

Questions

SGS Mathematics Faculty

UNIT 2: Algebra and Number

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Algebra: Simplifying, expanding brackets and factorising

1. (a) Factorise $10p - 4$
 $2(5p - 2)$ (1)

(b) Factorise $q^2 + 3q$
 $q(q + 3)$ (1)

(c) Factorise $r^2 - r$
 $r(r - 1)$ (1)

2. (a) Factorise $10a + 5$
 $5(2a + 1)$ (1)

(b) Factorise $c^2 - 4c$
 $c(c - 4)$ (2)

3. Factorise
 (a) $4x - 8$
 $4(x - 2)$ (1)

(b) $y^2 + 2y$
 $y(y + 2)$ (2)

4. (a) Simplify $5p + 2q - q + 2p$
 $7p + q$ (2)

(b) Multiply out $4(r - 3)$
 $4r - 12$ (1)

5. (a) Expand $3(y-4)$
 $3y - 12$ (1)
- (b) Simplify the expression $2c + 6d + 4c - 8c$
 $6d - 2c$ (2)
- (c) Factorise $x^2 + 5x$
 $x(x+5)$ (2)
6. Expand and simplify $4(3d-2e) - (2d-5e)$
 $12d - 8e - 2d + 5e = 10d - 3e$ (2)
7. Expand and simplify $5(2x+1) - 3(x-4)$
 $10x + 5 - 3x + 12 = 7x + 17$ (2)
8. (a) Simplify $2x + 3y + 5x - 2y - 4x$
 $3x + y$ (2)
- (b) Factorise $4c + 12$
 $4(c+3)$ (1)
- (c) Factorise $x^2 + 5x$
 $x(x+5)$ (2)
9. Expand and simplify
- (a) $5(2a-c) + 4(3a+2c)$
 $10a - 5c + 12a + 8c = 22a + 3c$ (2)
- (b) Expand and simplify $3(2x-1) + 2(3x+5)$
 $6x - 3 + 6x + 10 = 12x + 7$ (2)
- (c) Expand and simplify $(y+5)(y-1)$
 $y^2 - y + 5y - 5 = y^2 + 4y - 5$ (2)
- (d) Factorise $2xy - 6y^2$
 $2y(x - 3y)$ (2)

10. (a) Factorise $2x + 6$
 $2(x+3)$ (1)
- (b) Expand $3(4y+1)$
 $12y + 3$ (1)
- (c) Expand $4x(x^2+5)$
 $4x^3 + 20x$ (2)
11. Factorise fully $2x^2 - 50y^2$
 $2(x^2 - 25y^2)$
 $= 2(x-5y)(x+5y)$ (3)
12. Factorise $5x^2 + 20x$
 $5x(x+4)$ (1)
13. Factorise fully $6ab^2 - 2ab$
 $2ab(3b-1)$ (2)
14. Factorise fully $6a^2b + 9ab^2$
 $3ab(2a+3)$ (2)
15. Factorise completely $3x^2 - 6xy$
 $3x(x-2y)$ (2)
16. Factorise $r^6 - 3r^4$
 $r^4(r^2-3)$ (1)

Algebra: Solving Linear Equations

1. Solve the following equations

(a) $4(y-3) = 18$

$$4y - 12 = 18$$

$$4y = 30$$

$$y = \frac{30}{4} = 7.5$$

(3)

(b) $\frac{z+4}{2} = 11$

$$z + 4 = 22$$

$$z = 18$$

(2)

2. Solve the equation $2(x+5) = 7 - 4x$

$$2x + 10 = 7 - 4x$$

$$6x + 10 = 7$$

$$6x = -3$$

$$x = \frac{-3}{6} = -\frac{1}{2}$$

(3)

3. Solve the equation $4z + 8 = 3 - z$

$$5z + 8 = 3$$

$$5z = -5 \quad z = -1$$

(3)

4. Solve the equation $\frac{2t+5}{3} = 7$

$$2t + 5 = 21$$

$$2t = 16 \quad t = \frac{16}{2} = 8$$

(3)

5. Solve $\frac{x}{4} + 1 = 6$

$$\frac{x}{4} = 5$$

$$x = 4 \times 5$$

$$x = 20$$

(2)

6. Solve $\frac{4}{(y+1)} = 3$

$$4 = 3(y+1)$$

$$4 = 3y + 3$$

$$1 = 3y$$

$$y = \frac{1}{3}$$

(3)

7. Solve the equation $6y + 7 = 14y$

$$7 = 8y$$

$$y = \frac{7}{8}$$

(2)

8. Solve the equation $4(y-3) = 18$

$$4y - 12 = 18$$

$$y = \frac{30}{4} = 7.5$$

$$4y = 30$$

(3)

9. Solve the equation $7z - 3 = 6 + z$

$$7z = 9 + z$$

$$z = \frac{9}{6} = \frac{3}{2}$$

$$6z = 9$$

(3)

10. Solve the following equations

(a) $\frac{(z+4)}{2} = 11$

$$z + 4 = 22$$

$$z = 18$$

(2)

(b) $2x - 3 = 5x + 6$

$$2x = 5x + 9$$

$$x = \frac{9}{-3} = -3$$

$$-3x = 9$$

$$-3$$

(3)

11. Solve the equation $\frac{23-2x}{5} = 3$

$$23 - 2x = 15$$

$$x = \frac{8}{2} = 4$$

$$23 = 15 + 2x$$

$$8 = 2x$$

(3)

12. Solve the equation

$$9(x-1) = 5(x-2)$$

$$9x - 9 = 5x - 10$$

$$9x + 1 = 5x$$

$$1 = -4x$$

(3)

$$x = \frac{1}{-4} = -\frac{1}{4}$$

13. Solve the equation

$$\frac{x+1}{2} + \frac{x-3}{4} = 2$$

You must show all your working.

Multiply by 8

$$4(x+1) + 2(x-3) = 16$$

$$4x+4 + 2x-6 = 16$$

$$6x-2 = 16$$

$$6x = 18$$

$$x = \frac{18}{6} = 3$$

(4)

14. Solve the equation $\frac{y}{3} + 5 = 9$

$$\frac{y}{3} = 4$$

$$y = 3 \times 4 = 12$$

(2)

15. Solve the equation $4(z-1) = 2(z+3)$

$$4z-4 = 2z+6$$

$$4z = 2z+10$$

$$2z = 10 \quad z = \frac{10}{2} = 5$$

(3)

16. Solve the equation $2(3x-2) + 4(x+5) = 4(x-2)$

$$6x-4 + 4x+20 = 4x-8 \quad 6x = -24$$

$$10x+16 = 4x-8 \quad x = -4$$

$$6x+16 = -8$$

(3)

17. Solve the equation $3(2z-1) + 4(z+3) = 5(2z-1) + 4(3z-1)$

$$6z-3 + 4z+12 = 10z-5 + 12z-4$$

$$10z+9 = 22z-9$$

$$9 = 12z-9$$

$$18 = 12z$$

(3)

$$z = \frac{18}{12} = \frac{3}{2}$$

Algebra: Solving Simultaneous Linear Equations

1. Solve the simultaneous equations:

$$\begin{array}{r} 2x+3y=9 \quad \times 3 \\ 3x+2y=1 \quad \times 2 \end{array}$$

You must show all your working.
Do not use trial and improvement.

$$6x+9y=27$$

Subtract $6x+4y=2$

$$0x+5y=25 \quad y = \frac{25}{5} = 5$$

Find x $2x+3 \times 5 = 9$, $2x = -6$

Answer x = -3, y = 5

(4)

2. Solve the simultaneous equations

$$\begin{array}{r} 4x+3y=5 \quad \times 5 \\ 2x-5y=9 \quad \times 3 \end{array}$$

You must show your working.
Do not use trial and improvement.

Add $\begin{cases} 20x+15y=25 \\ 6x-15y=27 \end{cases}$

$$26x+0y=52 \quad x = \frac{52}{26} = 2$$

Find y $4 \times 2 + 3y = 5$

$$8+3y=5$$

$$3y = -3$$

Answer x = 2, y = -1

(4)

Algebra: Changing the Subject of a Formula

1. Make t the subject of the formula $w = 2t + v$

$$w - v = 2t$$

$$t = \frac{w - v}{2} \quad (2)$$

2. Make c the subject of the formula $E = mc^2$

$$\frac{E}{m} = c^2 \quad c = \pm \sqrt{\frac{E}{m}} \quad (2)$$

3. Make c the subject of the formula $d = \frac{c}{5} + e$

x 5

$$5d = c + 5e$$

$$c = 5d - 5e \quad (2)$$

4. Make x the subject of $w = \frac{x}{2} + 3$

x 2

$$2w = x + 6$$

$$x = 2w - 6 \quad \text{or} \quad x = 2(w - 3) \quad (2)$$

5. Make u the subject of the formula $s = \frac{1}{2}(u + v)t$

x 2

$$2s = (u + v)t$$

$$2s = ut + vt$$

$$2s - vt = ut$$

$$u = \frac{2s - vt}{t} \quad (3)$$

6. Make t the subject of the formula $w = \sqrt{t} - v$

$$(w + v) = \sqrt{t}$$

$$t = (w + v)^2 \quad (2)$$

7. Make r the subject of the formula $p = 3 + 2r$

$$p - 3 = 2r$$

$$r = \frac{p - 3}{2} \quad (2)$$

8. Make x the subject of the formula

$$3x + 2y = 8y - 3$$

Simplify your answer as much as possible.

$$(-2y) \quad 3x = 6y - 3$$

$$x = \frac{6y - 3}{3} = 2y - 1 \quad (3)$$

Number: Rules of Indices

1. Simplify

(a) $y^4 \times y^{-3}$

$y^{4-3} = y^1$

(1)

(b) $y^4 \div y^5$

$y^{4-5} = y^{-1}$ or $\frac{1}{y}$

(1)

2. Simplify

(a) $c \times c \times c \times c$

c^4

(1)

(b) $d^3 \times d^2$

$d^{3+2} = d^5$

(1)

(c) $\frac{e}{e^8}$

$e^{1-8} = e^{-7}$ or $\frac{1}{e^7}$

(1)

(d) $(2g^2h^4) \times (3g^3h)$

$6g^5h^5$

(2)

3. (a) Simplify

(i) $y^7 \times y^2$

y^9

(1)

(ii) $y^7 \div y^2$

y^5

(1)

(iii) $(y^7)^2$

$y^7 \times y^7 = y^{14}$

(1)

4. Simplify $(3xy^2)^4$

$3xy^2 \times 3xy^2 \times 3xy^2 \times 3xy^2$

$81x^4y^8$

(2)

5. Simplify

(i) $w^2 \times w^6$

$w^{2+6} = w^8$

(1)

(ii) $w^{10} \div w^4$

$w^{10-4} = w^6$

(1)

(iii) $(w^4)^3$

$w^4 \times w^4 \times w^4 = w^{12}$

(1)

6. If $x = 3^p$ and $y = 3^q$

Express in terms of x and/or y

(i) 3^{p-q}

$\frac{3^p}{3^q} = \frac{x}{y}$

(1)

(ii) 3^{2p}

$(3^p)^2 = x^2$

(1)

(iii) 3^{q+2}

$3^q \times 3^2 = 9x$

(1)

7. (a) Simplify $(2x^4y)^3$

$2x^4y \times 2x^4y \times 2x^4y$

$8x^{12}y^3$

(2)

8. (a) Simplify $x^3 \times x^5$

x^8

(1)

(b) Simplify $y^{12} \div y^4$

y^8

(1)

(c) Simplify $(3wt^2)^3$

$3wt^2 \times 3wt^2 \times 3wt^2$

$27w^3t^6$

(2)

11. (a) Work out the value of $5^7 \div 5^4$

$$5^{7-4} = 5^3$$

(2)

Number: Standard Form

1. A builder has 7200 kg of sand.

- (a) Write 7200 kg in grams.
Give your answer in standard form.

$$7.2 \times 10^3$$

(2)

- (b) One grain of this sand weighs 0.0006 g.
Write the weight of one grain of sand in standard form.

$$6 \times 10^{-4}$$

(1)

- (c) How many grains of sand are there in 7200 kg of sand?
Give your answer in standard form.

$$(7.2 \times 10^3) \div (6 \times 10^{-4})$$
$$= 1.2 \times 10^7$$

(2)

2. (a) Write 7 billion as a number in standard form.

1 billion = 1000 million

$$= 1000 \times 10^6 = 10^3 \times 10^6 = 10^9$$

(1)

- (b) Write the number 4.5×10^{-3} as an ordinary number.

$$0.0045$$

(1)

- (c) Find the value of $(2.7 \times 10^3) \div (3.375 \times 10^5)$
Give your answer in standard form.

$$(2.7 \times 10^3) \div (3.375 \times 10^5)$$

(2)

$$2.7 \div 3.375 = 0.8$$

$$0.8 \times 10^{-2}$$

$$8.0 \times 10^{-3}$$

3. Here are six numbers written in standard form.

2.6×10^5 1.75×10^6 5.84×10^{11} 8.2×10^{-3} 3.5×10^{-1} 4.9×10^{-2}

(a) Write down the largest number.

Answer 5.84×10^{11} (1)

(b) Write down the smallest number.

Answer 8.2×10^{-3} (1)

(c) Write 4.9×10^{-2} as an ordinary number.

Answer 0.049 (1)

(d) Work out $2.6 \times 10^5 + 0.1$
Give your answer in standard form.

Answer 2.6×10^6 (1)

4. (a) Work out $4 \times 10^7 \times 3 \times 10^4$

Give your answer in standard form.

12×10^{11}
 $= 1.2 \times 10 \times 10^{11} = 1.2 \times 10^{12}$ (2)

(b) Work out $\frac{4 \times 10^9}{8 \times 10^3}$

Give your answer in standard form.

$0.5 \times 10^4 = 5 \times 10^3$ (3)

5. Work out $(3 \times 10^2) \times (4 \times 10^5)$

Give your answer in standard form.

$12 \times 10^7 = 1.2 \times 10 \times 10^7$
 $= 1.2 \times 10^8$ (2)

6. (a) Work out $(3 \times 10^2) \times (4 \times 10^5)$

Give your answer in standard form.

12×10^7
 $= 1.2 \times 10^8$ (2)

(b) Work out $(3 \times 10^2) \div (4 \times 10^5)$

Give your answer in standard form.

0.75×10^{-3}
 $= 7.5 \times 10^{-4}$ (2)

7. Some large numbers are written below.

1 million = 10^6

1 billion = 10^9

1 trillion = 10^{12}

(a) How many millions are there in one trillion?

10^6 (1)

(b) Write 8 billion in standard form.

8×10^9 (1)

(c) Work out 8 billion multiplied by 3 trillion.
Give your answer in standard form.

$(8 \times 10^9) \times (3 \times 10^{12})$
 $= 24 \times 10^{21} = 2.4 \times 10^{22}$ (2)

8. (a) Work out $4 \times 10^8 \times 5 \times 10^{-6}$

Give your answer in standard form.

$20 \times 10^2 = 2 \times 10^3$ (2)

(b) Work out

$\frac{4 \times 10^8}{5 \times 10^{-6}}$

Give your answer in standard form.

0.8×10^{14}
 $= 8 \times 10^{13}$ (2)

9. Add together

$$5.1 \times 10^7 \text{ and } 3.89 \times 10^6$$

$$= (51 \times 10^6) + (3.89 \times 10^6)$$

$$= 54.89 \times 10^6$$

$$= 5.489 \times 10^7$$

10. (a) Write these numbers in standard form

(i) 9 170 000

$$9.17 \times 10^6$$

(ii) 0.000 048

$$4.8 \times 10^{-5}$$

(b) Find the value of $(1.8 \times 10^{12}) \div (2 \times 10^9)$

$$0.9 \times 10^4$$

$$= 9 \times 10^3$$

Algebra: Factorising and Solving Quadratic Equations

1. (a) Factorise $p^2 + 7p + 12$

$$(p+3)(p+4)$$

(b) Solve the equation $p^2 + 7p + 12 = 0$

$$(p+3) = 0 \quad \text{or} \quad (p+4) = 0 \quad p = -3$$

$$p = -4$$

2. Solve the equation $y^2 - 4y - 45 = 0$

$$(y+5)(y-9) = 0$$

$$y+5=0, \quad y=-5$$

$$y-9=0, \quad y=9$$

3. (a) Factorise $x^2 + 6x - 16$

$$(x-2)(x+8)$$

(b) Hence solve the equation $x^2 + 6x - 16 = 0$

$$(x-2) = 0, \quad x = 2$$

$$(x+8) = 0, \quad x = -8$$

4. Solve the equation $z^2 - 9z + 8 = 0$

$$(z-1)(z-8) = 0$$

$$(z-1) = 0 \quad z = 1 \quad \text{or} \quad z = 8$$

$$(z-8) = 0$$

5. Solve the equation $y^2 + 5y = 0$

$$y(y+5) = 0 \quad y = 0$$

$$\text{or } y = -5$$

6. (a) (i) Factorise $x^2 - 7x - 8$

$$(x-8)(x+1)$$

(ii) Hence solve the equation $x^2 - 7x - 8 = 0$

$$x-8=0 \quad \text{or} \quad x+1=0$$

$$x=8$$

$$x=-1$$

9. (a) Expand and simplify $(x+y)(x-y)$

$$\begin{aligned} & x^2 - xy + yx - y^2 \\ & = x^2 - y^2 \end{aligned}$$

(2)

- (b) (i) Factorise $x^2 - 13x + 36$

$$(x-4)(x-9)$$

(2)

- (ii) Hence, or otherwise, solve the equation $x^2 - 13x + 36 = 0$

$$\begin{aligned} \text{So } (x-4) &= 0 & x &= 4 \\ (x-9) &= 0 & x &= 9 \end{aligned}$$

(1)

10. (a) Factorise $x^2 + 5x - 14$

$$(x+7)(x-2)$$

(2)

- (b) Hence solve the equation $x^2 + 5x - 14 = 0$

$$\begin{aligned} (x+7) &= 0 & x &= -7 \\ (x-2) &= 0 & x &= 2 \end{aligned}$$

(1)

11. (i) Factorise $y^2 - 8y + 15$

$$(y-3)(y-5)$$

(2)

- (ii) Hence solve the equation $y^2 - 8y + 15 = 0$

$$\begin{aligned} \text{So } (y-3) &= 0 & y &= 3 \\ \text{or } (y-5) &= 0 & y &= 5 \end{aligned}$$

(1)

12. (a) Factorise $x^2 + 3x - 40$

$$(x+8)(x-5)$$

(2)

- (b) Hence, solve the equation $x^2 + 3x - 40 = 0$

$$\begin{aligned} \text{So } x+8 &= 0 & x &= -8 \\ \text{or } x-5 &= 0 & x &= 5 \end{aligned}$$

(1)

13. (a) Factorise $y^2 - 5y + 6$

$$(y-3)(y-2)$$

(2)

- (b) Hence solve the equation $y^2 - 5y + 6 = 0$

$$\begin{aligned} \text{So } (y-3) &= 0 & y &= 3 \\ (y-2) &= 0 & y &= 2 \end{aligned}$$

(1)

14. Mandy is x years old.
Her brother is 5 years older than Mandy.
The product of their ages is 84.

- (a) Show that $x^2 + 5x - 84 = 0$

$$\begin{aligned} \text{Mandy} &= x & x \times (x+5) &= 84 \\ \text{Brother} &= (x+5) & x^2 + 5x &= 84 \end{aligned}$$

(1)

- (b) Solve $x^2 + 5x - 84 = 0$

$$x^2 + 5x - 84 = 0$$

Do not use a trial and improvement method.

$$\begin{aligned} (x-7)(x+12) &= 0 & \text{Mandy} &= 7 \\ x &= 7, & x &= -12 \text{ (cannot have} \\ & & & \text{negative)} \end{aligned}$$

(3)

15. (a) (i) Factorise $x^2 - 10x + 25$

$$(x-5)(x-5)$$

(2)

- (ii) Hence, or otherwise, solve the equation

$$(y-3)^2 - 10(y-3) + 25 = 0$$

$$\text{From above } x = 5$$

$$\begin{aligned} y-3 &= x \\ y-3 &= 5 & y &= 8 \end{aligned}$$

(2)

Algebra: Inequalities and Regions

1. Solve the inequality

$$5x + 3 > 10$$

$$5x > 7$$

$$x > 7/5 \quad (\text{or } 1.4) \quad (2)$$

2. (a) x is an integer.

$$0 < x \leq 3$$

Write down all the possible values of x .

$$1, 2, 3 \quad (2)$$

(b) x and y are integers.

$$0 < x \leq 3 \quad 1, 2, 3$$

$$y < x$$

$$x + y < 5$$

Write down two pairs of values of x and y which satisfy all three inequalities.

$$x = 1, \text{ then } y: 0, -1, -2, -3, \dots$$

$$x = 2, \text{ then } y: 1, 0, -1, -2, -3, \dots$$

$$x = 3, \text{ then } y: 2, 1, 0, -1, -2, -3, \dots$$

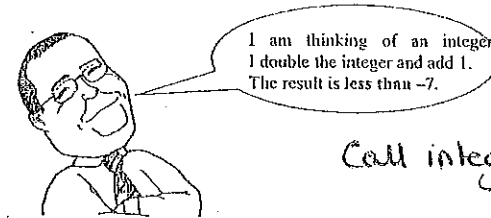
Answer (1, 0) and (2, 1) (2)

3. (a) Solve the inequality $3x + 7 \geq 13$

$$3x \geq 6$$

$$x \geq 2 \quad (2)$$

(b) A mathematics teacher says



Call integer x .

What is the largest integer the teacher could have thought of?

$$2x + 1 < -7$$

$$2x < -8$$

$$x < -4 \quad (2)$$

4. (a) Solve the inequality $3x + 5 \leq 16$

$$3x \leq 11 \quad x \leq \frac{11}{3} \quad (2)$$

(b) Write down the integer value satisfied by the inequality $5 < 2x < 7$

$$2.5 < x < 3.5$$

$$x = 3 \quad (2)$$

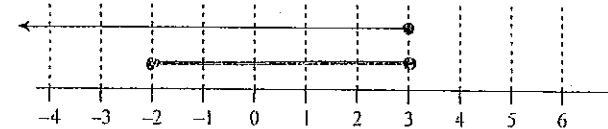
5. (a) Solve the inequality $3(x - 2) \leq 9$

$$3x - 6 \leq 9$$

$$3x \leq 15$$

$$x \leq 5 \quad (3)$$

(b) The inequality $x \leq 3$ is shown on the number line below.



Draw another inequality on the number line so that only the following integers satisfy both inequalities $\{-2, -1, 0, 1, 2, 3\}$

6. (a) List all the solutions of the inequality

$$4 < 2n \leq 11$$

where n is an integer. $(\div 2)$ $2 < n \leq 5.5$
 $3, 4, 5$

(2)

- (b) Solve the inequality

$$4x + 1 < 7$$

$$4x < 6 \quad x < \frac{3}{2}$$

(2)

- (c) Show that, for any value of n ,

$$n^2 + 2n + 1 > n^2 + n$$

$$(n+1)^2 > n(n+2)$$

$$(n+1)(n+1) = n^2 + 2n + 1$$

always one greater.

7. (a) List the integer values of x such that

$$n(n+2) = n^2 + 2n$$

$$-2 \leq x < 3$$

$-2, -1, 0, 1, 2$

(2)

- (b) Solve the inequality

$$x^2 > 64$$

$x > 8$ or $x < -8$

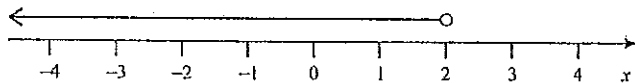
(2)

8. (a) Solve the inequality $2x + 3 \geq 1$

$$2x \geq -2 \quad x \geq -1$$

(2)

- (b) Write down the inequality shown by the following diagram.



$$x < 2$$

(1)

- (c) Write down all the integers that satisfy both inequalities shown in parts (a) and (b).

$-1, 0, 1$

(1)

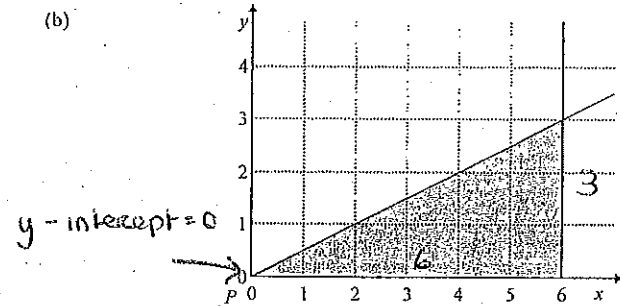
9. (a) List the integer values of n such that $3 \leq 3n < 18$

$$1 \leq n < 6$$

$(\div 3)$
 $1, 2, 3, 4, 5$

(3)

- (b)



$$\text{gradient} = \frac{3}{6} = \frac{1}{2}$$

- (i) Find the equation of the line PO .

$$y = \frac{1}{2}x$$

(1)

- (ii) Write down three inequalities which together describe the shaded area.

$$y \leq \frac{1}{2}x, \quad y \geq 0, \quad x \leq 6$$

(3)