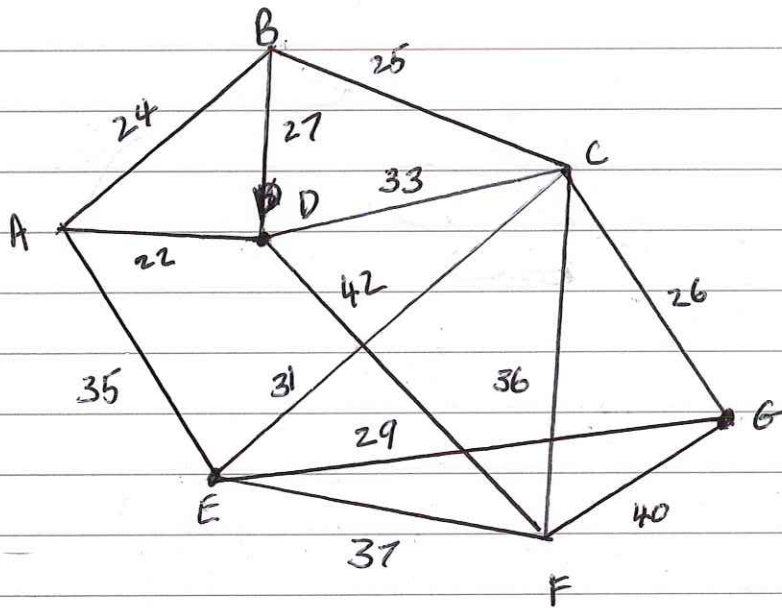


①

# October Further Maths Division 1

October 2020

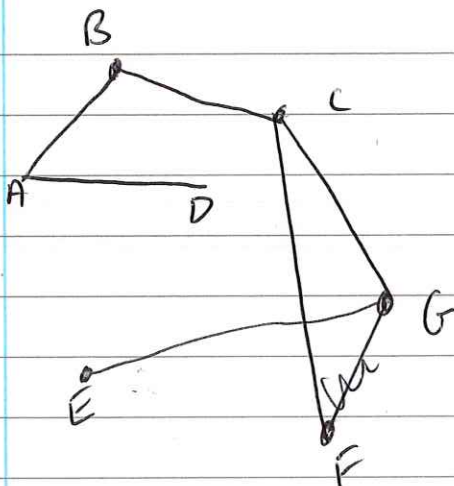
Q1



~~24, 27, 35, 25, 27, 33, 31, 36, 26, 42, 37, 29, 40~~

ascending order  $(22), (24), (25), (26), (27), (29), (31), (33), (35), (36), (40), 42$

~~AD, AB, BC, CG, GE, GF, CF~~



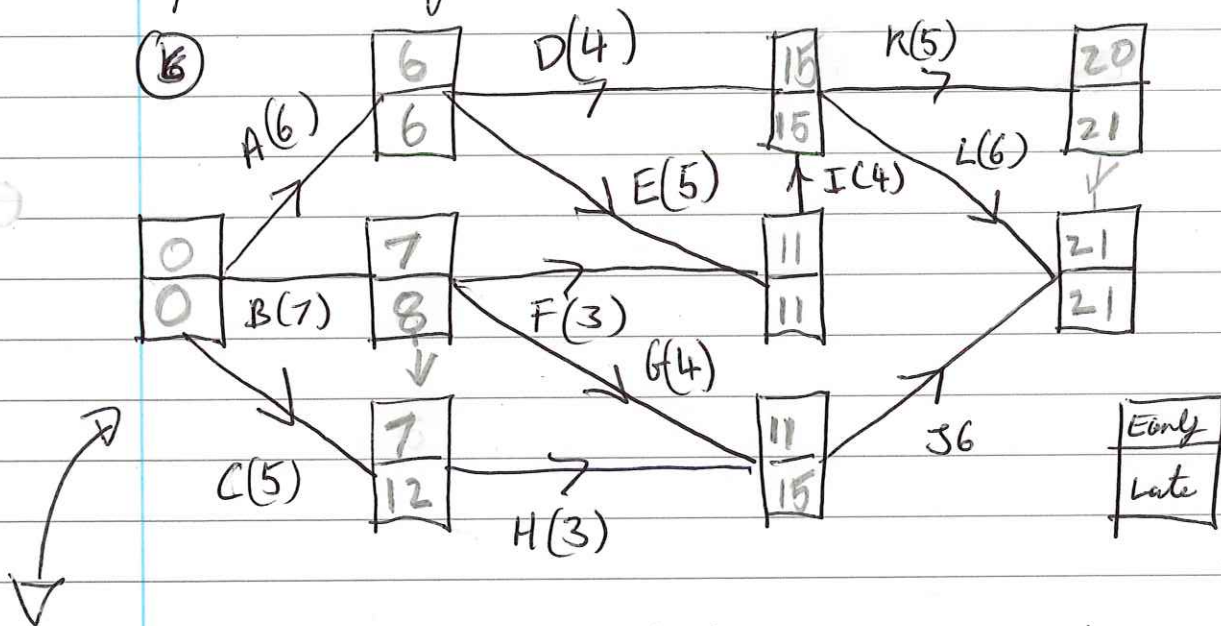
Total distance

$$22 + 24 + 25 + 26 + 29 + 36 = 162$$

(2) October Further Maths Decision 1

2. (a) Dummy at K needed as two activities can't start and finish at the same event.

It is required at H as H requires C and B, but G only requires activity B.



(b)

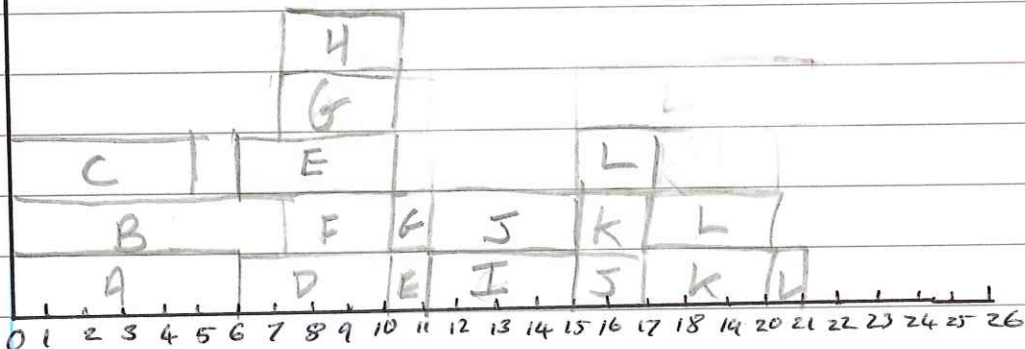
Activity	IPA	Activity	IPA	Activity	IPA
A	-	E	A	I	E, F
B	-	F	B	J	G, H
C	-	G	B	K	D, I
D	A	H	B, C	L	D, I

(ii) 21

(iii) A ~~and~~ E I L

Q2  
 Number of workers

7  
 6  
 5  
 4  
 3  
 2  
 1  
 0



Time in hours

Yes it can be completed with fewer workers in the minimum time.

H could be delayed to start at 10 rather than 7, as the late start time is 12. This would reduce by 1 the number of required workers.

(4)

Q3

October Further Maths Decision 1

Time Matrix

	A	B	C	D	E
A	-	8	4	7	$\infty$
B	8	-	3	$\infty$	10
C	4	3	-	$\infty$	6
D	7	$\infty$	1	-	1
E	$\infty$	10	6	1	-

Route Matrix

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	C	D	E
C	A	B	C	D	E
D	A	B	C	D	E
E	A	B	C	D	E

(b)

Time Matrix

	A	B	C	D	E
A	-	7	4	7	10
B	7	-	3	14	9
C	4	3	-	11	6
D	5	4	1	-	1
E	10	9	6	1	-

	A	B	C	D	E
A	A	C	C	D	C
B	C	B	C	C	C
C	A	B	C	A	E
D	C	C	C	D	E
E	C	C	C	D	E



5 Q3 b

Time Matrix

	A	B	C	D	E
A	-	7	4	7	8
B	7	-	3	14	9
C	4	3	-	11	6
D	5	4	1	-	1
E	6	5	2	1	-

	A	B	C	D	E
A	A	C	C	D	D
B	C	B	C	C	C
C	A	B	C	A	E
D	C	C	C	D	E
E	D	D	D	D	E

(i) A C B E D A

(ii)  $4 + 3 + 9 + 1 + 5 = 22$  mins

(iii) A C B C E D C A

6

4

# October Further Maths Decision 1

$$\begin{aligned}
 (a) \quad & 2y \leq 5x \\
 & y \geq x+1 \\
 & 6x + 5y \leq 30
 \end{aligned}$$

$$P = 3x + y$$

$$\begin{aligned}
 (b) \quad & 2y = 5x \quad (1) \\
 & 6x + 5y = 30 \quad (2) \Rightarrow 12x + 10y = 60
 \end{aligned}$$

$$\begin{aligned}
 \text{sub (1) in (2)} \quad & 12x + 25x = 60 \\
 & 37x = 60
 \end{aligned}$$

$$\begin{aligned}
 x &= 60/37 \\
 y &= 150/37 \\
 P &= \underline{\underline{330/37}}
 \end{aligned}$$


$$\begin{aligned}
 y &= x+1 \quad (1) \\
 6x + 5y &= 30 \quad (2)
 \end{aligned}$$

$$\begin{aligned}
 \text{sub (1) into (2)} \quad & 6x + 5(x+1) = 30 \\
 & 6x + 5x + 5 = 30 \\
 & 11x + 5 = 30
 \end{aligned}$$

$$\begin{aligned}
 11x &= 25 \\
 x &= 25/11 \\
 y &= 36/11 \\
 P &= \underline{\underline{111/11}}
 \end{aligned}$$

$$\begin{aligned}
 2y = 5x \quad \} \quad & 2(x+1) = 5x \\
 y = x+1 \quad \} \quad & 2x + 2 = 5x \\
 & 2 = 3x \\
 & x = 2/3 \Rightarrow y = 5/3
 \end{aligned}$$

$$P = \underline{\underline{11/3}}$$


  
Optimum
  
 $x = 25/11$ 
  
 $y = 36/11$

7

Q4

(c)

$$Q = 3x + ay$$

$$3 \times \frac{60}{37} + a \times \frac{150}{37} < 3 \times \frac{25}{11} + a \frac{36}{11}$$

$$\frac{150a - 36a}{37 \cdot 11} < \frac{795}{407} \quad \text{or} \quad \frac{114a}{407} < \frac{795}{407} \quad a < \frac{5}{2}$$

$$3 \times \frac{2}{3} + a \times \frac{5}{3} < 3 \times \frac{25}{11} + a \times \frac{36}{11}$$

$$\frac{5a - 36a}{3 \cdot 11} < \frac{53}{11}$$

$$a < -3$$

so  $-3 < a < 5/2$

⑧

5(a) If  $x \geq 21$  the extra 10 would not fit into the bin.

If  $x \leq 13$  the 18 would go in bin 2

So as discrete  $13 < x < 21$

⑥  $x \leq 24$  as swapped with 24

but  $x$  not swapped with 8 so

$$x \leq 8$$

$$8 \leq x < 24$$

Also  $x$  not swapped with 17  $x \leq 17$

$$8 \leq x \leq 17$$

But distinct numbers  $8 < x < 17$

when combine with other constraint

$$13 < x < 21$$



(9)

4C

To fit in bin 3 and satisfy  $13 < x < 21$

then  $x$  could be 14 or 15.

But only 1 bin full, so  $x$  must be 14.

(10) (6) (a) As the graph has two odd nodes

it is semi-Eulerian.

(see next page)

$$AG = 60 + y$$

$$\textcircled{1} \quad \begin{aligned} 409 &= 320 + x + y + 60 + y \\ 29 &= x + 2y \end{aligned}$$

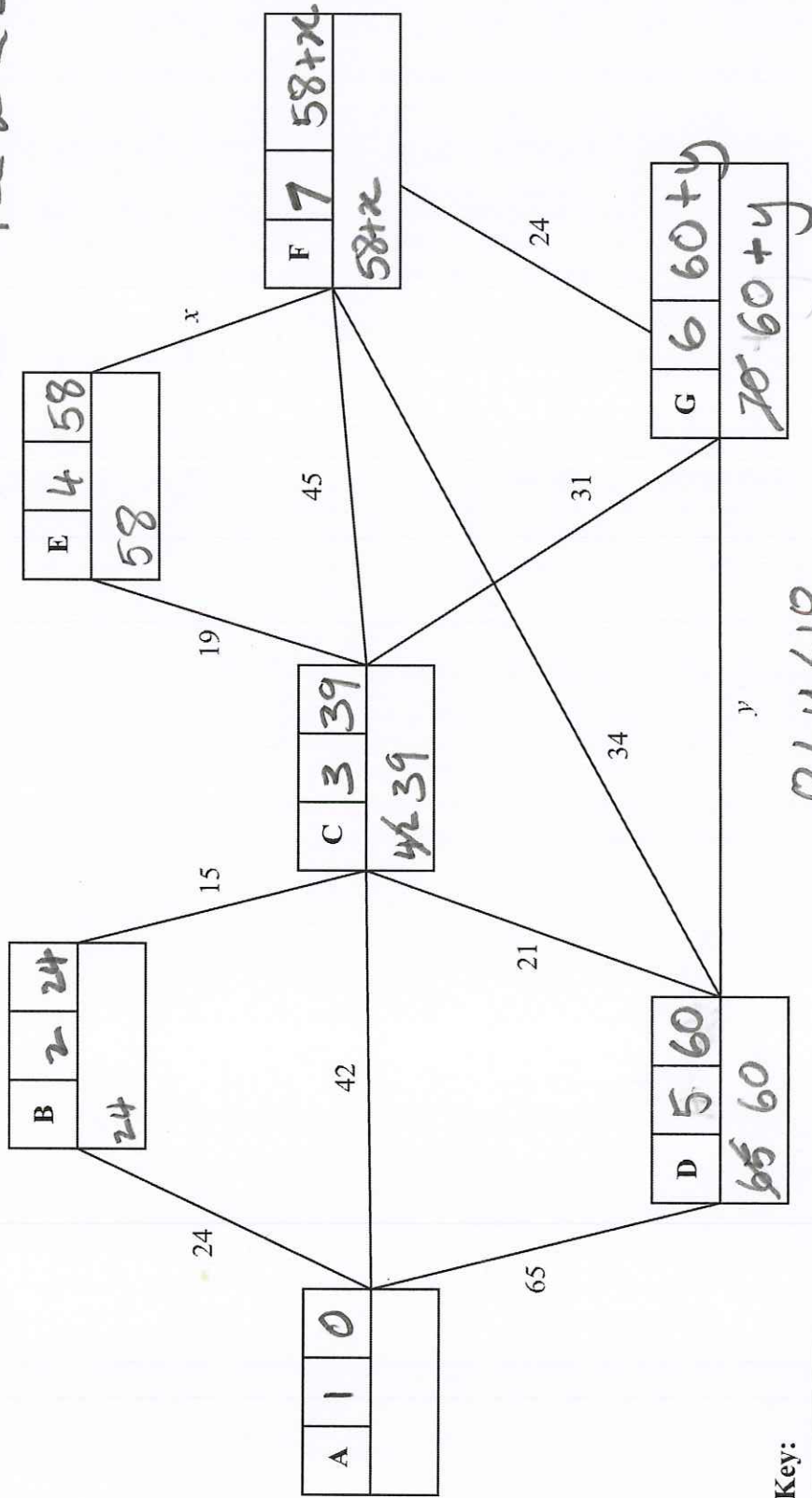
$$F \text{ to } G \text{ via } A = 58 + x + 60 + y = 140$$

$$\textcircled{2} \quad 22 = x + y$$

$$\begin{array}{r} - 29 = x + 2y \\ 22 = x + y \\ \hline 7 = y \end{array} \Rightarrow x = 15$$

6. (a)

122x < 226



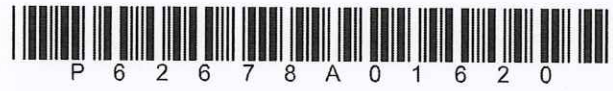
Key:

Vertex	Order of labelling	Final value
Working values		

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



12

7.

$$x + 2y + 3z \leq 45$$

$$3x + 2y \geq 9$$

$$-x + 4z \geq 4$$

$$P = 2x + y + 3z$$

Basic variable	$x$	$y$	$z$	$s_1$	$s_2$	$s_3$	$a_1$	$a_2$	Value
$s_1$	1	2	3	1	0	0	0	0	45
$a_1$	3	2	0	0	-1	0	1	0	9
$a_2$	-1	0	4	0	0	-1	0	1	4
$P$	-2	-1	-3	0	0	0	0	0	0
$A$	-2	-2	-4	0	1	1	0	0	13

Table 1

$$\begin{aligned} x + 2y + 3z + s_1 &= 45 \\ 3x + 2y - s_2 + a_1 &= 9 \\ -x + 4z - s_3 + a_2 &= 4 \end{aligned}$$

$$A = -(a_1 + a_2)$$

$$\begin{aligned} A &= -9 - s_2 + 2y + 3x \\ &\quad -4 - s_3 - x + 4z \end{aligned}$$

$$-13 = A - 2x - 2y - 4z + s_2 + s_3$$





13

(7) (i) As the value for the basic objective ~~feasible solution~~ of A is zero then there is a basic feasible solution

c(ii)  $x = 3$      $y = 0$      $z = 7/4$   
 $S_1 = 14\frac{1}{4}$      $S_2 = S_3 = 0$

(d)

Basic Variable	x	y	z	$S_1$	$S_2$	$S_3$	Value	Row Ops
$S_2$	0	$10/7$	0	$12/7$	1	$9/7$	63	$R_1 \div 7/12$
x	1	$8/7$	0	$4/7$	0	$3/7$	24	$R_2 + 1/3 R_1$
z	0	$2/7$	1	$1/7$	0	$-1/7$	7	$R_3 + 1/12 R_1$
P	0	$15/7$	0	$11/7$	0	$3/7$	69	$R_4 + 11/12 R_1$

e(i) As no negatives in objective row then this is an optimal solution

(ii) 69

(d)  $S_2 = 63$      $x = 24$      $z = 7$