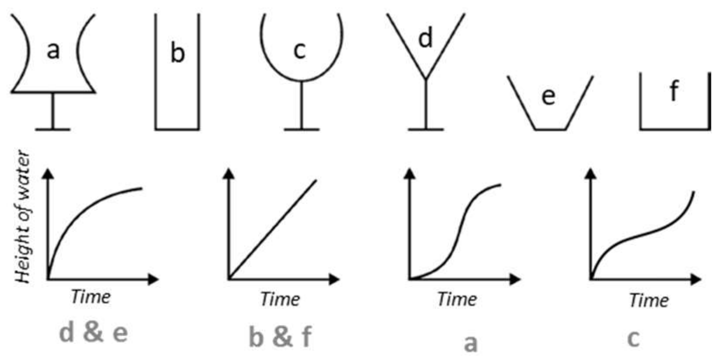
(d) How long did it take the train to travel from Redville to Leek? 3 hours

(e) How far is Leek from Redville? 120 miles

(f) Work out the speed of the train for the journey from Redville to Leek. $120 \div 3 = 40$mph



Example: The water comes out of the tap at a constant rate and fills the glasses below.
Which of these graphs shows how the depth of water in the glass changes with time?



Keywords

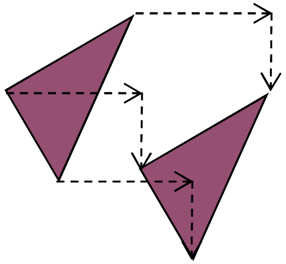
Transformations involve mathematically altering the position, and sometimes the size of a shape.
There are four main types of transformation:

1. **translations**
2. **rotations**
3. **reflections**

} the image is congruent to the object (the size is the same, orientation and position can be different)

4. **enlargements** → image is similar to the object (image is enlarged by the scale factor, position and orientation can be different)

TRANSLATION



The translation is described by:
1. Vector or movement description

Translations are made using a **VECTOR**

$\begin{pmatrix} x \\ y \end{pmatrix}$	The amount to move the shape **horizontally** The amount to move the shape **vertically**

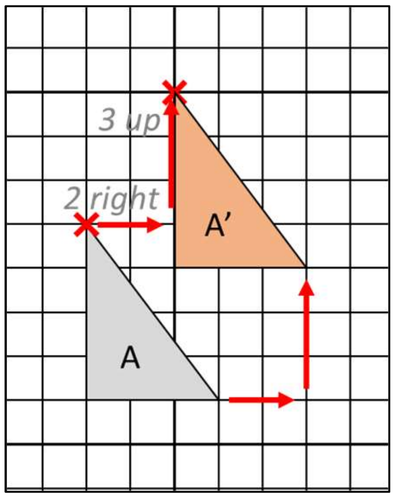
Movement of the shape:

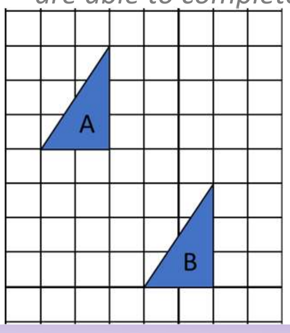
If x is - positive ... to the **right**
- negative ... to the **left**

If y is - positive ... **up**
- negative ... **down**

Example: Translate shape A by vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

- *Start by taking one vertex of the shape to be translated.*
- *Draw in the translation vector and mark the position of the corresponding vertex of the translated shape.*
- *Repeat with the other vertices (corners) until you are able to complete the shape.*





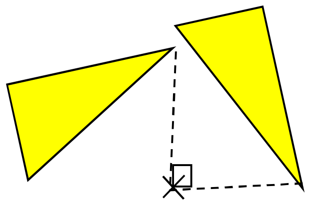
Example: Describe the transformation which takes shape A to B and shape B to A.

A to B: translation $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

B to A: translation $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

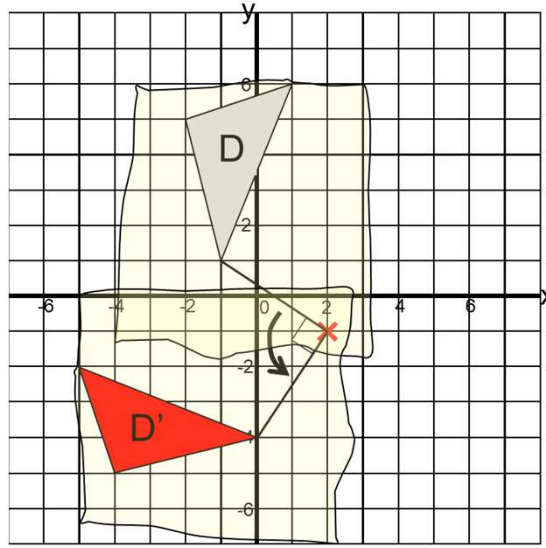
ROTATION

Rotation is described by:
1. Angle of rotation (90°, 180° or 270°)
2. Direction of rotation (clockwise, anticlockwise)
3. Centre of rotation ('at the origin' means point (0,0))

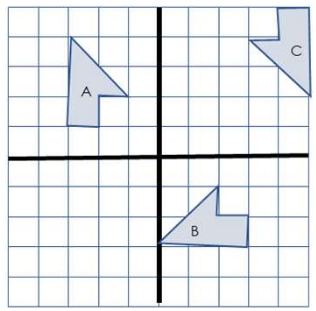


Example: Rotate shape D anticlockwise through 90° about the point (2,-1).

- *Mark the centre of rotation and draw a straight line connecting it to one vertex on the shape. Draw a 90° angle from this line with vertex in the Centre of rotation.*
- *Put tracing paper over both, the shape AND the centre of rotation.*
- *Trace the shape and line onto the paper.*
- *Put your pencil point on the centre of rotation and turn the paper through the appropriate angle.*
- *Draw the transformed shape in its new position.*



Example: Describe the transformation which takes shape A to B and C.



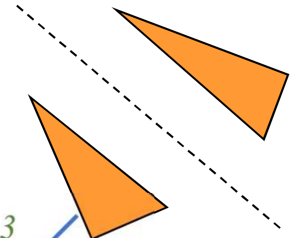
A to B: rotation, 90° anticlockwise, around centre of rotation (2,2)

A to C: rotation 180° anticlockwise/clockwise, around centre of rotation (1,3)

REFLECTION

Reflection is described by:

1. Equation of the 'mirror line' or 'line of reflection'

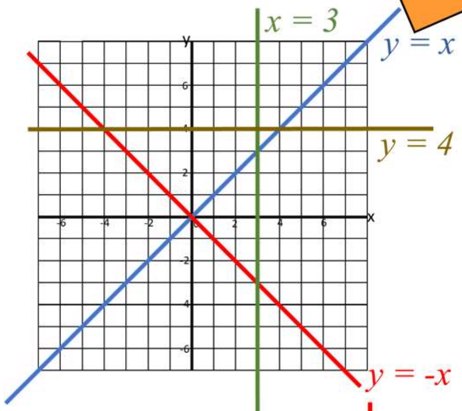


Examples of the lines of reflection:

Horizontal lines: $y = c$

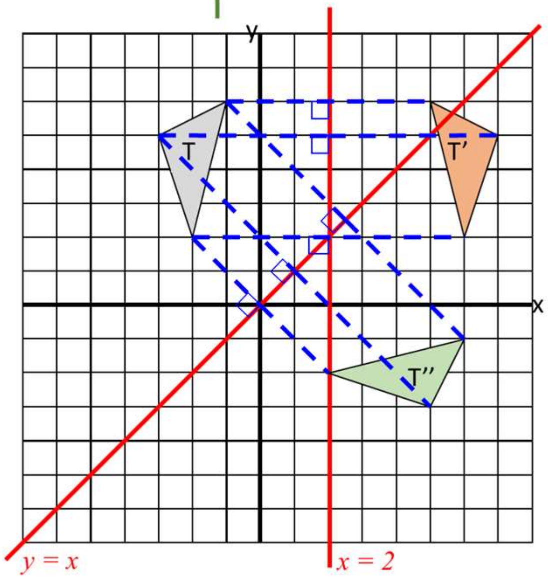
Vertical lines: $x = c$

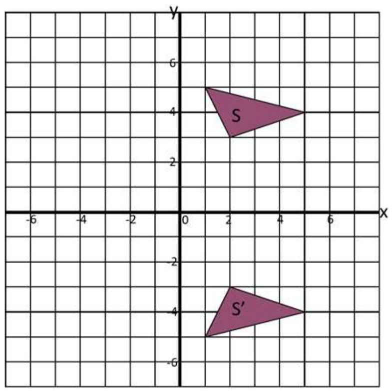
Diagonal lines: $y = x$ or $y = -x$



Example: Reflect shape T in lines $x = 2$ and $y = x$

- *Draw the mirror line.*
- *Draw a perpendicular line from one vertex of the original shape to the mirror line, and extend **it for the same distance** on the other side.*
- *Repeat with the other vertices until you can draw the whole shape.*





Example: Describe the transformation which takes shape S to S'.

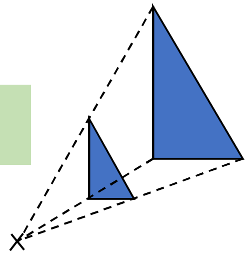
Reflection in the line of reflection $y = 0$.

ENLARGEMENT

Enlargement is defined by:

1. Scale factor (SF)
2. Centre of enlargement

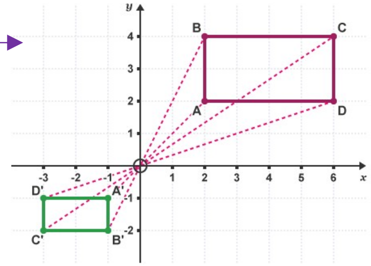
$$\text{Scale factor} = \frac{\text{new length}}{\text{old length}}$$



If the positive SF **is more than 1**, the shape will get bigger

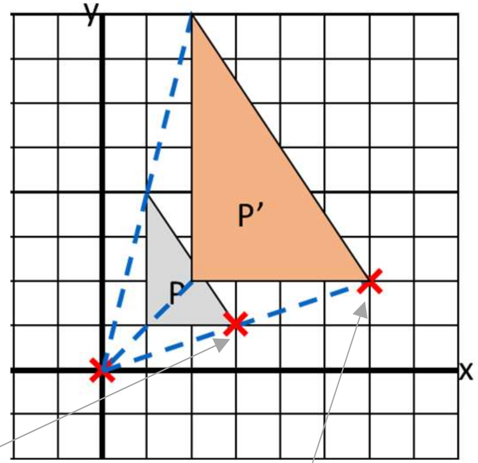
If the positive SF **is less than 1**, the shape will get smaller – but it's still called an enlargement!

Negative scale factor produces an image on the other side of the centre of enlargement.



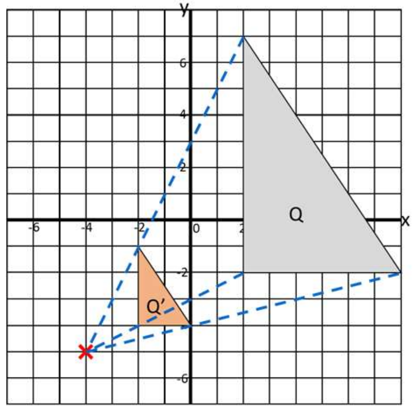
Example: Enlarge shape P about the origin with scale factor 2.

- *Mark the centre of enlargement (CoE).*
- *Draw a line from the CoE to one vertex on the shape.*
- *Multiply the length of line by the SF and extend it to find the position of the corresponding vertex on the enlarged shape.*
- *Repeat with the other vertices until you have enough information to draw the new shape.*



Original distance from CoE: 3→ and 1↑

Enlarged distance from CoE: 6→ and 2↑



Example: Describe the transformation which takes shape Q to Q'.

Enlargement with the scale factor $\frac{1}{3}$ and centre of the enlargement (-4, -5).

Keywords

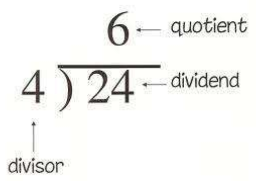
Factors are numbers which multiplied together get another number. *Example: 2 and 3 are factors of 6, because 2 × 3 = 6*

Product is the answer when two or more values are multiplied together. *Example: 6 is a product of 2 and 3, because 2 × 3 = 6*

Dividend a number to be divided.

Divisor a number by which another number is to be divided.

Quotient the answer after one number is divided by another.



CONVERTING BETWEEN FRACTIONS, TERMINATING DECIMALS AND PERCENTAGES

D ⟶ F

1. Write the decimal as a fraction 'over one'.
2. Convert the fraction into a fraction with a whole number in the numerator by multiplying both the numerator and denominator by the powers of 10.
3. Simplify.

Example: Convert 0.84 into a fraction. $0.84 = \frac{0.84}{1} = \frac{84}{100} = \frac{21}{25}$

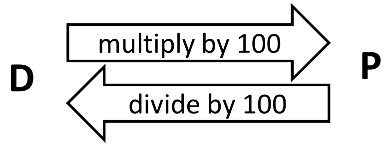
F ⟶ D

1. Divide the numerator by denominator.
2. Sometimes, you can help yourself by converting the fraction into the fraction with multiple of 10 in the denominator.

Example1: Convert $\frac{2}{9}$ *to decimal* $2 \div 9 = 0.22222.....$

Example2: Convert $\frac{2}{25}$ *to decimal* $\frac{2}{25} = \frac{8}{100}$

$8 \div 100 = 0.08$



Examples:

$0.04 = (0.04 \times 100)\,\% = 4\%$

$1.2 = (1.2 \times 100)\% = 120\%$

$23\% = \frac{23}{100} = 0.23$

$5\% = \frac{5}{100} = 0.05$

RECURRING DECIMALS

Recurring decimal is a decimal number that has digit or digits that repeat forever.
The part that repeats is shown by placing a dot above the repeated digit, or dots over the first and last digit of the repeating pattern.

Examples:

$\frac{1}{3} = 0.333\ldots = 0.\dot{3}$

$\frac{1}{7} = 0.\underbrace{142857}\underbrace{142857}\ldots = 0.\dot{1}4285\dot{7}$

$0.\dot{2}\dot{3} = 0.232323....$

$0.5\dot{3} = 0.533333.....$

$0.\dot{1}2\dot{3} = 0.123123123...$

CONVERTING RECURRING DECIMALS TO FRACTIONS

1. Create the equation $x = recurring\ number.$
2. Multiply the both sides of the equation by the power of 10 (*10, 100, 1000…*) until the decimal parts of multiples match up.
3. Subtract two equations with recurring digits after the decimal point matched up.
4. Rearrange.

Example: Convert $0.\dot{4}\dot{7}$ *to fraction.*

$0.\dot{4}\dot{7} = 0.474747....$

Let $x = 0.474747...$

$100x = 47.474747...$

$100x - x = 47.4747... - 0.4747...$

$99x = 47$

$x = \frac{47}{99}$

$0.\dot{4}\dot{7} = \frac{47}{99}$

Example: Convert $0.\dot{3}$ *to fraction.*

$0.\dot{3} = 0.33333....$

Let $x = 0.333333...$

$10x = 3.333333...$

$10x - x = 3.3333... - 0.3333...$

$9x = 3$

$x = \frac{3}{9} = \frac{1}{3}$

$0.\dot{3} = \frac{1}{3}$

Example: Convert $0.3\dot{4}$ *to fraction.*

$0.3\dot{4} = 0.3444444....$

Let $x = 0.344444...$

$10x = 3.444444...$

$100x = 34.444444...$

$100x - 10x = 34.444... - 3.444...$

$90x = 31$

$x = \frac{31}{90}$

$0.3\dot{4} = \frac{31}{90}$

Keywords

'Per cent' means out of 100. *Example:* 3% means 3 out of 100, which can be written in a form of a fraction $\frac{3}{100}$ or as a decimal 0.03

Multiplier is a decimal that represents the percentage change.

VAT stands for Value Added Tax. This is (usually) 20% tax added to the price of most of the things that you can buy.

Increase / reduce means to make something bigger / smaller (in size or quantity).

% OF AN AMOUNT

Finding 'easy' %s (without calculator)

50% by halving an amount
25% by dividing an amount by 4
10% by dividing an amount by 10
5% by halving 10%
1% by dividing an amount by 100
…and adding them together

100%
50% | 50%
25% | 25% | 25% | 25%
20% | 20% | 20% | 20% | 20%
10% | 10% | 10% | 10% | 10% | 10% | 10% | 10% | 10% | 10%

Example: 35% of 50

10% of 50 = 5
5% of 50 = 2.5
35% = 10% + 10 % + 10% + 5% =
5 + 5 + 5 + 2.5 = 17.5

÷ 2

100%	10%	30%	5%	35%
50	5	15	2.5	17.5

÷ 10   × 3

Using multiplier (with calculator)

1. Change % into decimal (multiplier).
2. Multiply.

Example: 35% of 50 = 0.35 × 50 = 17.5

'of' means '×'

% INCREASE AND DECREASE

Finding % of an amount and adding (increase) or subtracting (decrease)

Example1:
Increase 40 by 25%.
25% of 40 = 10
*40 + 10 = **50***

100% = 40
25% 10 | 25% 10 | 25% 10 | 25% 10
40 | 10
125% = 40 + 10 = 50

Example2:
Decrease 40 by 25%.
25% of 40 = 10
*40 - 10 = **30***

100% = 40
25% 10 | 25% 10 | 25% 10 | 25% 10
75% = 40 - 10 = 30

Using multiplier

Find the multiplier and **multiply**.

Example1:
Increase 40 by 25%.
Multiplier = (100 + 25)% = 125% = 1.25
40 × 1.25 = 50

100% | + 25%

Example2:
Decrease 40 by 25%.
Multiplier = (100 - 25)% = 75% = 0.75
40 × 0.75 = 30

100%
75% | -25%

FINDING AN ORIGINAL AMOUNT

Using multiplication table

Example1: T-shirt costs £30 after VAT (20% increase). What is its original price?

20% increase:
(100 + 20)% = 120% of the original price

÷12   ×10

120%	10%	100%
30	2.5	25

÷12   ×10

Example2: T-shirt costs £24 after 20% discount. What is its original price?

20% discount:
(100 – 20)% = 80% of the original price

÷8   ×10

80%	10%	100%
24	3	30

÷8   ×10

Using multiplier

Find the multiplier and **divide**.

Example1: Multiplier = (100 + 20)% = 120% = 1.2
£30 ÷ 1.2 = £25

Example2: Multiplier = (100 - 20)% = 80% = 0.8
£24 ÷ 0.8 = £30

EXPRESSING ONE NUMBER AS A % OF ANOTHER

Example: What is 17 as a percentage of 25?

$\frac{17}{25} \times 100 = 68$

17 is 68% of 25.

ORIGINAL AMOUNT → × by multiplier → AMOUNT AFTER % INCREASE/ DECREASE

AMOUNT AFTER % INCREASE/ DECREASE → ÷ by multiplier → ORIGINAL AMOUNT

Year 8 Mathematics Knowledge Organiser – Unit 6: Fractions, Decimals and Percentages

Keywords

Interest rate means how much is paid for the use of money, as a percent.
Depreciate means to decrease the value of something over time.
Annual means happening once a year.

FIND % CHANGE

Formula $$\% \text{ change} = \frac{\text{numerical change}}{\text{original amount}} \times 100$$

Example: Tom bought a pen for £5 and sold it for £8, what is the % increase of the price?

Change = 8 – 5 = £3 (Tom gained £3)

$\% \text{ change} = \frac{3}{5} \times 100 = 60\%$ **PROFIT**

Example: Tom bought a laptop for £200 and sold it for £50, what is % decrease of the price?

Change = 50 – 200 = -£150 (Tom lost £150)

$\% \text{ change} = \frac{-150}{200} \times 100 = -75\%$ **LOSS**

SIMPLE INTEREST

The simple interest is earned by certain % of the original amount paid regularly (annually, monthly…). The amount which is paid is the same every time.

Example: Tom invests £200 in an account which pays simple interest 2% yearly. How much interest will he earn in 3 years?

2% of £200 = 0.02 × 200 = £4
3 × £4 = £12, the interest is £12.
The final amount on the account is £212.

	Start of the year	Interest	End of the year
1	£200	*2% of £200 = £4*	£204
2	£204	*2% of £200 = £4*	£208
3	£208	*2% of £200 = £4*	£212
		Total interest = £12	

COMPOUND GROWTH AND DECAY

COMPOUND INTEREST is earned by **recalculating** interest each time from a new total.

Example1: Tom invests £200 in an account with compound interest 2% yearly. How much interest will he earn in 3 years?

	Start of the year	Interest	End of the year
1	£200	*2% of £200 = £4*	£204
2	£204	*2% of £204 = £4.08*	£208.08
3	£208.08	*2% of £208.08 = £4.16*	£212.24
		Total interest = £12.24	

Using multiplier: *(100 + 2)% = 102% = 1.02*
1.year: £200 × 1.02= £204
2.year: £204 × 1.02= £208.08
3.year: £208.08 × 1.02= £212.24 (amount on the account)
Interest is 212.24 – 200 = £12.24

DECAY (DEPRECIATION) means gradual decrease in value.

Example2: £2000 car decreases its value by 5% each year. What is its value after 3 years?

	Start of the year	Interest	End of the year
1	£2000	*5% of £2000 = £100*	£1900
2	£1900	*5% of £1900 = £95*	£1805
3	£1805	*5% of £1805 = £90.25*	£1714.75

Using multiplier: *(100 - 5)% = 95% = 0.95*
1.year: £2000 × 0.95= £1900
2.year: £1900 × 0.95= £1805
3.year: £1805 × 0.95= £1714.75 (the new value of the car)

COMPOUND GROWTH/DECAY FORMULA

$$FA = OA \times multiplier^n$$

FA = final amount
OA = original amount
n = number of times the interest is calculated

Solving examples above using the formula:

Example1: Compound interest $200 \times 1.02^3 = £212.24$
Example2: Decay $2000 \times 0.95^3 = £1714.75$

Keywords

TRIANGLE A polygon with three sides.

EQUILATERAL TRIANGLE A triangle with equal side lengths and equal angles of 60°.

ISOSCELES TRIANGLE A triangle in which two sides have the same length and consequently two angles are equal.

SCALENE TRIANGLE A triangle with no two sides equal and consequently no two angles equal.

ARC A portion of a curve. Often used for a portion of a circle.

CONSTRUCT To draw accurately.

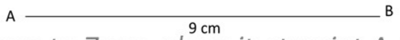
LINE SEGMENT Part of a line bounded by two distinct end points.

CONSTRUCT A TRIANGLE KNOWING THREE SIDES

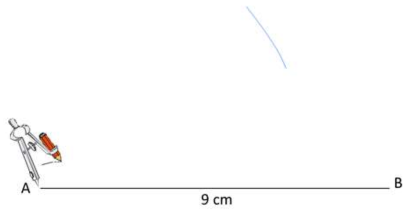
SSS

Example: Construct a triangle ABC with sides AB = 9cm, BC = 4m and AC = 7cm.

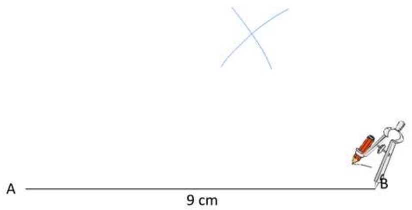
1. *Draw the line segment AB, 9 cm long.*



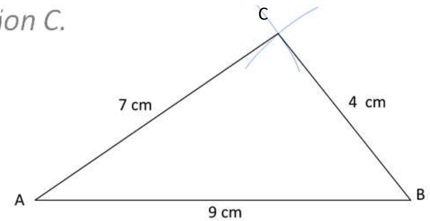
2. *Set compass to 7 cm, place it at point A and draw an arc.*



3. *Set compass to 4 cm, place it at point B and draw an arc to intersect the first one.*



4. *Draw straight lines from points A and B to point of intersection C.*



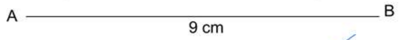
CONSTRUCT A TRIANGLE GIVEN ONE SIDE AND TWO ANGLES

ASA

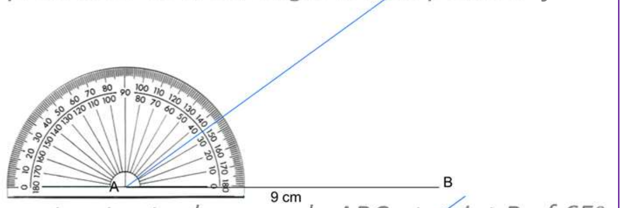
The known side has to be between two known angles.

Example: Construct a triangle ABC with side AB = 9cm between angles BAC = 35º and ABC = 65º.

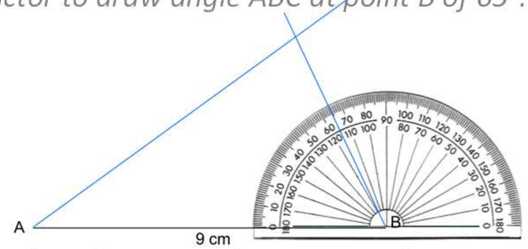
1. *Draw the line segment AB, 9 cm long.*



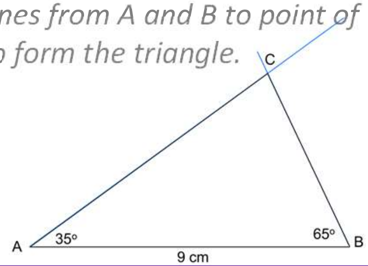
2. *Use a protractor to draw angle BAC at point A of 35º.*



3. *Use a protractor to draw angle ABC at point B of 65º.*



4. *Draw straight lines from A and B to point of intersection C to form the triangle.*



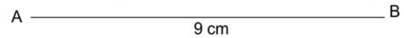
CONSTRUCT A TRIANGLE GIVEN TWO SIDES AND ONE ANGLE

SAS

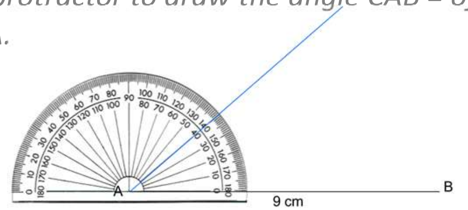
The known angle has to be between two known sides.

Example: Construct a triangle ABC with side AB = 9cm, side AC = 7cm and angle between them, CAB = 40º.

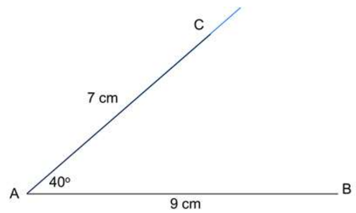
1. *Draw the line segment AB, 9 cm long.*



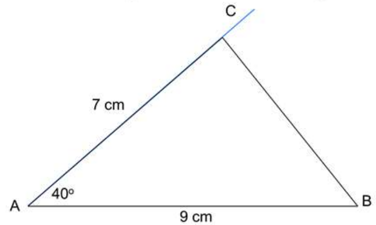
2. *Use a protractor to draw the angle CAB = of 40° at point A.*



3. *Measure line AC, 7 cm long, mark point C.*



4. *Join points A, B and C to form the triangle.*



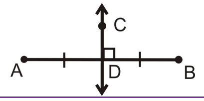
Always leave construction lines (the arcs and extended lines that you have added) when you have completed the diagram.

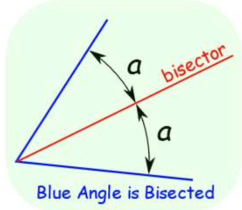
Keywords

PERPENDICULAR A line or plane that is at the right angle to another line or plane.

BISECTOR A point, line or plane that divides (a line, an angle or a solid shape) into two equal parts.

PERPENDICULAR BISECTOR This is a line that cuts a line segment into two equal parts at 90 degrees. If the ends of the line segment are A and B then any point on the line will always be the same distance from A and B. The points will be **EQUIDISTANT** from A and B.

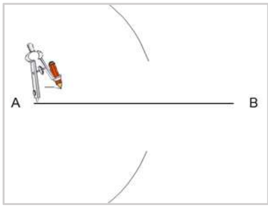




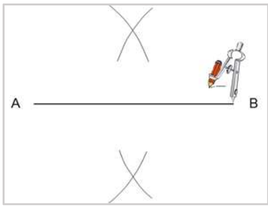
CONSTRUCT PERPENDICULAR BISECTOR OF A LINE SEGMENT.

Example: Construct perpendicular bisector of the line AB.

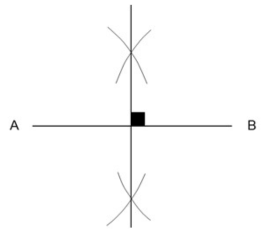
1. *Place compass at point the A, set over halfway the line segment AB and draw 2 arcs, one above and one below the line AB.*



2. *Without changing the setting of the compass, place it at the point B and draw 2 arcs, one above and one below the line AB, to intersect the first two arcs.*

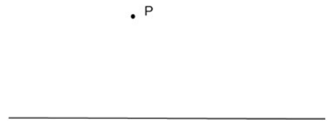


3. *Connect intersections of arcs with the straight line, this line is the perpendicular bisector of the line AB.*

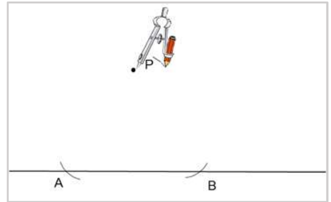


CONSTRUCT PERPENDICULAR FROM THE POINT TO THE LINE.

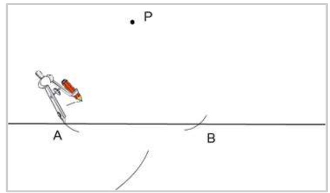
Example: Construct perpendicular from point P which is above the line.



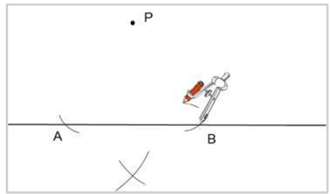
1. *Place compass at point P and draw two arcs A and B on the line. Length PA is equal to the length PB.*



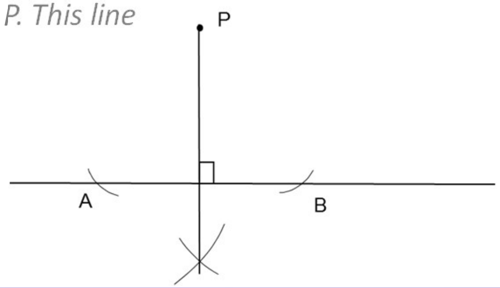
2. *With compass at point A and distance set greater than half of the distance between A and B, draw the arc bellow line AB.*



3. *Without changing the setting on the compass, place it at point B and draw the arc below the line AB.*



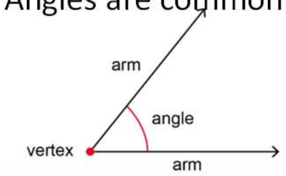
4. *Draw a line through intersection of arcs to P. This line is perpendicular to line AB.*

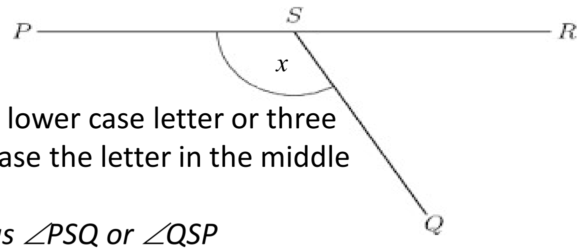


Always leave construction lines (the arcs and extended lines that you have added) when you have completed the diagram.

Keywords

An **ANGLE** is the **amount of turn** between two **straight lines** joined (or **intersected**) at a point called a **VERTEX**. The two lines are called **arms**. Angles are commonly marked by an **arc** (part of a circle) between the arms.





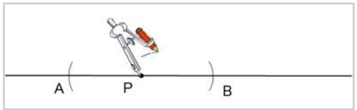
Angles are labelled using one lower case letter or three upper case letters, in which case the letter in the middle always represents the vertex.

Example: Angle x is labelled as ∠PSQ or ∠QSP

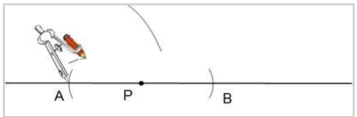
CONSTRUCT PERPENDICULAR FROM THE POINT TO THE LINE

Example: Construct perpendicular from point P which is at the line.

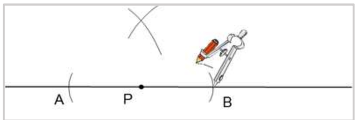
1. *Place compass at point P and draw two arcs A and B, distance PA is equal to PB.*



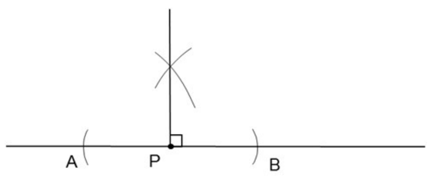
2. *With the compass at point A and distance set greater than AP, draw arc above the line AB.*



3. *Repeat point 2) with the compass at B and same distance set.*



4. *Draw the line through intersection of arcs to P. This line is perpendicular to line AB.*

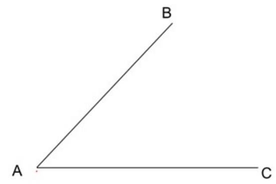


NOTE: The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. This is the **shortest** distance from the point to the line.

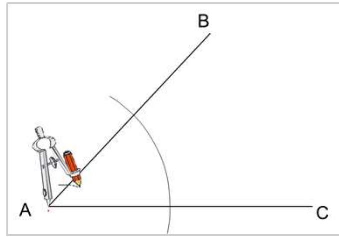
Distance from a point to a line

CONSTRUCT BISECTOR OF AN ANGLE

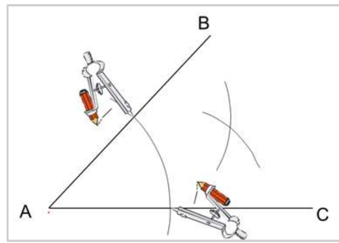
Example: Bisect angle BAC.



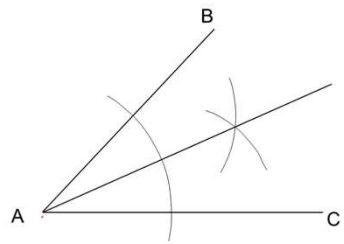
1. *Place compass at point A and draw an arc crossing arms AB and AC.*



2. *Place compass at the intersections and (with the same distance set) draw 2 arcs that intersect.*



3. *Draw the angle bisector from A through the point of intersection.*



Always leave construction lines (the arcs and extended lines that you have added) when you have completed the diagram.

Keywords

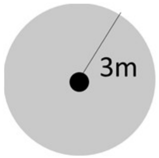
A **LOCUS** (plural **LOCI**) is a set of points or regions ("pathway") that satisfy given conditions.

Example: A Circle is "the locus of points on a plane that are a certain distance from a central point".

EQUIDISTANT means equal distance.

LOCUS AROUND A POINT.

A circle is the locus of points on a plane that are a certain distance from a central point.

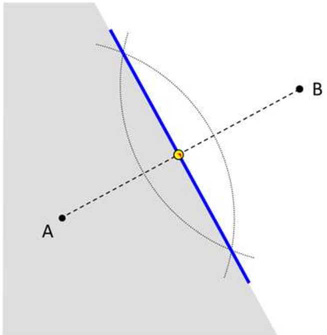


Region inside the circle is locus of points which are less than 3m from the centre.

Region outside the circle is locus of points that are more that 3 m from the centre.

SAME DISTANCE FROM TWO POINTS

The locus of points equidistant from two given points is the perpendicular bisector of the line segment that joins the two points.



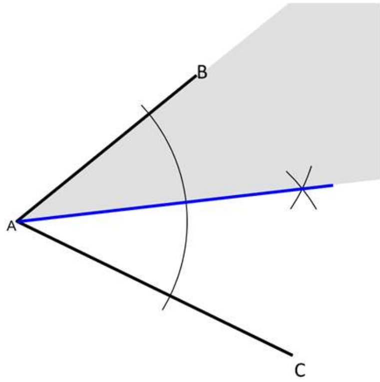
Any point on the perpendicular bisector is equidistant from A and B.

Any point 'left' of the bisector is closer to A than B (shaded grey).

Any point 'right' of the bisector is closer to B than A.

SAME DISTANCE FROM TWO INTERSECTING LINES

The locus of points equidistant from two intersecting lines is the bisector of the angle formed by these two lines.



Any point on the bisector is equidistant from AB and AC.

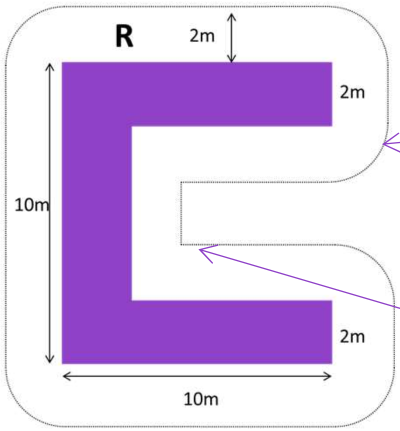
Any point 'above' the line is closer to AB (shaded grey).

Any point 'below' the line is closer to AC.

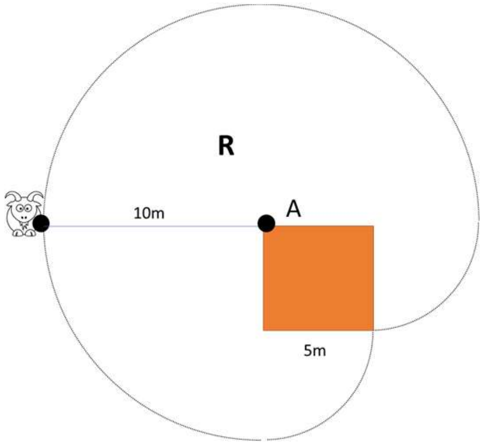
SAME DISTANCE FROM A LINE

The locus of points less than 3 m away from a line segment is shaded grey on the picture.

Example: Mark the locus of points less than 2 m away from purple building with R.



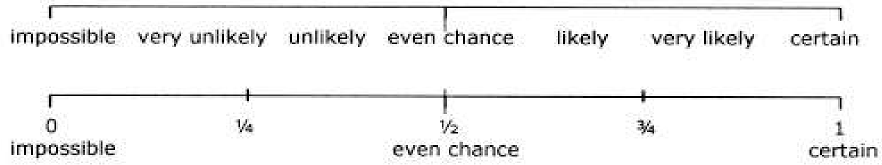
Example: A goat is attached to a fixed point A on a square building, of 5m × 5m, by a piece of rope 10m in length. What region can the goat reach?



Keywords

Probability is **how likely** something is to happen (**chance** of something happening).
Probability can be expressed as **fractions, decimals or percentages, or on a probability scale** between 0 (impossible) and 1 (certain).

- Higher the probability of something, more likely it is going to happen.
- Probability can not be greater than 1 or less than 0.



Fair means that all outcomes are equally likely.

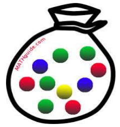
Example: Fair dice has an equally likely chance of landing on any face.

Bias means an opposite of fair, a built-in error which makes all values wrong by a certain degree.

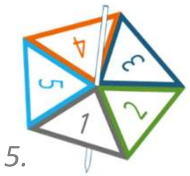
PROBABILITY OF SINGLE EVENTS

$$\textbf{Probability } P = \frac{\text{number of favourable outcomes}}{\text{number of all possible outcomes}}$$

Example: What is the probability that out of the bag on the right I will choose



1. *a green bead?*
   The number of all possible outcomes (beads) is 10.
   The number of favourable outcomes (green beads) is 4.
   P(green) = $\frac{4}{10}$ *= 0.4 = 40%.*
2. *a red bead?*
   P(red) = $\frac{3}{10}$ *= 0.3 = 30%*
3. *a blue OR yellow bead?*
   P(blue or yellow) = $\frac{2+1}{10} = \frac{3}{10}$
4. *black bead?*
   P(black) = 0 (no black beads)



Example: What is the probability that

a) *the spinner will land on 3?*
   The number of all possible outcomes is 5.
   The number of favourable outcomes (number 3) is 1.
   P(3) = $\frac{1}{5}$ *= 0.2 = 20%.*

b) *the spinner will land on even number?*
   P(even) = $\frac{2}{5}$ *= 0.4 = 40%.*

c) *the spinner will land on number less than 6?*
   All the numbers on the spinet are less than 6, so
   P(less than 6) = 1

MUTUALLY EXCLUSIVE EVENTS

Events are mutually exclusive if they cannot happen at the same time.
Probabilities of mutually exclusive events can be added together.

P(A or B) = P(A) + P(B)

If mutually exclusive events cover all possible outcomes, their probabilities add up to 1, or 100%.

Example: The table shows probabilities of choosing different colours of beads from the bag. What is the probability of choosing a yellow bead?

Colour	blue	green	yellow	red
P	0.2	0.4		0.3

P(blue) + P(green) + P(red) = 0.2 + 0.4 + 0.3 = 0.9
P(yellow) = 1 – 0.9 = 0.1

P(event happening) + P(event not happening) = 1

Example: Probability it will rain tomorrow is 0.7, what is the probability it will not rain?
P(not raining) = 1 – P(rain) = 1 – 0.7 = 0.3

RELATIVE FREQUENCY

When you do an experiment over and over again and count outcomes, you find a **frequency** of outcomes.
From the experiment, you can calculate **relative frequency**:

$$\textbf{Relative frequency} = \frac{\text{frequency}}{\text{number of all times you did the experiment}}$$

Relative frequency is used to **estimate the probability, EXPERIMENTAL PROBABILITY.**

Example: Here are the results of a survey of cars passing a school:

Colour	Number of cars
Red	3
Black	10
Silver	15
Other	2

Based on this experiment, what is the relative frequency of the silver cars?
How many silver cars would you expect to see if 1000 cars passed by school?
What is the experimental probability that the car will be silver?

Total number of cars = 3 + 10 + 15 + 2 = 30.
Frequency of silver cars = 15.
Relative frequency = $\frac{15}{30} = \frac{1}{2}$.
If 1000 cars passed by the school ½ would be silver, that means 500 car.
Experimental probability that the next car passing by school is silver equals ½.

EXPECTED FREQUENCY

Expected frequency estimates number of times outcome happens in an experiment.

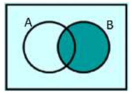
Expected frequency = probability × number of trials

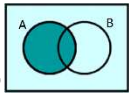
Example: The coin is tossed 1000 times, how many times do I expect the result to be a head?

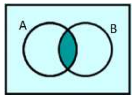
P(head) = ½
Number of trials = 1000
Expected frequency = 1000 × ½ = 500

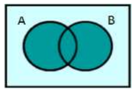
VENN DIAGRAMS

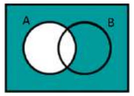
Set is the collection of things called **elements**.
Universal set ξ is a complete group that the elements are selected from.
In **Venn diagrams**, the sets are represented by circles.


Probability of B, P(B)

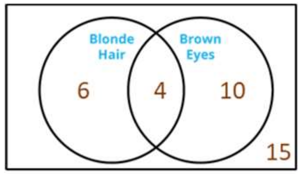

Probability of A, P(A)


Probability of A **AND** B, P(A∩B), intersection


Probability of A **OR** B, P(A∪B), union


Probability of **NOT** A, P(A'), complement

Example: From the Venn diagram, find probability that student chosen at random will have brown eyes and blond hair.



P (blond and brown)= $\frac{4}{6+4+10+15} = \frac{4}{35}$

SPACE DIAGRAMS

Sample space diagrams show all the possible outcomes, mostly in the form of two way table.

Example: If you throw two dice and add the numbers on the dice, which sum is the most probable to be thrown? What is probability of getting a sum greater or equal 9?

Sample space = 36
The most probable sum is 7,
P(total = 7) = $\frac{6}{36}$
P(total ≥ 9) = $\frac{10}{36}$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Example: Two fair spinners are spun.
1 contains the colours red, yellow, blue and white.
The other contains the numbers 1,2,3, and 4.
1) Draw a sample space diagram .

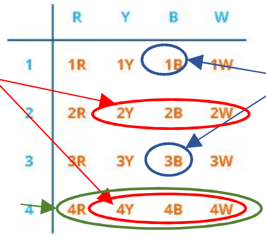
	R	Y	B	W
1	1R	1Y	1B	1W
2	2R	2Y	2B	2W
3	3R	3Y	3B	3W
4	4R	4Y	4B	4W

2) Work out the probability of spinning

a) An odd number and a blue. $\frac{2}{16}$

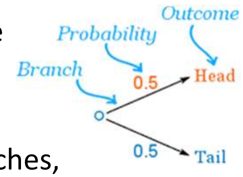
b) An even number and not a red. $\frac{6}{16}$

c) A number greater than 3 (with any colour). $\frac{4}{16}$



PROBABILITY TREES

A tree diagram is a way to represent the probabilities of two or more events.

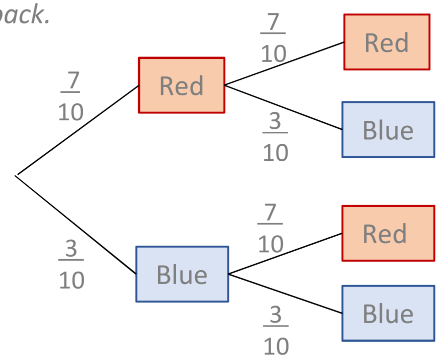


To find probabilities:

- multiply probabilities along the branches,
- add probabilities at the end of the branches together.

*Example (**WITH REPLACEMENT**): I have a bag of 3 blue and 7 red counters. I pick one counter out, record the colour and put it back. Then again I pick out a counter, record the colour and put it back.*
What is the probability that I picked up two red counters?

Start with 7 red and 3 blue counters. After the first pick, there are still 7 R and 3 B counters, because counter was returned back.



$P(RR) = \frac{7}{10} \times \frac{7}{10} = \frac{49}{100}$

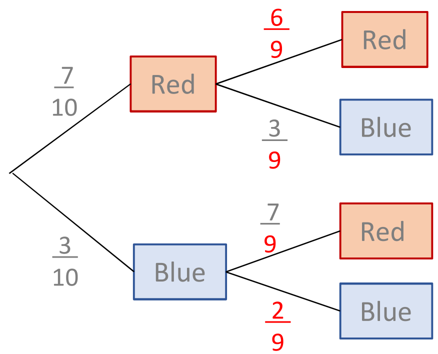
$P(RB) = \frac{7}{10} \times \frac{3}{10} = \frac{21}{100}$

$P(BR) = \frac{3}{10} \times \frac{7}{10} = \frac{21}{100}$

$P(BB) = \frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$

*Example (**WITHOUT REPLACEMENT**): I have a bag of sweets, 3 blue and 7 red. I pick one sweet out without looking, eat it and then pick out and eat a second sweet.*
What is the probability that I eat two red sweets?

Start with 7 red and 3 blue sweets. After the first pick:
- *if I picked RED sweet, there are 6 red and 3 blue sweets left,*
- *if I picked BLUE sweet, there are 7 red and 2 blue sweets left.*



$P(RR) = \frac{7}{10} \times \frac{6}{9} = \frac{42}{90}$

$P(RB) = \frac{7}{10} \times \frac{3}{9} = \frac{21}{90}$

$P(BR) = \frac{3}{10} \times \frac{7}{9} = \frac{21}{90}$

$P(BB) = \frac{3}{10} \times \frac{2}{9} = \frac{6}{90}$