June 2012 (C1)

① $x^3 + 3x - 5x^2 + 15 - (x^2 + 3x - 4)$

$= x^3 - 6x^2 - 11$

② i) $\sqrt[4]{7} = 7^{1/4}$

ii) $\frac{1}{7\sqrt{7}} = \frac{1}{7^1 \times 7^{1/2}} = \frac{1}{7^{3/2}} = 7^{-3/2}$

iii) $7^4 \times 49^{10} = 7^4 \times (7^2)^{10} = 7^4 \times 7^{20} = 7^{24}$

③ i) ~~$y = 3 - 5$~~ $3x - 5y - 20 = 0$

$\frac{3}{5}x - 4 = y$

~~$\frac{dy}{dx}$~~ gradient $= 3/5$

ii) @ $x = 0$ (y-axis)

$y = -4 \quad \therefore Q(0, -4)$

@ $y = 0$ (x-axis) $\quad \frac{3}{5}x = 4$

$x = 20/3$

$P(20/3, 0)$

midpoint $= \left(\frac{\frac{20}{3}+0}{2}, \frac{-4+0}{2}\right) = \left(\frac{20}{6}, -2\right)$

$= \left(\frac{10}{3}, -2\right)$

④ i) $2x^2 - 20x + 49$

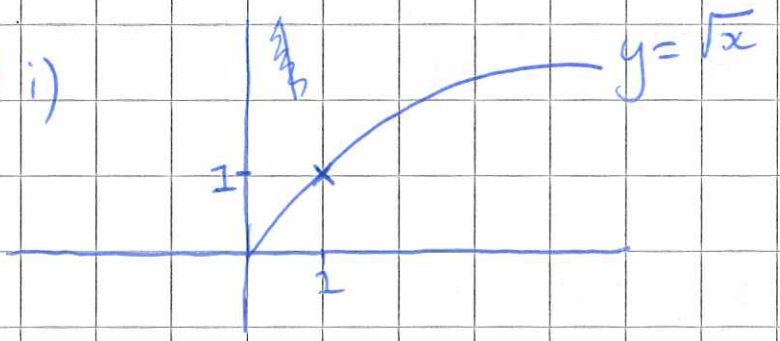
$= 2\left(x^2 - 10x\right) + 49$

$= 2\left((x-5)^2 - 25\right) + 49$

$= 2(x-5)^2 - 1$

ii) vertex is @ $(5, -1)$

⑤ i)



ii) Translation by vector $\begin{bmatrix} 4 \\ 0 \end{bmatrix}$, 4 units in positive x-direction

iii) $y = \sqrt{\frac{1}{5}x}$

⑥ $y = 6x^{-2} - 5$

$\frac{dy}{dx} = -12x^{-3}$ @ $x = 2$ $\frac{dy}{dx} = -12(2)^{-3}$

$= \frac{-12}{8} = -\frac{3}{2}$

gradient of normal $m_1 \times m_2 = -1$

hence $2/3$

eqn: $y = \frac{2}{3}x + c$

(6) continued

@ $x = 2$ $\quad y = 6(2)^{-2} - 5$

$= -7/2$

$y - y_1 = m(x - x_1)$

$y + \frac{7}{2} = \frac{2}{3}(x - 2)$

$6y + 21 = 4x - 8$

$0 = 4x - 6y - 29$

(7) $x - 6x^{1/2} + 2 = 0$ $\quad$ let $x = k^2$

$k^2 - 6k + 2 = 0$

$k = \dfrac{6 \pm \sqrt{6^2 - 4(1)(2)}}{2}$

$k = \dfrac{6 \pm \sqrt{28}}{2}$

$k = 3 \pm \sqrt{7}$

$x = (3 \pm \sqrt{7})^2$

$x_1 = 9 + 6\sqrt{7} + 7$

$= 16 + 6\sqrt{7}$

$x_2 = 9 - 6\sqrt{7} + 7$

$= 16 - 6\sqrt{7}$

$\Rightarrow x = 16 \pm 6\sqrt{7}$

(8) i) $y = x^4 + 32x$

$\frac{dy}{dx} = 4x^3 + 32$ @ stationary point $\frac{dy}{dx} = 0$

$\therefore \quad 4x^3 + 32 = 0$

$4x^3 = -32$

$x^3 = -8$

$\underline{x = -2}$

$y = (-2)^4 + 32(-2)$

$y = 16 - 64$

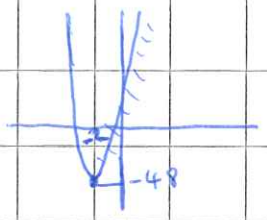
$\underline{y = -48}$

ii) $\frac{d^2y}{dx^2} = 12x^2$

@ $x = -2$ $\quad \frac{d^2y}{dx^2} = 12(-2)^2$

$= 48$

$48 > 0$ Hence a min point.



iii) $x > -2$

(9) $4x(x+3) < 112$

$4x^2 + 12x < 112$

$4x^2 + 12x - 112 < 0$

$4(x^2 + 3x - 28) < 0$

$4(x+7)(x-4) < 0$

$-7 < x < 4$

$\therefore \underline{0 < x < 4}$

ii) $20 < 4y + 3 + 2y + y + 2y + y + 3 < 54$

$20 < 10y + 6 < 54$

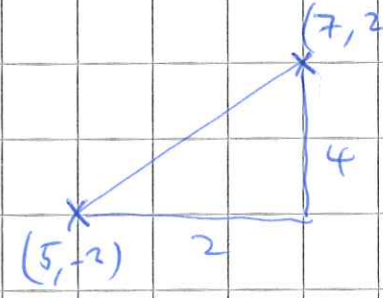
$14 < 10y < 48$

$1.4 < y < 4.8$

(10) $(x-5)^2 + (y+2)^2 = 25$

i) Centre: $(5, -2)$ diameter $= 10$

ii)



$m = \frac{2 - -2}{7 - 5} = \frac{4}{2} = 2$

$y - 2 = 2(x - 7)$

$y - 2 = 2x - 14$

$y = 2x - 12$

iii) $CP = \sqrt{2^2 + 4^2}$

$= \sqrt{20}$

$\sqrt{20} < 5$ hence inside the circle.

iv) $y = 2x$ sub into circle eqn.

$(x-5)^2 + (2x+2)^2 = 25$

$x^2 - 10x + 25 + 4x^2 + 8x + 4 = 25$

$5x^2 - 2x + 4 = 0$

$b^2 - 4ac$

$4 - 4(5)(4) < 0$ hence no roots

line and circle do not meet.