SOLUTIONS.

Sale Grammar School

Mathematics Faculty

UNIT 3 Exam Booster

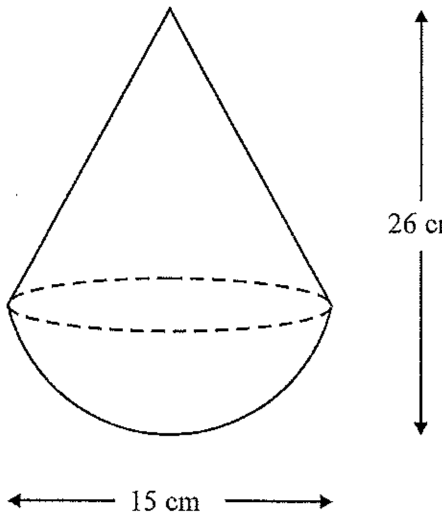
Geometry and Algebra

*Moving from A to A**

Contents **Page**

Volumes of Solids

1. A child's toy is in the shape of a cone on top of a hemisphere.
The diameter of the hemisphere is 15 cm and the overall height of the toy is 26 cm.



Not to scale

Cone $V = \frac{1}{3}\pi r^2 h$

Sphere $V = \frac{4}{3}\pi r^3$

Calculate the volume of this toy.

Full Sphere $(r = 7.5)$ $V = \frac{4}{3} \times \pi \times (7.5)^3 = 1767.1\text{cm}^3$

Cone $(r = 7.5, h = 18.5)$ $V = \frac{1}{3} \times \pi \times (7.5)^2 \times 18.5 = 1089.7\text{cm}^3$

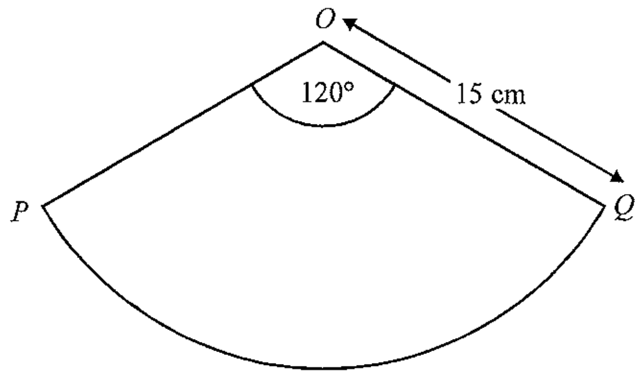
Total $V = (1767.1 \div 2) + 1089.7 \sim 1973\text{cm}^3$

Answer ....1973cm^3. .... cm^3

(5 marks)

2. *OQP* is a sector of a circle of radius 15 cm.

The angle of the sector is 120°.



Not drawn accurately

Full $C = 2\pi r$

Full $C = 2 \times \pi \times 15$

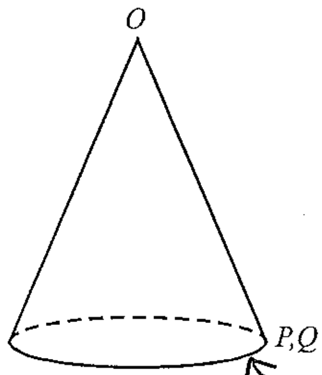
$= 30\pi$ cm.

(a) Show that the length of the arc *PQ* is 10π cm.

$\frac{120}{360} \times 30\pi = \underline{\underline{10\pi \text{ cm}}}$

(2 marks)

The sector is folded to form a cone.



Not to scale

arc PQ.

(b) Calculate the radius of the base of the cone.

Arc PQ = Circumference of cone

$2\cancel{\pi}r = 10\cancel{\pi}$

$\underline{\underline{r = 5\text{cm}}}$

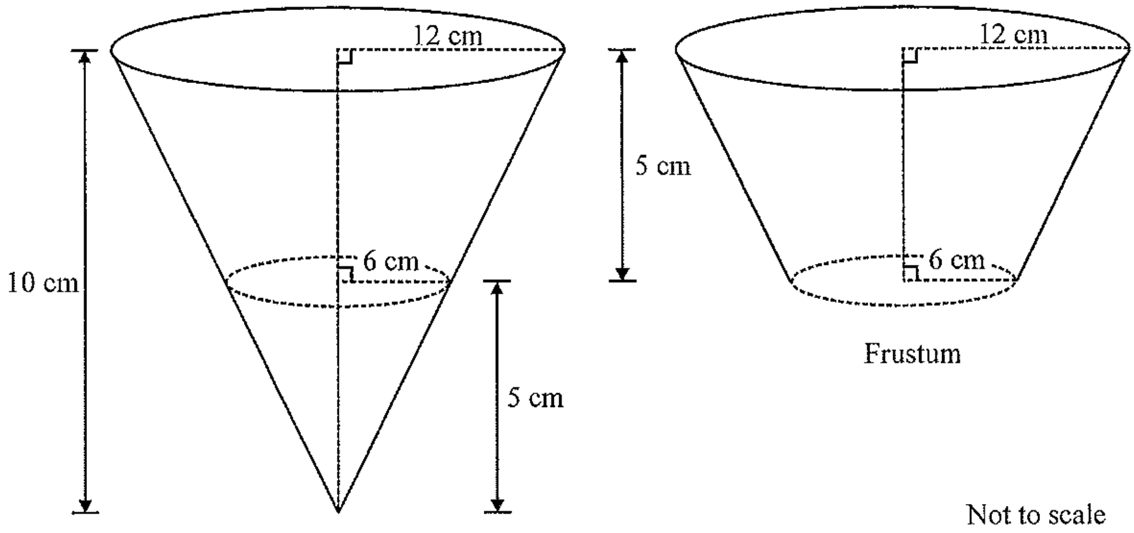
Answer ....5cm.......... cm

(2 marks)

3. The first diagram shows a cone of base radius 12 cm and perpendicular height 10 cm.

A small cone of base radius 6 cm and perpendicular height 5 cm is cut off the bottom to leave a frustum.

The frustum has a lower radius of 6 cm, an upper radius of 12 cm and a perpendicular height of 5 cm (see second diagram).



Find the volume of the frustum, giving your answer in terms of π.

Volume of Large $= \frac{1}{3}\pi r^2 h$

$(r = 12, h = 10)$ $= \frac{1}{3} \times \pi \times 12^2 \times 10$

$= 480\pi \text{ cm}^3.$

Volume of Small $= \frac{1}{3} \times \pi \times 6^2 \times 5$

$(r = 6, h = 5)$ $= 60\pi$

Answer 420π cm³

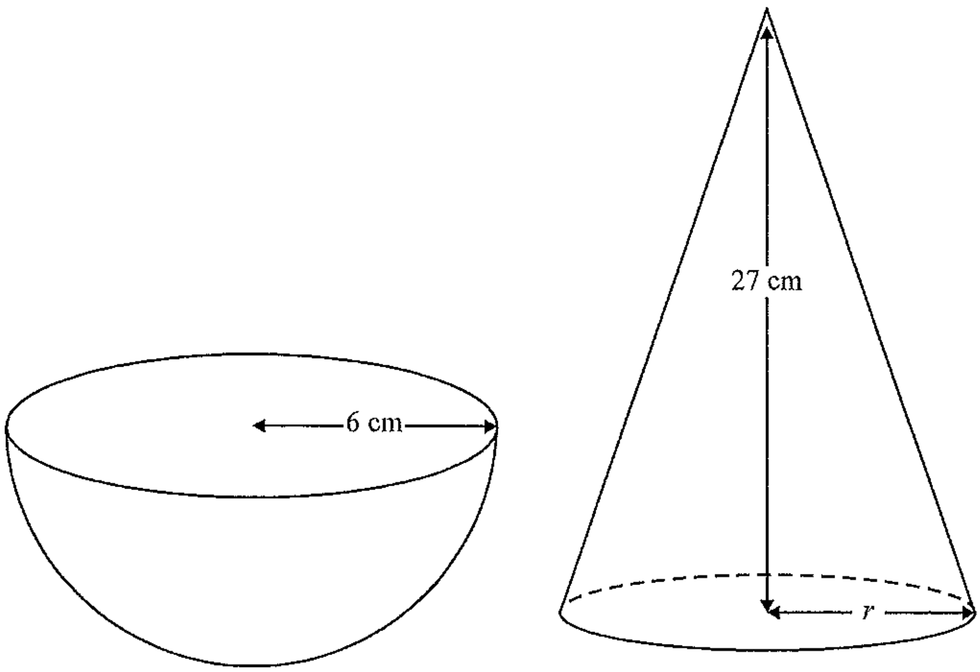
(4 marks)

Frustum V $= 480\pi - 60\pi$

$= \underline{420\pi \text{ cm}^3}$

4. A hemispherical bowl of radius 6 cm has the same volume as a cone of perpendicular height 27 cm.

Not drawn accurately



Calculate the base radius, *r*, of the cone.

Volume of hemisphere $= \dfrac{\frac{4}{3}\pi r^3}{2} = \left(\frac{4}{3} \times \pi \times 6^3\right) \div 2$

$= 144\pi$

Vol of Cone $= \frac{1}{3}\pi r^2 h \quad (r = ?\ ,\ h = 27)$

Answer .... 4 .... cm

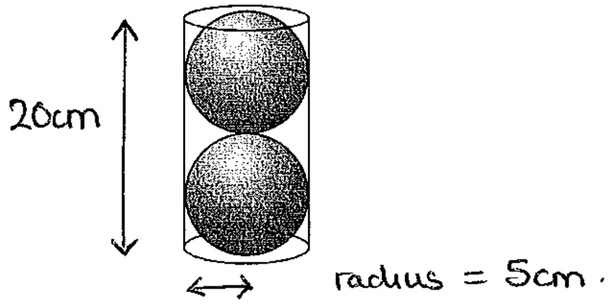
(4 marks)

$144\cancel{\pi} = \frac{1}{3}\cancel{\pi} r^2 \times 27$

$9r^2 = 144$

$r = \sqrt{\frac{144}{9}} \qquad \underline{\underline{r = 4\text{cm.}}}$

5. Two spheres of radius 5 cm just fit inside a tube.



Calculate the volume inside the tube not filled by the spheres.

Volume of Cylinder $= \pi r^2 h$ (prism)

$= \pi \times 5^2 \times 20$

$= 500\pi$

Volume of Sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 5^3 = \frac{500}{3}\pi$

Answer ....524...............cm^2

(5 marks)

Empty Space $= 500\pi - \left(2 \times \frac{500}{3}\pi\right)$

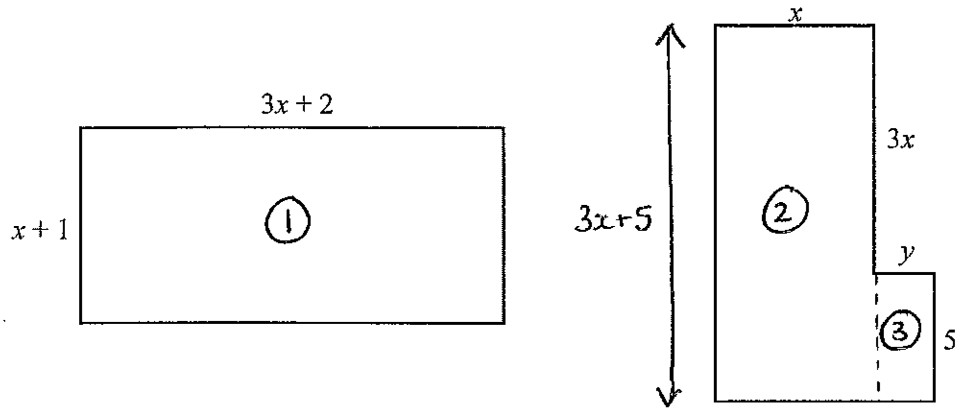
$= \frac{500}{3}\pi$

$\sim 524\text{cm}^3$

Geometry Problems involving Quadratic Equations

1. The diagrams show a rectangle and an L shape
All the angles are right angles.
All lengths are in centimetres.
The shapes are equal in area.

Diagrams not to scale



Calculate the value of y.

$$\text{Area } ① = (3x+2)(x+1) = 3x^2 + 5x + 2$$

$$\text{Area } ② = x(3x+5) = 3x^2 + 5x.$$

$$\text{Area } ③ = 5y$$

$$\therefore \quad \text{Area } ① = \text{Area } ② + \text{Area } ③$$

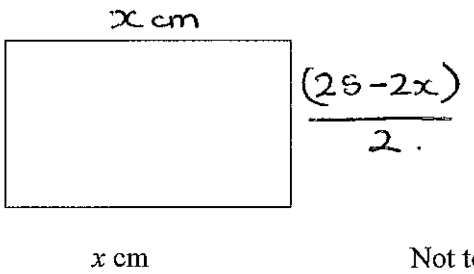
$$\cancel{3x^2} + \cancel{5x} + 2 = \cancel{3x^2} + \cancel{5x} + 5y$$

Answer .... $2/5$ .... cm

(6 marks)

$$5y = 2, \quad y = 2/5$$

2. The perimeter of a rectangle is 25 cm.
The length of the rectangle is x cm.



Not to scale

(a) Write down an expression for the width of the rectangle in terms of x.

Answer $(25-2x)/2$ cm

(1 mark)

(b) The area of the rectangle is 38 cm^2.
Show that $2x^2 - 25x + 76 = 0$

$$\text{Area} = x \times \frac{(25-2x)}{2} = 38$$

(×2)

$$x(25-2x) = 76$$

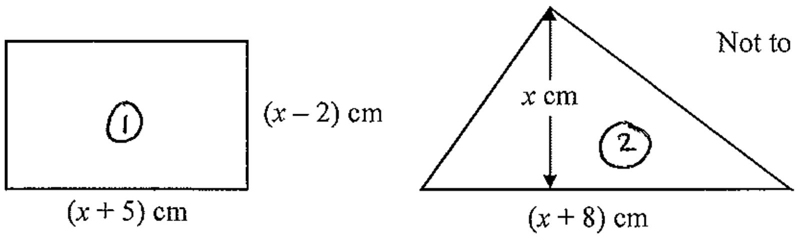
$$25x - 2x^2 = 76$$

move to RHS

$$2x^2 - 25x + 76 = 0.$$

(2 marks)

3. A rectangle has length $(x + 5)$ cm and width $(x - 2)$ cm.
A triangle has base $(x + 8)$ cm and height x cm.



Not to scale

The area of the rectangle is equal to the area of the triangle.

Show that $x^2 - 2x - 20 = 0$ (You are **not** required to solve this equation.)

$$\text{Area } 1 = \text{length} \times \text{width}$$

$$= (x+5)(x-2)$$

$$= x^2 + 3x - 10$$

$$\text{Area } 2 = \frac{\text{base} \times \text{height}}{2}$$

$$= \frac{(x+8)x}{2}$$

Equal areas

$$x^2 + 3x - 10 = \frac{x(x+8)}{2}$$

(4 marks)

(×2)

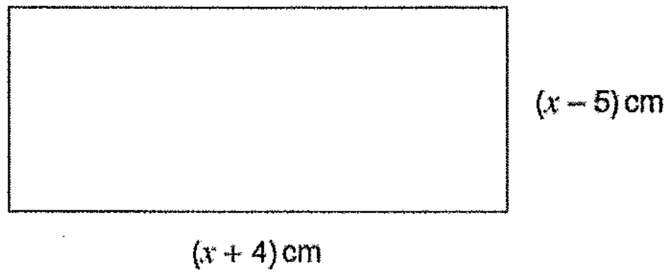
$$2x^2 + 6x - 20 = x^2 + 8x$$

(−x^2)
(−$8x$)

$$x^2 - 2x - 20 = 0.$$

4.

The diagram shows a rectangle.



The area of the rectangle is $90\,\text{cm}^2$.

Set up and solve a quadratic equation to work out the value of x.

Area = length × width

$90 = (x+4) \times (x-5)$

$90 = x^2 - x - 20$

$x^2 - x - 110 = 0$

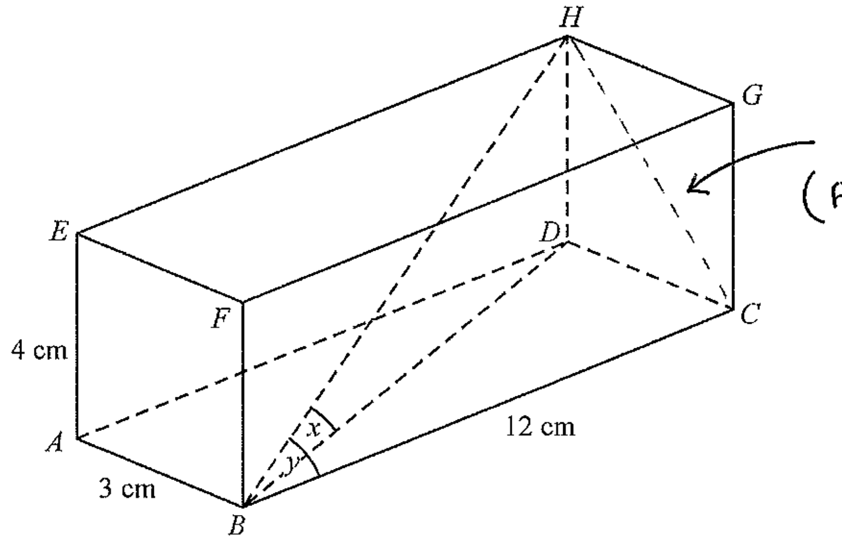
$(x-11)(x+10)$ $\quad x = 11$ or $x = \cancel{-10}$

would give negative length.

$x =$ 11 cm (5 marks)

3D Pythagoras and Trigonometry

1. The diagram shows a cuboid.
$AB = 3$ cm, $AE = 4$ cm, $BC = 12$ cm.



5cm
(from $3^2 + 4^2$)

Not drawn accurately

(a) Find the length of *BH*.

$BD^2 = BC^2 + CD^2$

$BD^2 = 12^2 + 3^2$

$BD^2 = 153$

$BD = 12.4$ (or $3\sqrt{17}$)

$BH^2 = BD^2 + DH^2$

$= (3\sqrt{17})^2 + 4^2$

$= 169$

$BH = \sqrt{169}$

Answer ....13.......... cm

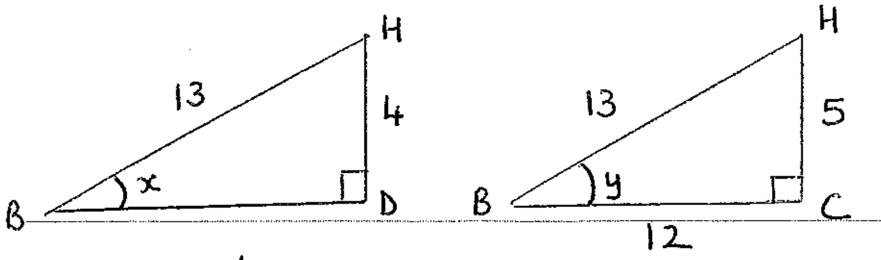
(2 marks)

(b) The angle between *BH* and *BD* is *x* and the angle between *BH* and *BC* is *y*.

Which angle is bigger, *x* or *y*?
You **must** show your working.

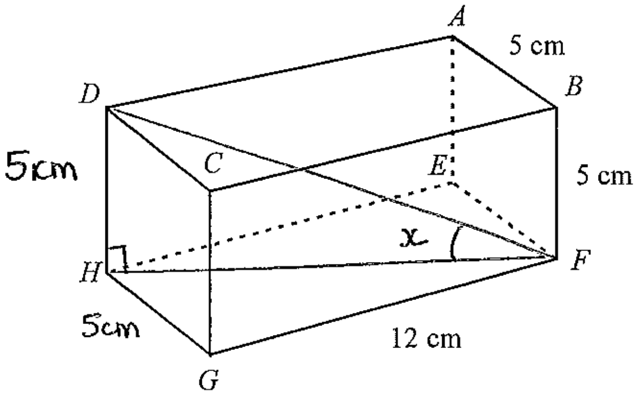
Answer ....y..........

(3 marks)



$\sin x = \frac{4}{13}$ ($x = 17.9°$) $\sin y = \frac{5}{12}$ ($y = 22.6°$)

2. *ABCDEFGH* is a cuboid with sides of 5 cm, 5 cm and 12 cm as shown.



Not to scale

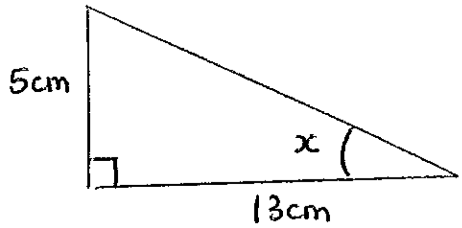
Calculate angle *DFH*. = angle x

$FH^2 = HG^2 + GF^2$

$FH^2 = 5^2 + 12^2$

$FH^2 = 169$

$FH = 13\text{cm}$

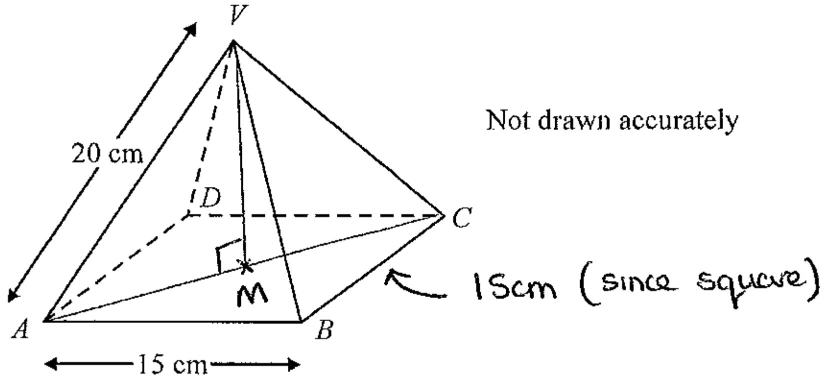


$\tan x = \dfrac{5}{13}$, $x = \tan^{-1}\left(\dfrac{5}{13}\right)$

Answer ....21.0°........ degrees

(5 marks)

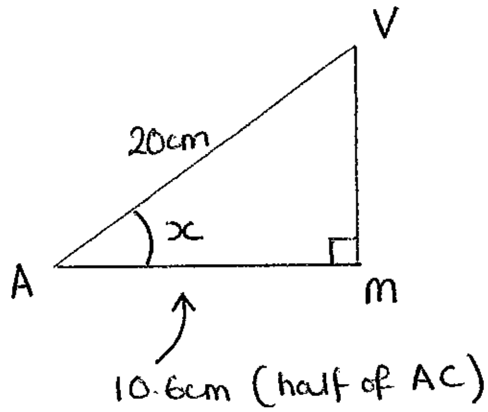
3. *VABCD* is a right pyramid on a square base.
V is vertically above the centre of the square.
$VA = VB = VC = VD = 20$ cm
$AB = 15$ cm



Not drawn accurately

Calculate the angle between the edge *VA* and the base *ABCD*.

$AC^2 = AB^2 + BC^2$
$= 15^2 + 15^2$
$AC = \sqrt{450}$
$= 21.2\text{cm}$



$\cos x = \frac{10.6}{20}$

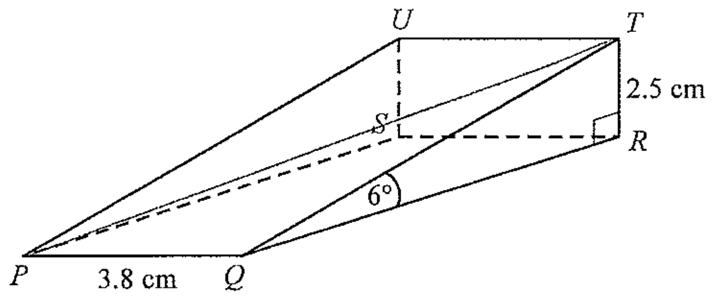
$x = \cos^{-1}\left(\frac{10.6}{20}\right)$

$= 57.97°$

Answer ......58°...... degrees

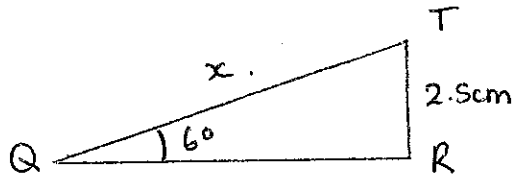
(5 marks)

4. The diagram shows a door-wedge with a rectangular horizontal base *PQRS*.
The sloping face *PQTU* is also rectangular.
$PQ = 3.8$ cm and angle $TQR = 6°$
The height *TR* is 2.5 cm.



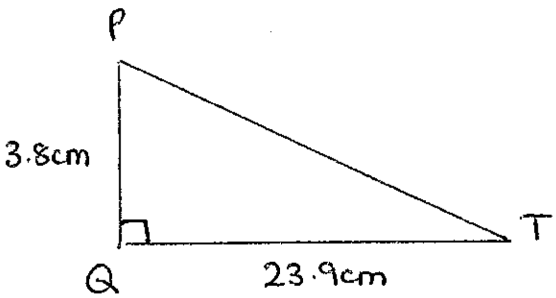
Not drawn accurately

Calculate the length of the diagonal *PT*.



$\sin 6 = \frac{2.5}{x}$

$x = \frac{2.5}{\sin 6}$ $\quad$ $\underline{x = 23.9\text{cm}.}$



$PT^2 = PQ^2 + QT^2$

$= (3.8)^2 + (23.9)^2$

$= 586.46$

$\underline{\underline{PT = 24.2\text{cm}.}}$

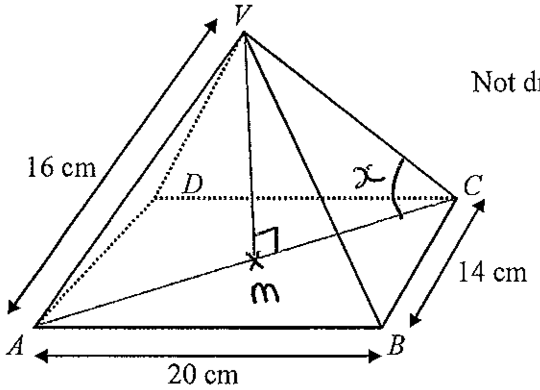
Answer ....24.2........ cm

(5 marks)

5. *VABCD* is a right pyramid on a rectangular base.

$VA = VB = VC = VD = 16$ cm.

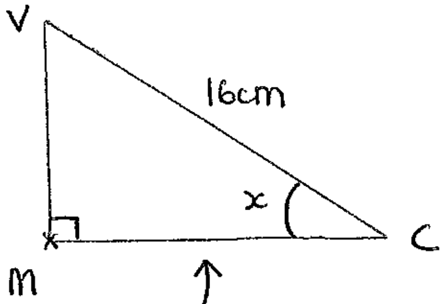
$AB = 20$ cm and $BC = 14$ cm.



Not drawn accurately

Calculate the angle between the edge *VC* and the base *ABCD*.

$$AC^2 = AB^2 + BC^2$$
$$= 20^2 + 14^2$$
$$AC^2 = 596$$
$$\underline{\underline{AC = 24.4\text{cm}}}$$



12.2cm (half of AC)

$\cos x = \frac{12.2}{16}$, $x = \cos^{-1}\left(\frac{12.2}{16}\right)$, $x = 40.3^\circ$

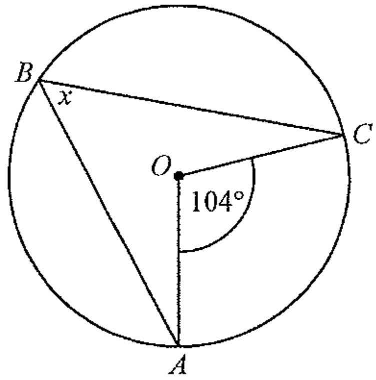
Answer ....40.3........ degrees

(5 marks)

Circle Theorems (Alternate Segment Theorem)

1. (a) O is the centre of the circle.
A, B and C are points on the circumference.

Write down the value of angle x.

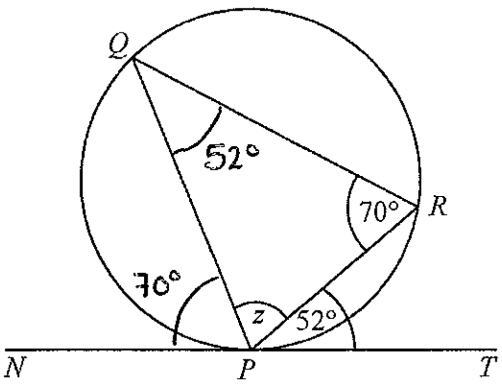


angle at centre is twice angle at circumference

Answer x = ...52°........degrees

(1 mark)

(b) P, Q and R are points on the circumference of the circle.
NPT is the tangent to the circle at P.



Not drawn accurately

Calculate the value of z.
Give a reason for each step of your working.

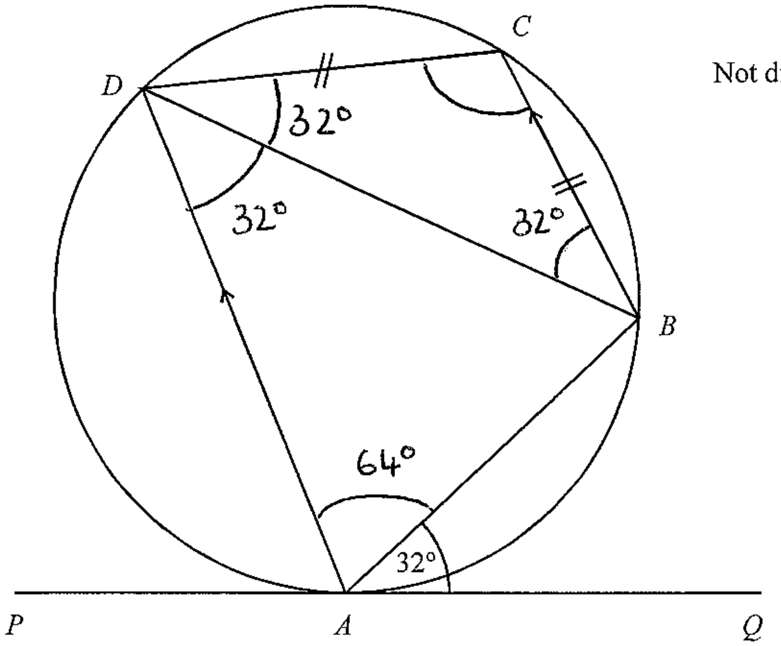
Angle PQR = 52° (alternate segment theorem)

$180 - (52+70) = 58°$

Answer ....58........degrees

(3 marks)

2. *ABCD* is a cyclic quadrilateral.
PAQ is a tangent to the circle at *A*.
$BC = CD$.
AD is parallel to *BC*.
Angle $BAQ = 32°$.



Not drawn accurately

Find the size of angle *BAD*.

You **must** show all your working.

Angle ADB = 32° (alternate segment)
Angle DBC = 32° (alternate angles)
Angle BDC = 32° (isosceles triangle)

Angle DCB = 180 − (2×32)
= 116°

∴ BAD = 64° (opposite angles in a cyclic quadrilateral sum to 180°)

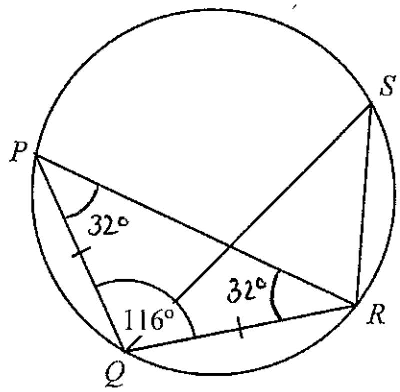
Answer Angle *BAD* = 64° degrees

(5 marks)

3. (a) Points P, Q, R and S lie on a circle.

$PQ = QR$

Angle $PQR = 116°$



Not drawn accurately

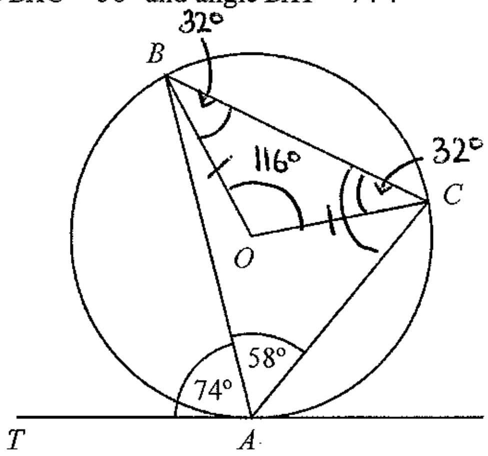
Angles QPR and QRP both 32° (isosceles triangle)

QSR = QPR since angles in the same segment.

Explain why angle $QSR = 32°$.

(2 marks)

(b) The diagram shows a circle, centre O.
TA is a tangent to the circle at A.
Angle $BAC = 58°$ and angle $BAT = 74°$.



Not drawn accurately

(i) Calculate angle BOC.

(angle at the centre is twice angle at circumference)

Answer Angle BOC = ...116°.......... degrees

(1 mark)

(ii) Calculate angle OCA.

Triangle BOC (isosceles) so angle OCB = 32°

Alternate segment theorem ACB = 74°

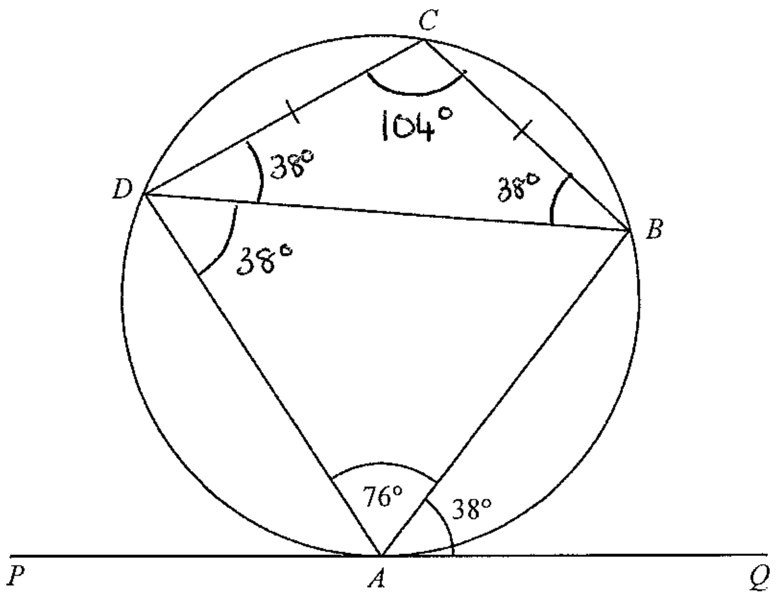
Angle OCA = 74 − 32 = 42°

Answer Angle OCA = ...42.......... degrees

(3 marks)

4. *ABCD* is a cyclic quadrilateral.
PAQ is a tangent to the circle at *A*.
BC = *CD*
Angle *QAB* = 38° and angle *BAD* = 76°

Not drawn accurately



Show that *AD* is parallel to *BC*.
Give reasons to justify any values you write down or calculate.

Angle ADB = 38° (alternate segment theorem)

Angle BCD = 104° (opposite angles in a cyclic quadrilateral sum to 180°)

Angle CBD = 38° (isosceles triangles)

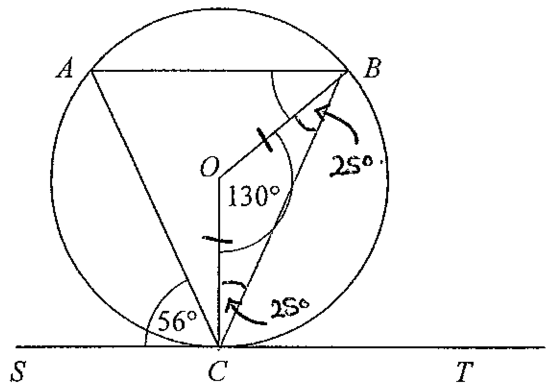
CBD = ADB (alternate angles)

So AD is parallel to BC.

(4 marks)

5. ABC are three points on the circumference of a circle centre O.
SCT is a tangent to the circle.
$\angle SCA = 56°$ $\angle COB = 130°$

Not drawn accurately



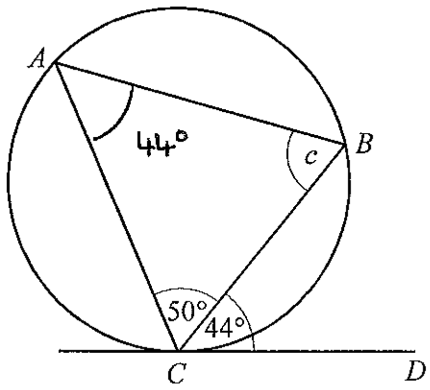
Find the size of angle OBA.

Triangle COB is isosceles (angles are 25°
Angle CBA is 56° (alternate segment theorem)

$OBA = 56 - 25$
$= 31°$

Answer Angle OBA = ...31.............. degrees
(3 marks)

6. CD is a tangent to the circle at C.



Not drawn accurately

Calculate the value of c.

Give reasons for your answer.

Angle CAB = 44° (alternate segment theorem)

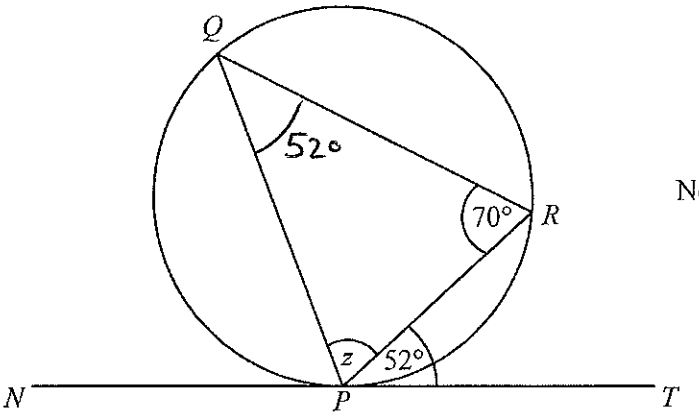
c = 180 − (50 + 44)

= 86°

Answer ....86°.......... degrees

(3 marks)

7. P, Q, and R are points on the circumference of the circle.
NPT is the tangent to the circle at P.



Not drawn accurately

Calculate the value of z.
Give a reason for each step of your working.

PQR = 52° (alternate segment theorem)

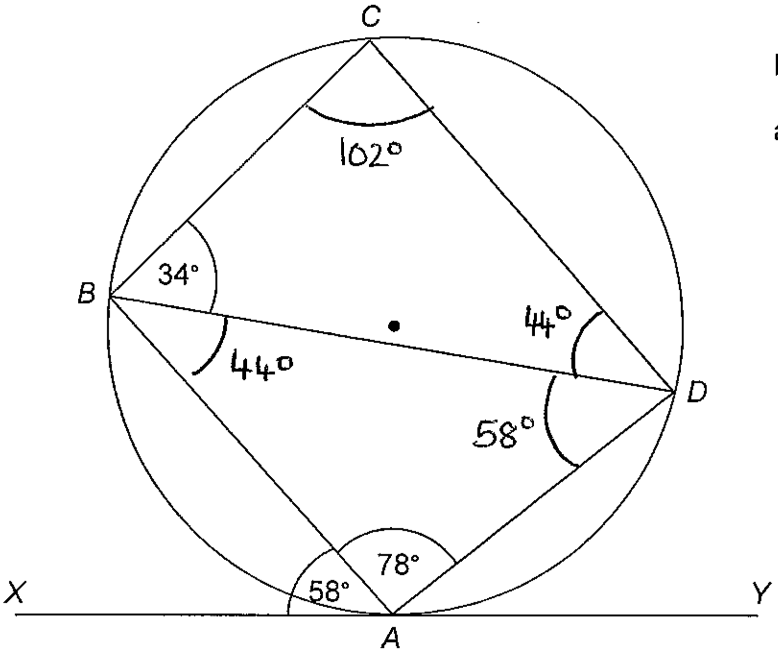
180 − (52 + 70)

Answer ....58°.......... degrees

(3 marks)

8. *ABCD* is a cyclic quadrilateral within a circle centre *O*.

XY is the tangent to the circle at *A*.
Angle *XAB* = 58°
Angle *BAD* = 78°
Angle *DBC* = 34°



Not drawn accurately

Prove that *AB* is parallel to *CD*.

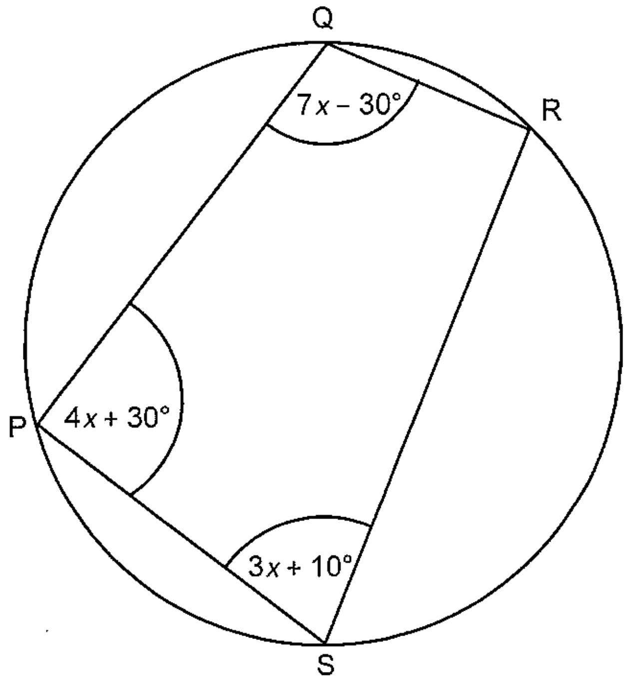
Angle BCD = 102° (opposite angles in a cyclic quadrilateral sum to 180°)
Angle CDB = 44° (angles in a triangle)
Angle ADB = 58° (alternate segment theorem)
Angle ABD = 44° (angles in a triangle)

ABD = CDB (alternate angles)

AB is parallel to CD.

(5 marks)

9. *PQRS* is a cyclic quadrilateral as shown.



Not drawn accurately

Prove that *PQ* is parallel to *SR*.

opposite angles in a cyclic quadrilateral sum to 180°

$(7x-30) + (3x+10) = 180°$

$10x - 20 = 180$

$10x = 200$

$x = 20°$

$P = 110°$

$Q = 110°$

$S = 70°$

$R = 70°$

$P + S = 180°$

interior (allied) angles $= 180$

$\therefore$ PQ parallel to SR.

(5 marks)

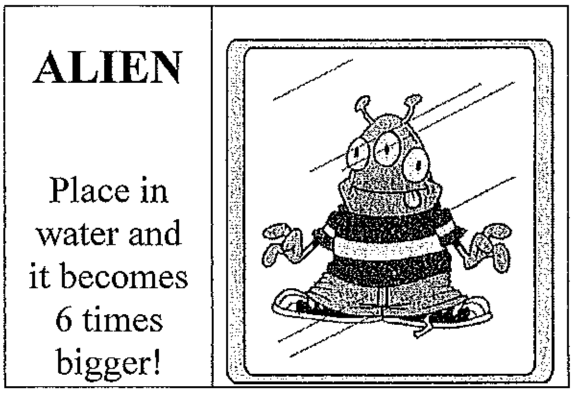
Areas and Volumes of Similar Shapes

1. (a) Explain why the volume of a cube increases by a factor of 8 when the side length is doubled.

Volume scale factor $= 2^3 = 8$.

(2 marks)

(b) June recently bought a small toy in the local shop.



Scale factor $= 14.5 \div 8$
$= 1.8125$.

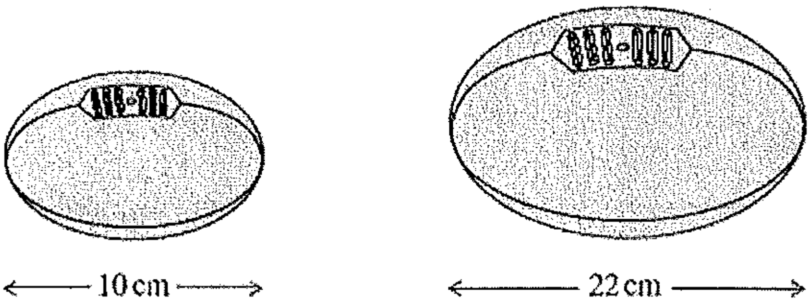
Volume scale factor
$= (1.8125)^3$
~ 5.95

So volume increases by about 6 so claim justified.

It was originally 8 cm tall.
After she placed it in water it grew to a similarly shaped alien.
The height was then 14.5 cm.
Is the claim on the pack justified?

(3 marks)

2. A child's rugby ball is 10 cm long and has a volume of 200 cm^3.
It is similar in shape to a full-size rugby ball.
A full-size rugby ball is 22 cm long.



Find the volume of the full-size ball.

Answer ....2129.6........ cm^3

Scale factor $= \frac{22}{10} = 2.2$

(2 marks)

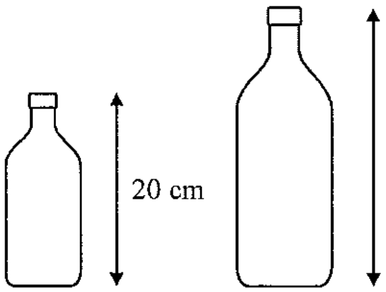
Volume scale factor $= (2.2)^3$

$\therefore$ Volume of full ball $= 200 \times (2.2)^3$
$= 2129.6$

3. Two similar bottles are shown below.

The smaller bottle is 20 cm tall and holds 480 ml of water.

The larger bottle is 30 cm tall.



Not to scale

Scale factor $= \frac{30}{20} = 1.5$

Volume scale factor $= (1.5)^3$

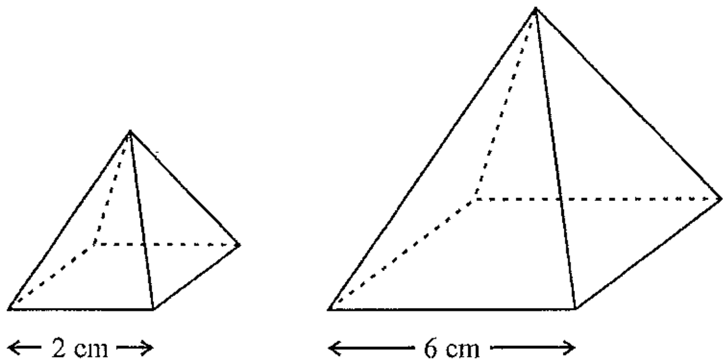
How much water does the larger bottle hold?

Volume of large $= 480 \times (1.5)^3$

Answer ......1620...... ml

(2 marks)

4. A square-based pyramid with a base of side 2 cm has a volume of 2.75 cm^3.



Scale factor $= \frac{6}{2} = 3$

Volume Scale Factor $= (3)^3$

Not to scale

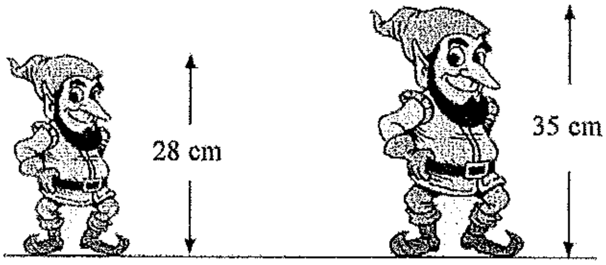
What is the volume of a similar square-based pyramid with a base of side 6 cm?

Volume of large $= 2.75 \times (3)^3$

Answer ......74.25...... cm^3

(2 marks)

5. Gnomes 'R'Us makes garden gnomes in two sizes.
The gnomes are similar in shape.
The smaller gnome is 28 cm high and the larger one is 35 cm high.



It takes 7936 cm^3 of plaster to make a small gnome.

How much plaster is needed to make a large gnome?

Scale factor $= \frac{35}{28} = \frac{5}{4}$

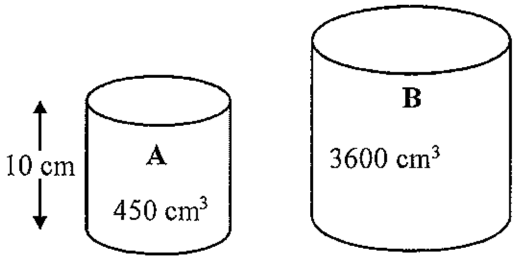
Volume scale factor $= \left(\frac{5}{4}\right)^3$

Vol of large $= 7936 \times (5/4)^3$

Answer.......15,500.......cm^3

(3 marks)

6. *A* and *B* are two similar cylinders.



The height of cylinder *A* is 10 cm and its volume is 450 cm^3.
The volume of cylinder *B* is 3600 cm^2.

Calculate the height of cylinder *B*.

Volume scale factor $= 3600/450 = 8$

But scale factor $^3 = 8$

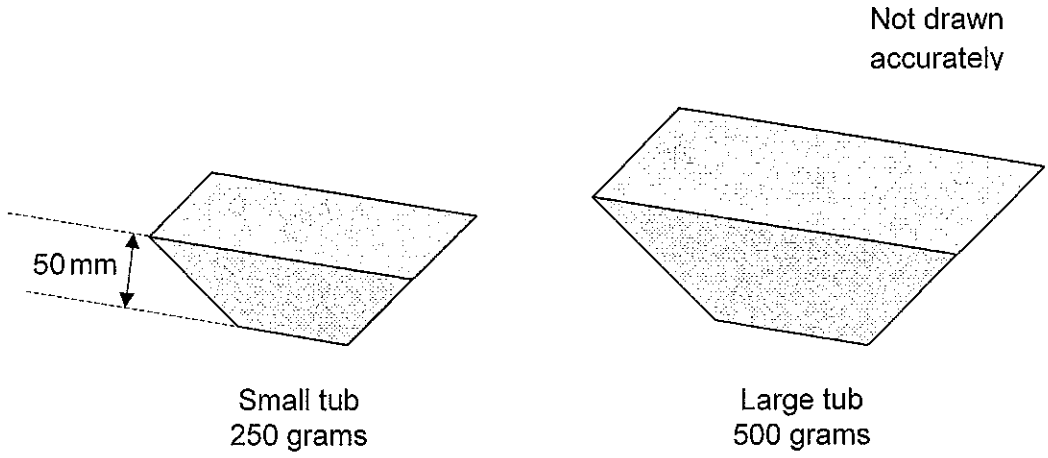
$\therefore$ scale factor $= 2$.

Answer ....20.......cm

(3 marks)

Height of B $= 2 \times 10$
$= 20$cm

7. A large tub holds 500 grams of butter.



The height of the 250 gram size tub is 50 mm.

Work out the height of the 500 gram tub.

Volume scale factor $= \frac{500}{250} = 2$.

$(\text{Scale factor})^3 = 2$

Scale factor $= \sqrt[3]{2}$

$= 1.26$

Height $= 50 \times 1.26$

$= 62.996\ldots$

Answer 63.0 mm

(3 marks)

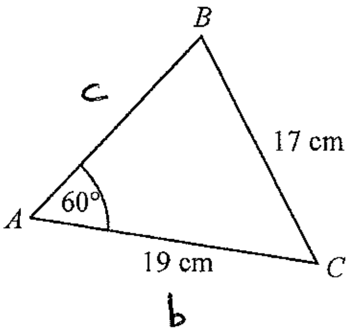
Sine and Cosine Rule

1. (a) *ABC* is a triangle.
$AC = 19$ cm, $BC = 17$ cm and angle $BAC = 60°$

Sine Rule.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 60}{17} = \frac{\sin B}{19}$$



$$\sin B = \frac{19 \times \sin 60}{17}$$

$\sin B = 0.968$

$B = \sin^{-1}(0.968)$

$B = 75.4$

Not to scale

Calculate the size of angle *ABC*.

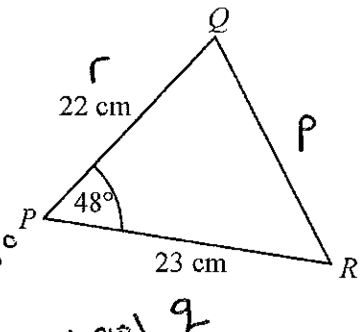
Answer ......75.4...... degrees
(3 marks)

(b) *PQR* is a triangle.
$PR = 23$ cm, $PQ = 22$ cm and angle $QPR = 48°$

Cosine Rule.

$p^2 = q^2 + r^2 - 2qr \cos P$

$p^2 = 23^2 + 22^2 - 2 \times 23 \times 22 \times \cos 48°$



$p^2 = 529 + 484 - (1012 \cos 48°)$

$p^2 = 1013 - (1012 \cos 48°)$

Not to scale

Calculate the length of *QR*.
Give your answer to an appropriate degree of accuracy.

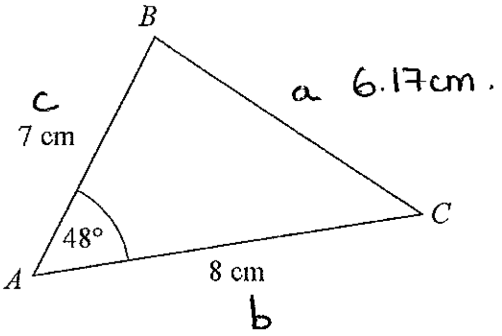
$p^2 = 335.8$

$p = 18.33$

Answer ......18.3...... cm
(4 marks)

2. *ABC* is a triangle.

Not drawn accurately



Cosine Rule.

$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 8^2 + 7^2 - (2 \times 8 \times 7 \times \cos 48°)$

$a^2 = 64 + 49 - (112 \cos 48°)$

$a^2 = 113 - (112 \cos 48°)$

$a^2 = 38.06$

$a = 6.169...$

(a) Calculate the length of side *BC*.

Answer ....6.17........ cm

(3 marks)

(b) Find the size of angle *BCA*.

Sine Rule.

$\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin C}{7} = \frac{\sin 48°}{6.17}$

Answer ....57.5........ degrees

(3 marks)

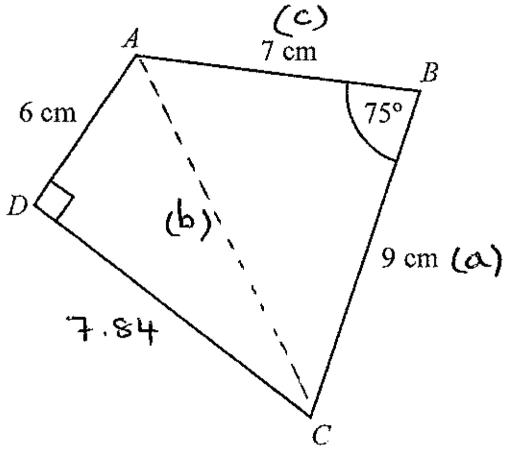
$\sin C = 7 \times \frac{\sin 48}{6.17}$

$\sin C = 0.843$

$C = \sin^{-1}(0.843)$

$= 57.5°$

3. *ABCD* is a quadrilateral.
$AB = 7$ cm, $AD = 6$ cm and $BC = 9$ cm.
Angle $ABC = 75°$ and angle $ADC = 90°$



Not drawn accurately

Calculate the perimeter of *ABCD*.

Cosine Rule (triangle ABC)

$b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 9^2 + 7^2 - (2 \times 9 \times 7 \times \cos 75°)$

$b^2 = 81 + 49 - (126 \cos 75°)$

$b^2 = 97.39$

$b = \sqrt{97.39}$

$b = 9.87\text{cm}$

Pythagoras (triangle ACD)

$DC^2 = AC^2 - AD^2$

$= 9.87^2 - 6^2$

$DC^2 = 61.4$

$DC = 7.84\text{cm}.$

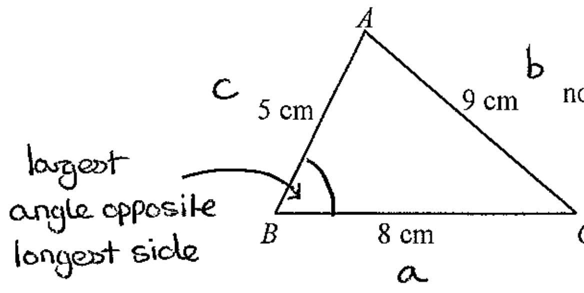
Answer ....29.84........ cm

~ 29.8.

(5 marks)

Total Perimeter = 6 + 7.84 + 7 + 9

4. In triangle ABC, $AB = 5$ cm, $BC = 8$ cm and $AC = 9$ cm.



not drawn accurately

Use the cosine rule to show that triangle ABC does **not** contain an obtuse angle.

$b^2 = a^2 + c^2 - 2ac\cos B$

$9^2 = 8^2 + 5^2 - 2 \times 8 \times 5 \times \cos B$

$81 = 64 + 25 - 80\cos B$

$81 = 89 - 80\cos B$

$-8 = -80\cos B$

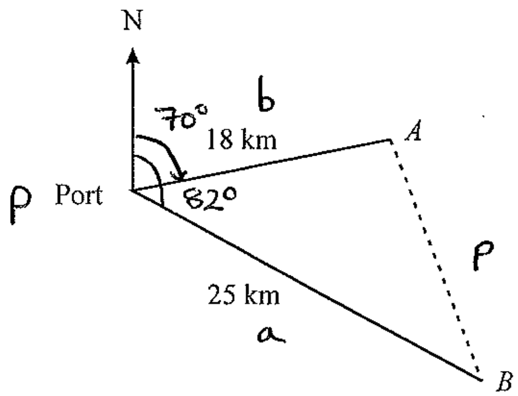
$\cos B = \dfrac{-8}{-80}$

$B = \cos^{-1}(1/10)$

$\underline{\underline{B = 84.3^\circ}}$ not obtuse $(< 90^\circ)$

(3 marks)

5. Two ships, A and B, leave port at 13 00 hours.
Ship A travels at a constant speed of 18 km per hour on a bearing of 070°.
Ship B travels at a constant speed of 25 km per hour on a bearing of 152°.



Not drawn accurately

Calculate the distance between A and B at 14 00 hours.

$P^2 = a^2 + b^2 - 2ab\cos P$

$P^2 = 25^2 + 18^2 - 2 \times 25 \times 18 \times \cos 82^\circ$

$= 625 + 324 - 900\cos 82^\circ$

$= 949 - (900\cos 82^\circ)$

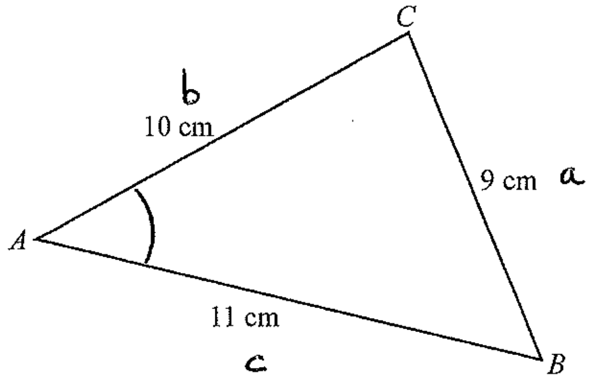
$= 823.7$

$P = \sqrt{823.7}$

Answer ....28.7........ km

(4 marks)

6.



Not to scale

Find the area of triangle *ABC*.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$9^2 = 10^2 + 11^2 - (2 \times 10 \times 11 \times \cos A)$$

$$81 = 100 + 121 - (220\cos A)$$

$$81 = 221 - (220\cos A)$$

$$-140 = -220\cos A$$

$$\cos A = \frac{-140}{-220} = \frac{7}{11}$$

$$A = \cos^{-1}(7/11)$$

$$= \underline{\underline{50.5^\circ}}$$

Answer ......42.4...... cm^2 **(5 marks)**

Area $= \frac{1}{2}ab\sin C$ (Formula Sheet)

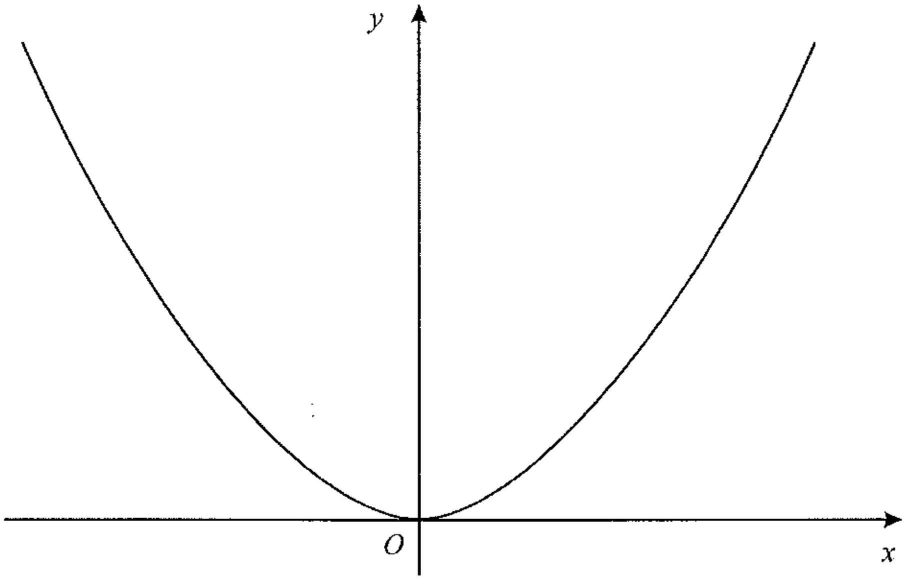
For this triangle Area $= \frac{1}{2}bc\sin A$

$$= \frac{1}{2} \times 10 \times 11 \times \sin 50.5$$

$$= \underline{\underline{42.4\,cm^2}}$$

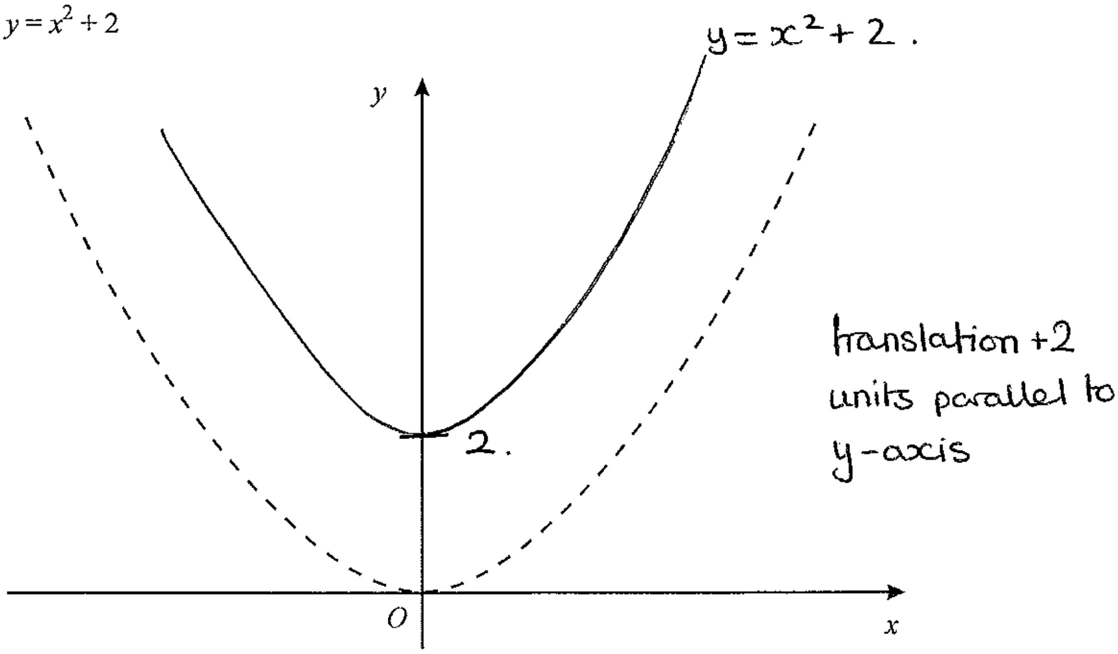
Transformations of Graphs

1. The sketch below is of the graph of $y = x^2$



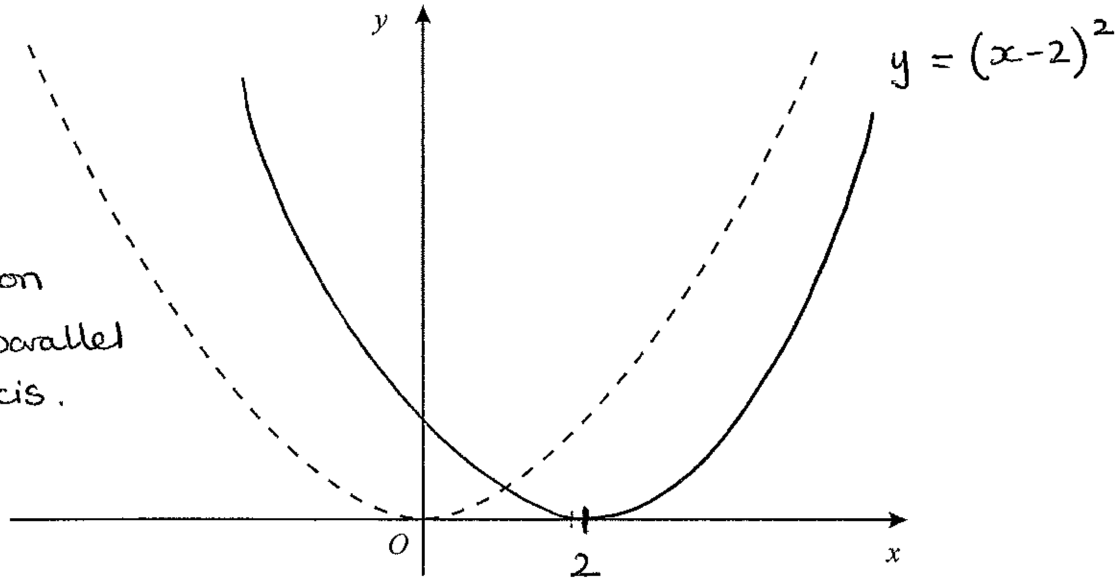
On the axes provided, sketch the following graphs.
The graph of $y = x^2$ is shown dotted on each set of axes to act as a guide.

(a) $y = x^2 + 2$



(1 mark)

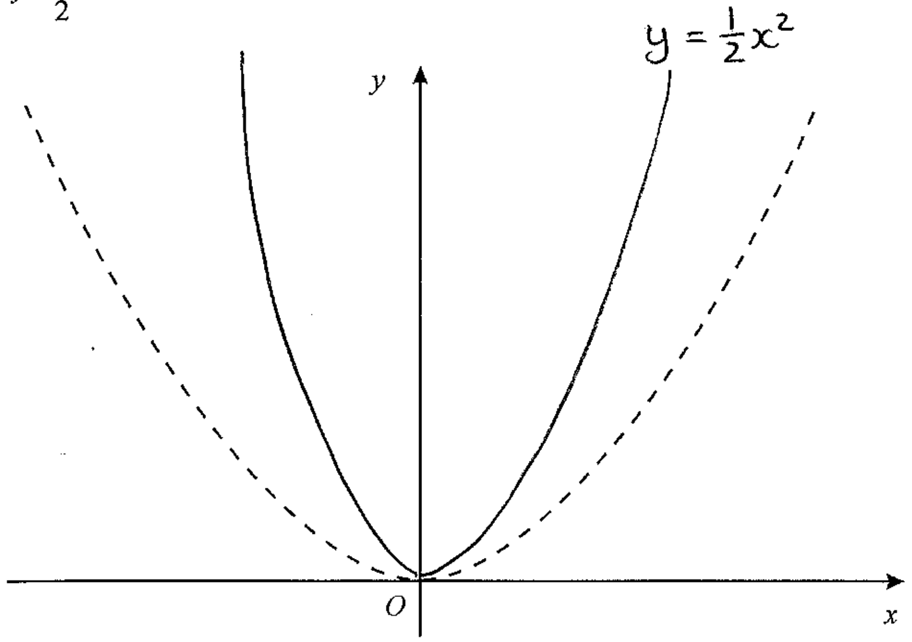
(b) $y = (x - 2)^2$



translation
+2 units parallel
to x-axis.

(1 mark)

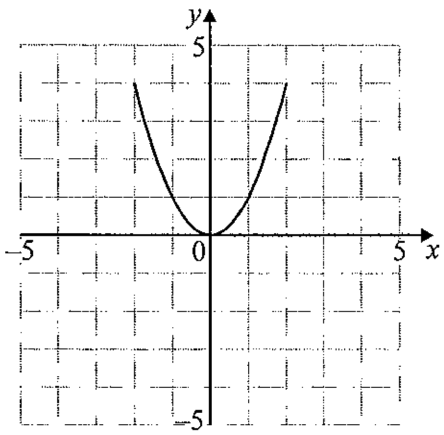
(c) $y = \frac{1}{2}x^2$



(1)

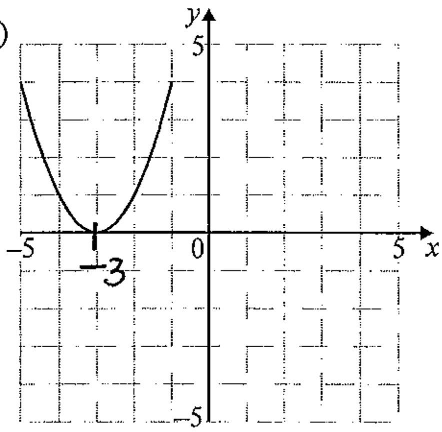
Stretch, scale factor 1/2, parallel to y-axis.

2. The diagram shows the graph of $y = x^2$ for $-2 \le x \le 2$.

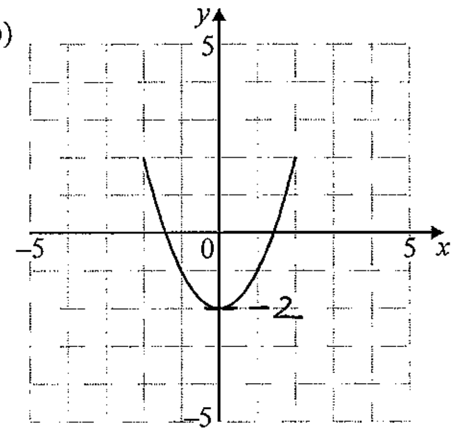


Each of the graphs below is a transformation of this graph.
Write down the equation of each graph.

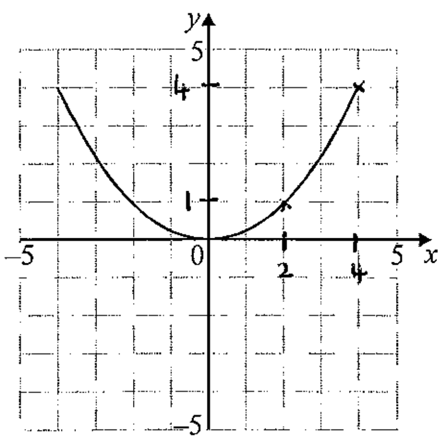
(a)



(b)



(c)



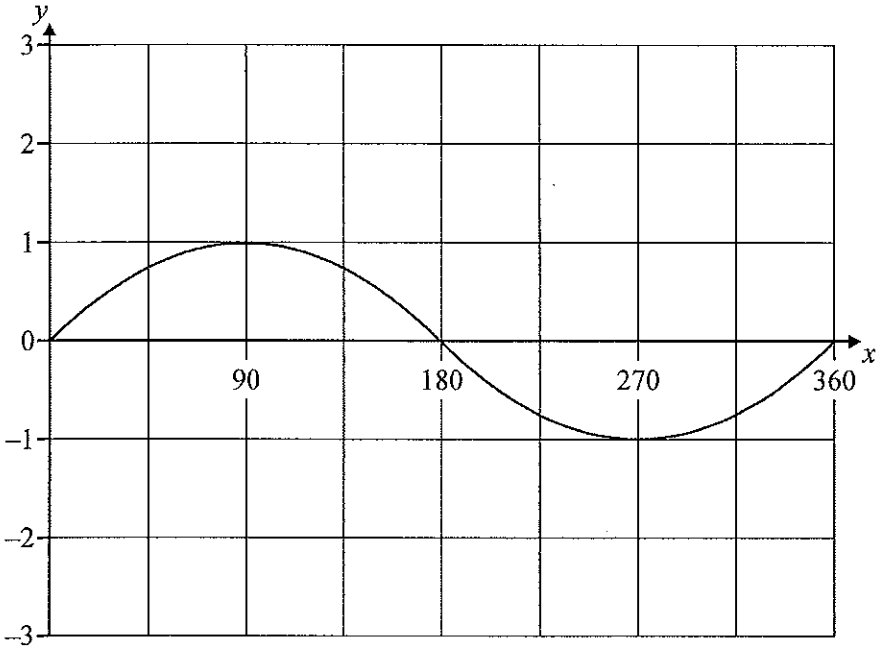
Answer (a) $y =$ .... $(x+3)^2$ ....

Answer (b) $y =$ .... $x^2 - 2$ ....

Answer (c) $y =$ .... $\left(\frac{x}{2}\right)^2$ or $\frac{1}{4}x^2$

(3 marks)

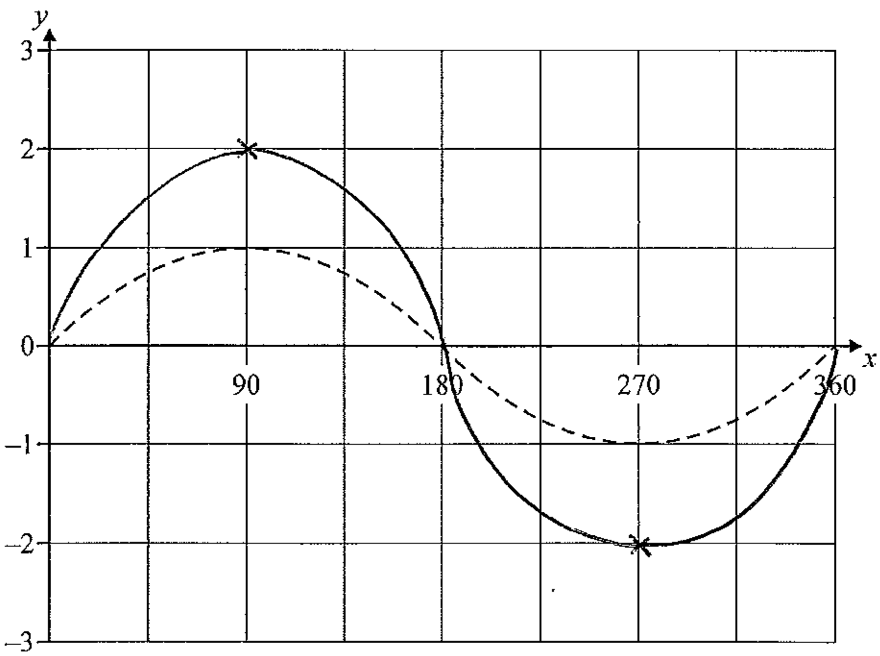
3. This is the graph of $y = \sin x$ for $0° \leq x \leq 360°$



Draw the graphs indicated for $0° \leq x \leq 360°$

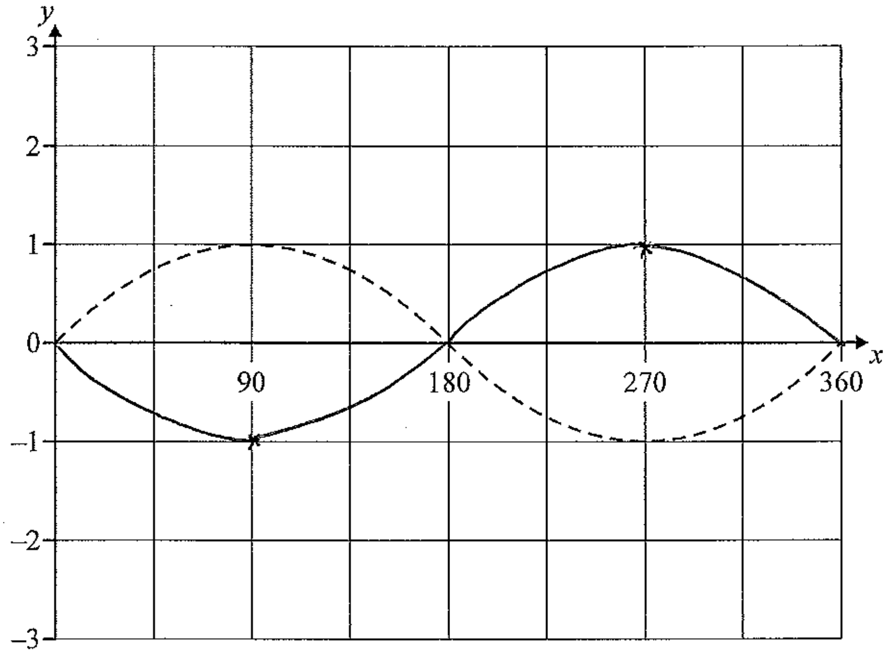
In each case the graph of $y = \sin x$ is shown to help you.

(a) $y = 2 \sin x$



(1 mark)

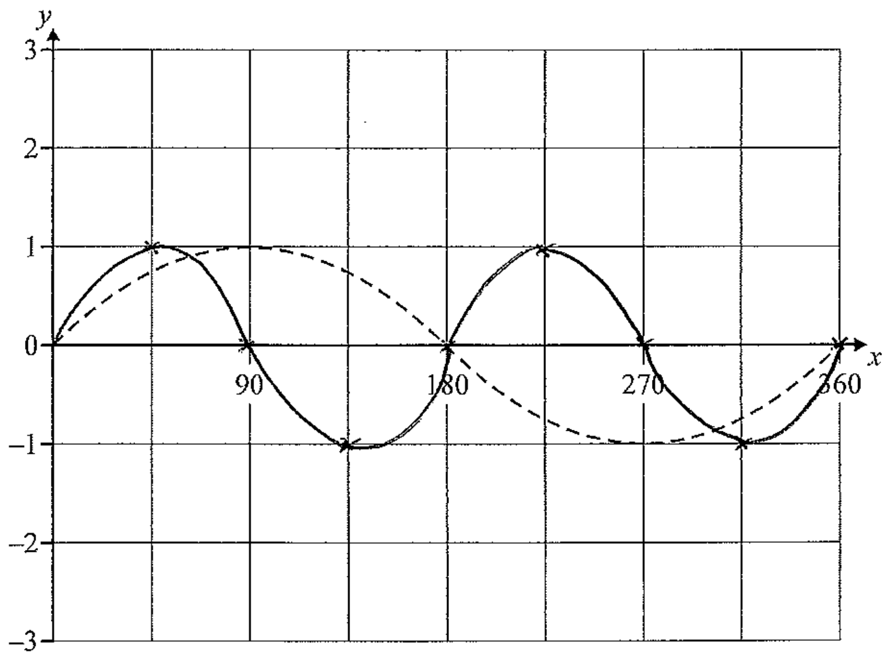
(b) $y = -\sin x$



reflection in x-axis.

(1 mark)

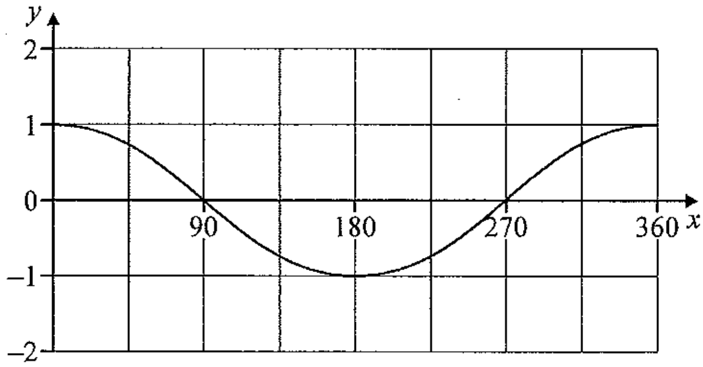
(c) $y = \sin 2x$



(1 mark)

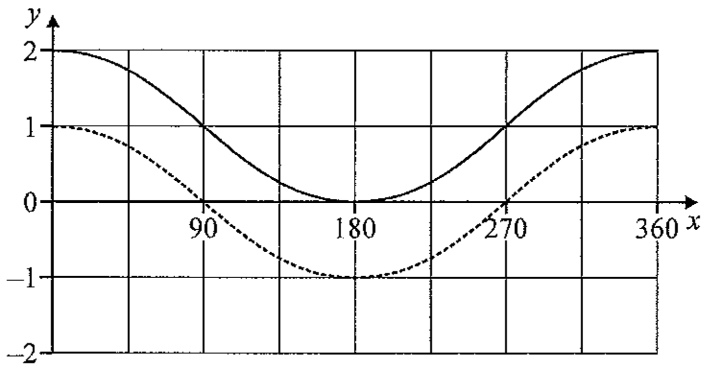
stretch scale factor $\frac{1}{2}$ parallel to x-axis

4. This is the graph of $y = \cos x$ for $0° \le x \le 360°$



Write the equation of each of the transformed graphs.
In each case the graph of $y = \cos x$ is shown dotted to help you.

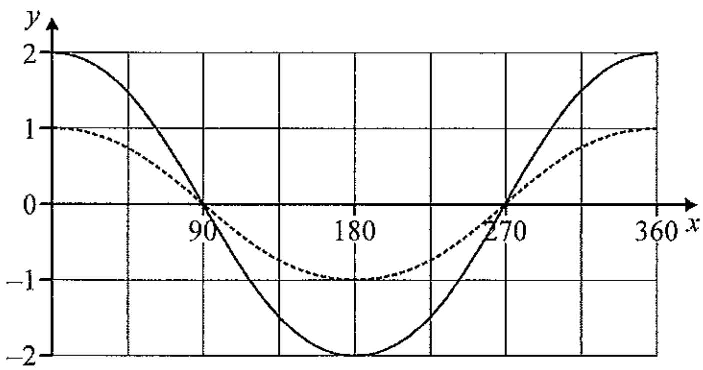
(a)



Equation $y =$ .... $(\cos x) + 1$

(1 mark)

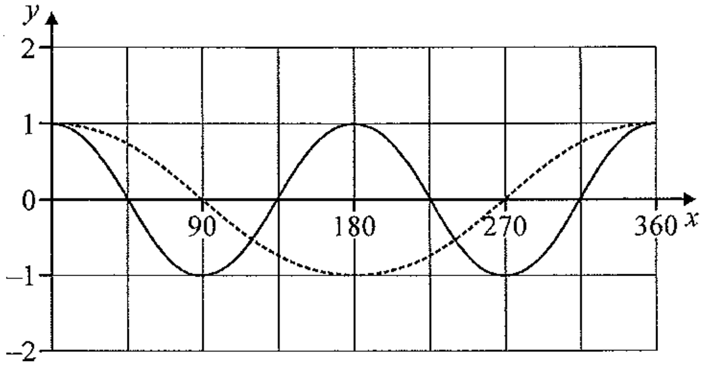
(b)



Equation $y =$ .... $2\cos x$

(1 mark)

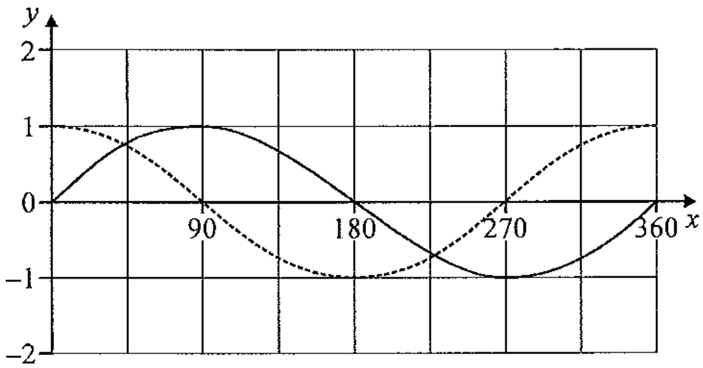
(c)



Equation $y =$ cos (2x)

(1 mark)

(d)

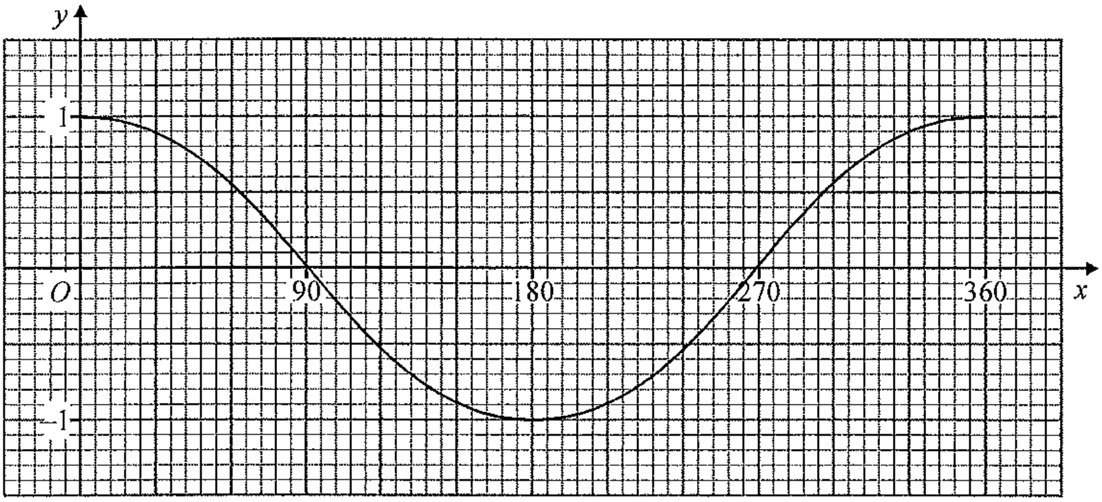


Equation $y =$ cos (x − 90)

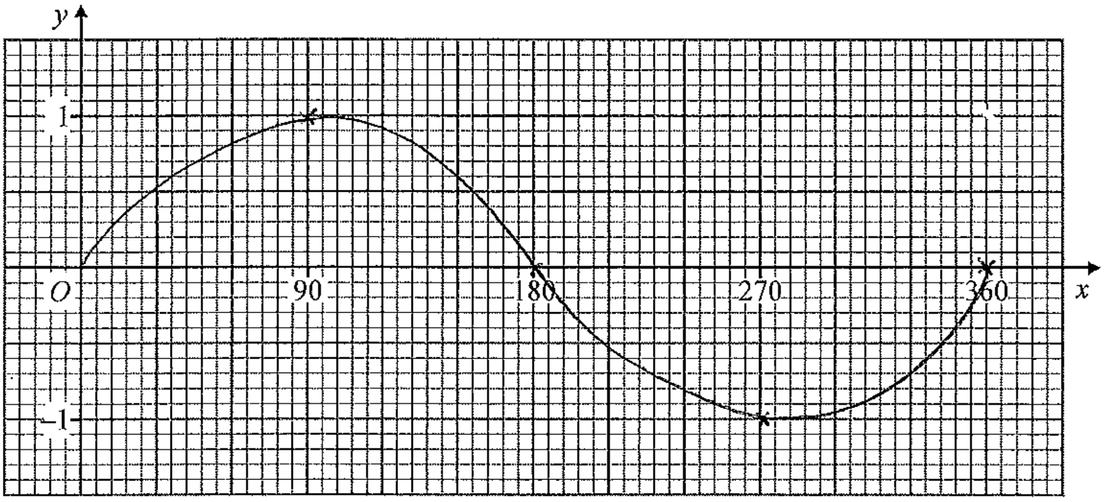
(1 mark)

or y = sin x

5. This is the graph of $y = \cos x$ for $0° \leq x \leq 360°$

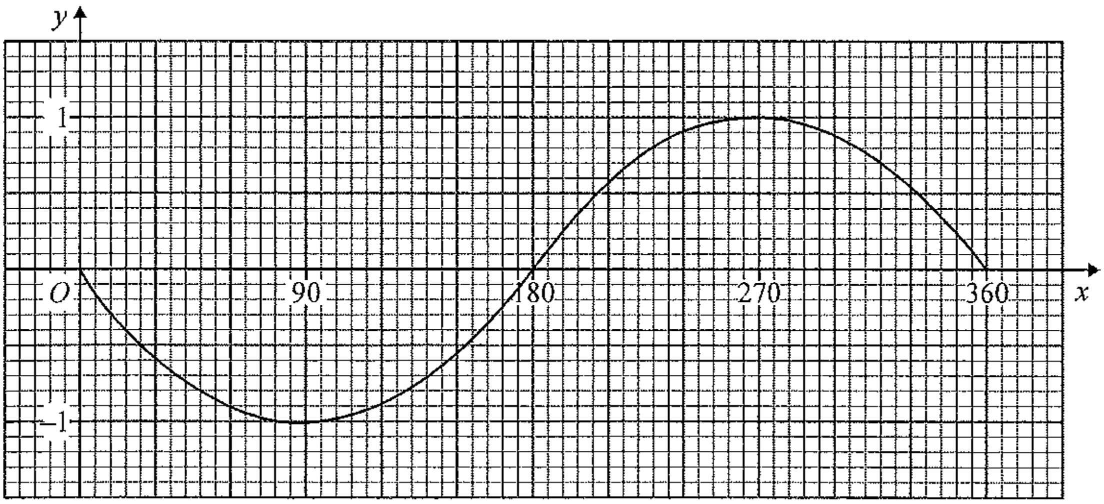


(a) On the axes below draw the graph of $y = \cos(x - 90)$ for $0° \leq x \leq 360°$



translation +90 parallel to x-axis

(b) Write down a possible equation of the following graph.



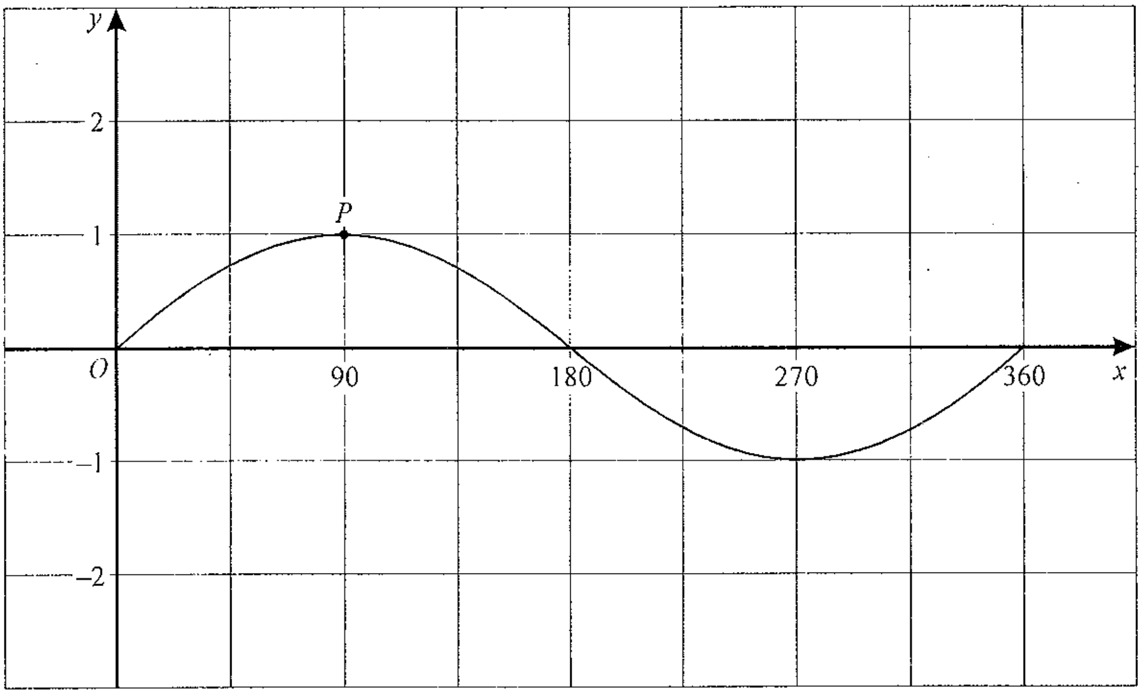
Answer ......$y = - \cos(x - 90)$......

reflection of (a) in x-axis

(3 marks)

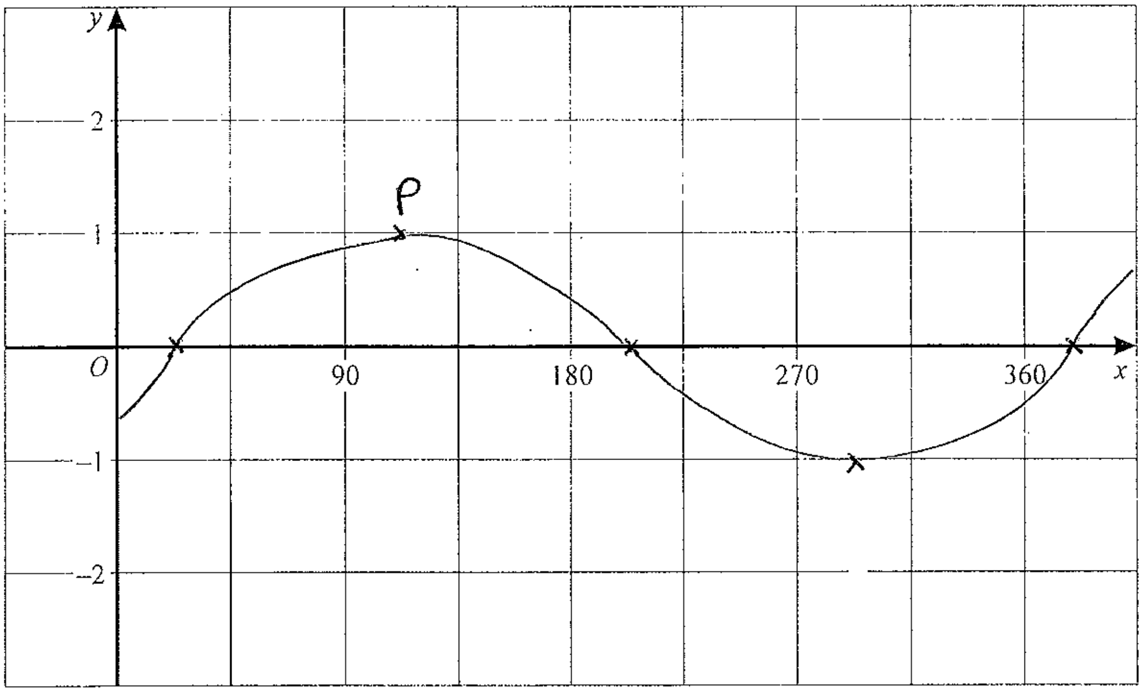
OR $y = -\sin(x)$.

6. The graph of $y = \sin x$ for $0° \leq x \leq 360°$ is shown on the grid below.
The point $P(90, 1)$ lies on the curve.



On both of the grids that follow, sketch the graph of the transformed function.
In both cases write down the coordinates of the transformed point P.

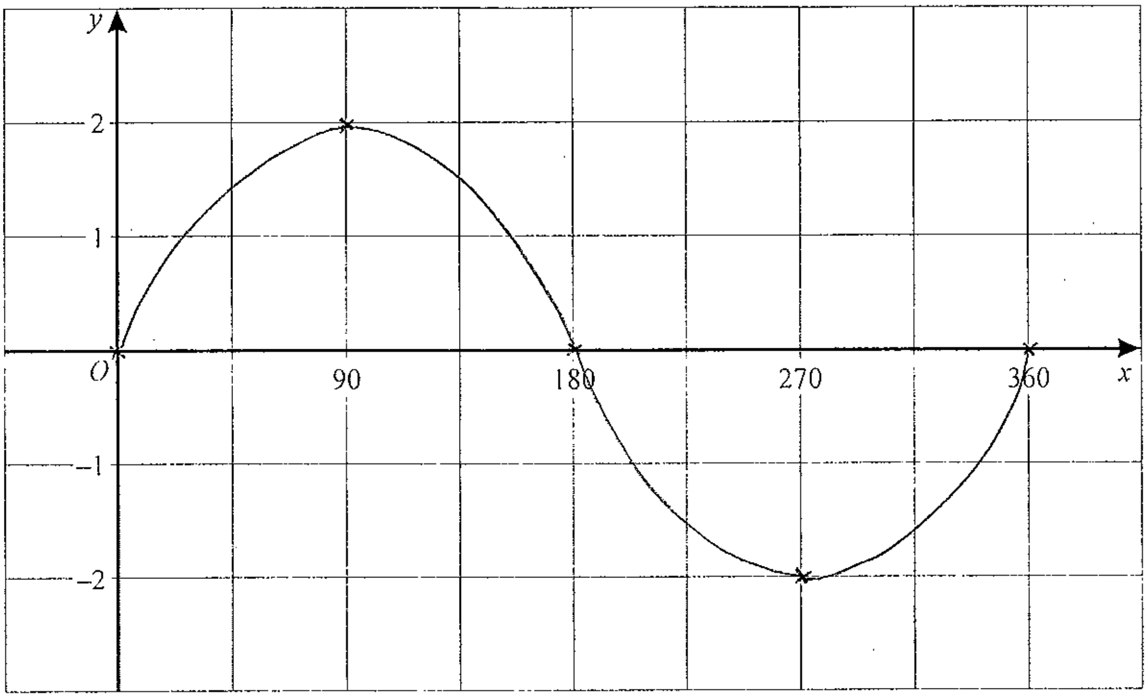
(a) $y = \sin(x - 45)$ translation of +45° parallel to x-axis



P(135° , 1)

(2 marks)

(b) $y = 2\sin x$ Stretch scale factor 2 parallel to y-axis.



P (90°, 2)

(2 marks)

Other Graphs

1. Below are three graphs.

Match each graph with one of the following equations.

Equation A: $y = 3x - p$
Equation B: $y = x^2 + p$
Equation C: $3x + 4y = p$
Equation D: $y = px^3$

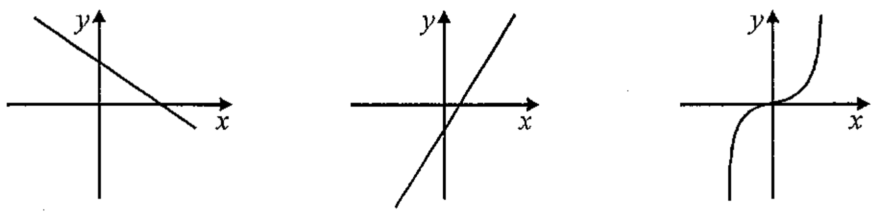
Rearrange C $4y = -3x + p$

$y = -\frac{3}{4}x + \frac{p}{4}$

negative gradient

In each case p is a **positive** number.

(i) (ii) (iii)



Answer Graph (i) Equation ....C........

Graph (ii) Equation ....A........

Graph (iii) Equation ....D.........

(3 marks)

2. Each of the graphs represents one of the following equations.

A $y = 3x + 4$	B $2x + 3y = 12$
C $y = x^2 - 2$	D $y = x^3$

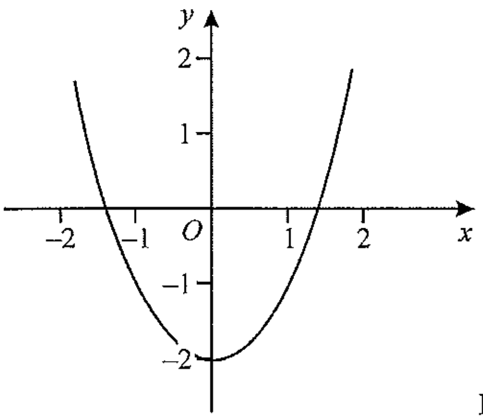
Write down the letter of the equation represented by each graph

Rearrange B

$3y = -2x + 12$

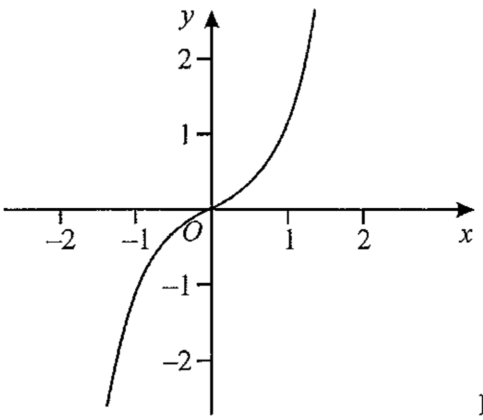
$y = -\frac{2}{3}x + 4$

negative gradient



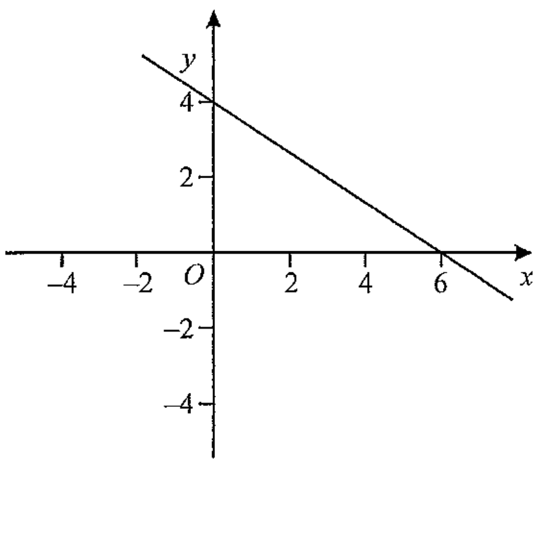
Equation ...C...

(1 mark)



Equation ...D...

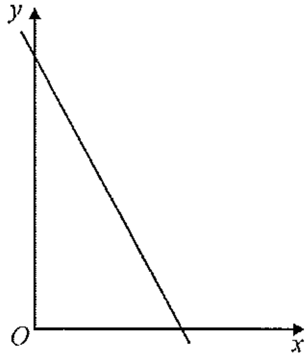
(1 mark)



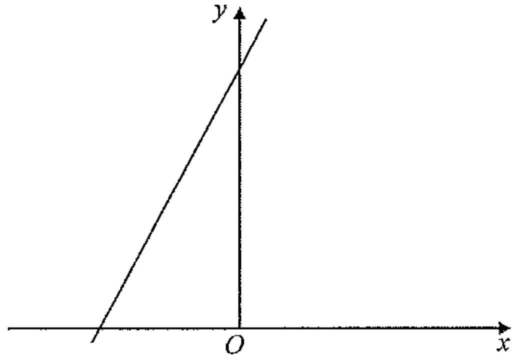
Equation ...B...

(1 mark)

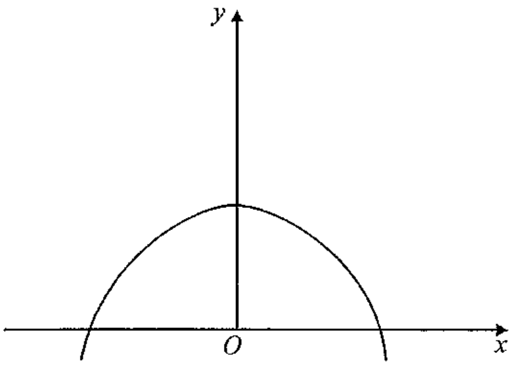
3. (a) Four graphs are sketched.



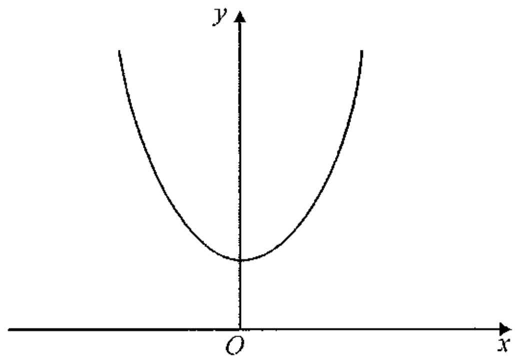
Graph A



Graph B



Graph C



Graph D

Complete the following statements.

rearrange

$y = -2x + 4$

negative gradient.

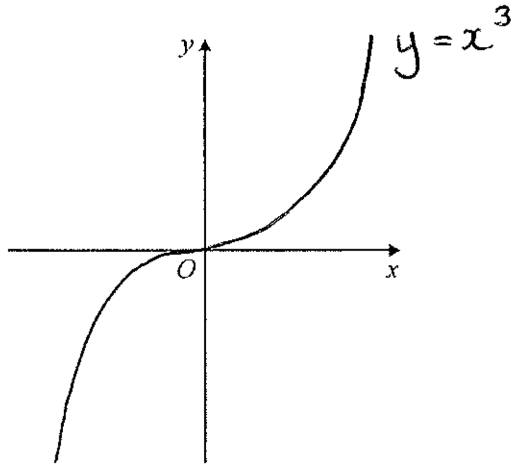
$y = 2x + 4$ matches graph ......B..............

$y = x^2 + 4$ matches graph ......D..............

$y + 2x = 4$ matches graph ......A..............

(3 marks)

(b) Sketch the graph of $y = x^3$ on the axes below.

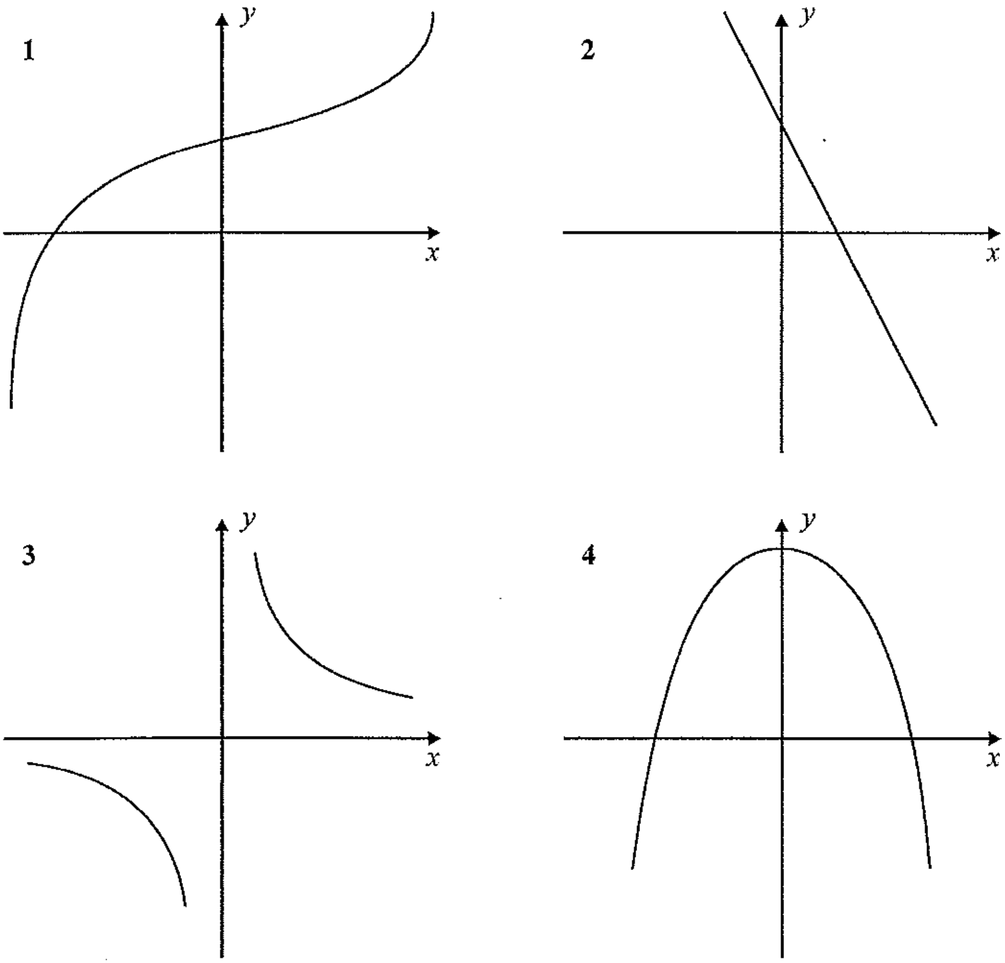


(2 marks)

4. Match each of the sketch graphs to one of these equations.

A $y = 2 - 2x$ **B** $y = 2x + 2$ **C** $y = 3 - x^2$ **D** $y = x^3 + 4$ **E** $y = \frac{2}{x}$

negative gradient



Graph **1** represents equation ....D........

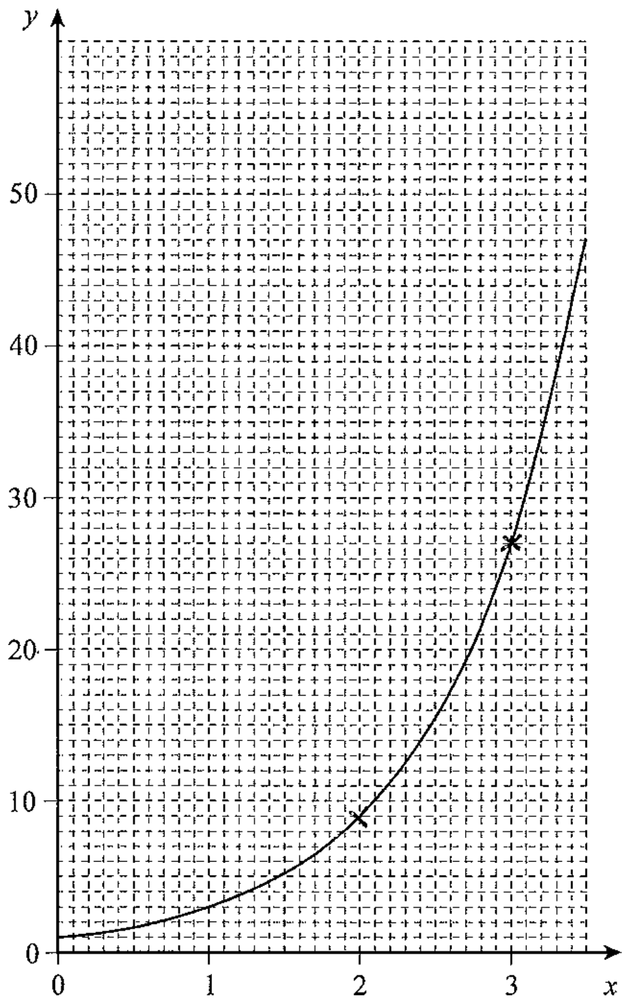
Graph **2** represents equation ....A........

Graph **3** represents equation ....E........

Graph **4** represents equation ....C........

(4 marks)

5. The graph shows the function $y = a^x$



(a) Write down the coordinates of the point where the graph intersects with the y-axis.

Answer (........0........ , ...1........)

(1 mark)

b) Find the value of a.

Answer ....$a = 3$........

(2 marks)

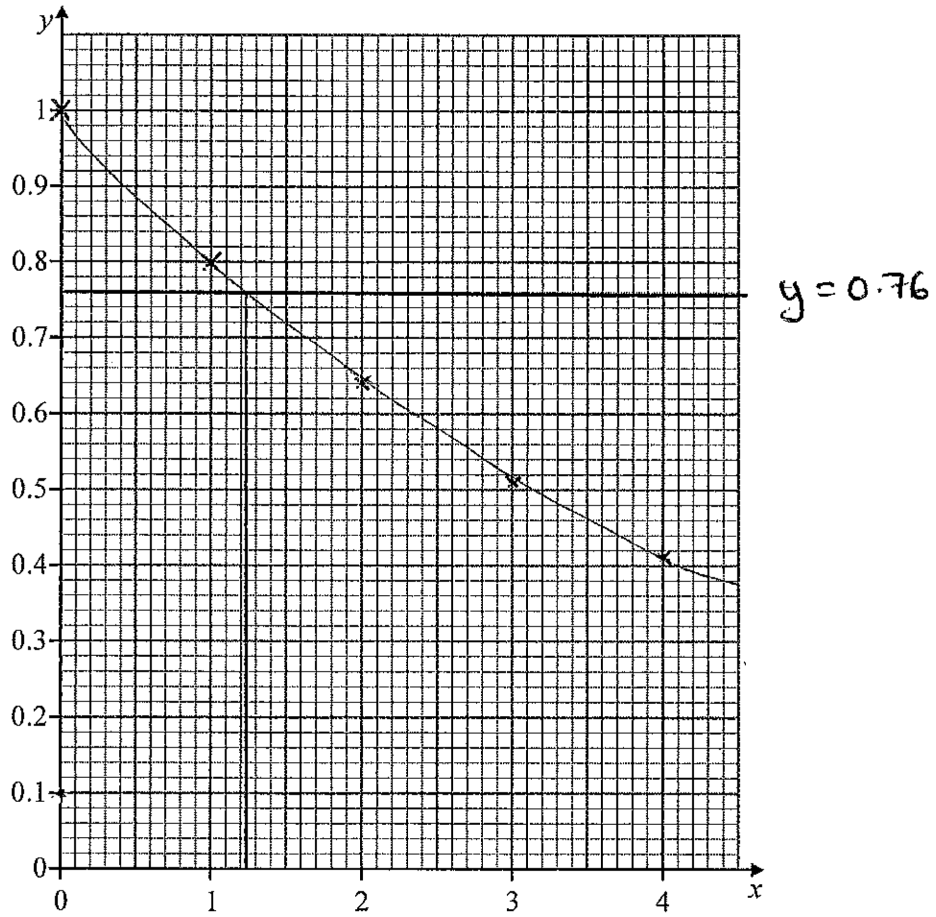
$a^2 = 9$ and $a^3 = 27$

6. (a) Complete the table of values for $y = (0.8)^x$

x	0	1	2	3	4
y	1	0.8	0.64	0.512	0.41

(1 mark)

(b) On the grid below, draw the graph of $y = (0.8)^x$ for values of x from 0 to 4.



(2 marks)

(c) Use your graph to solve the equation $(0.8)^x = 0.76$

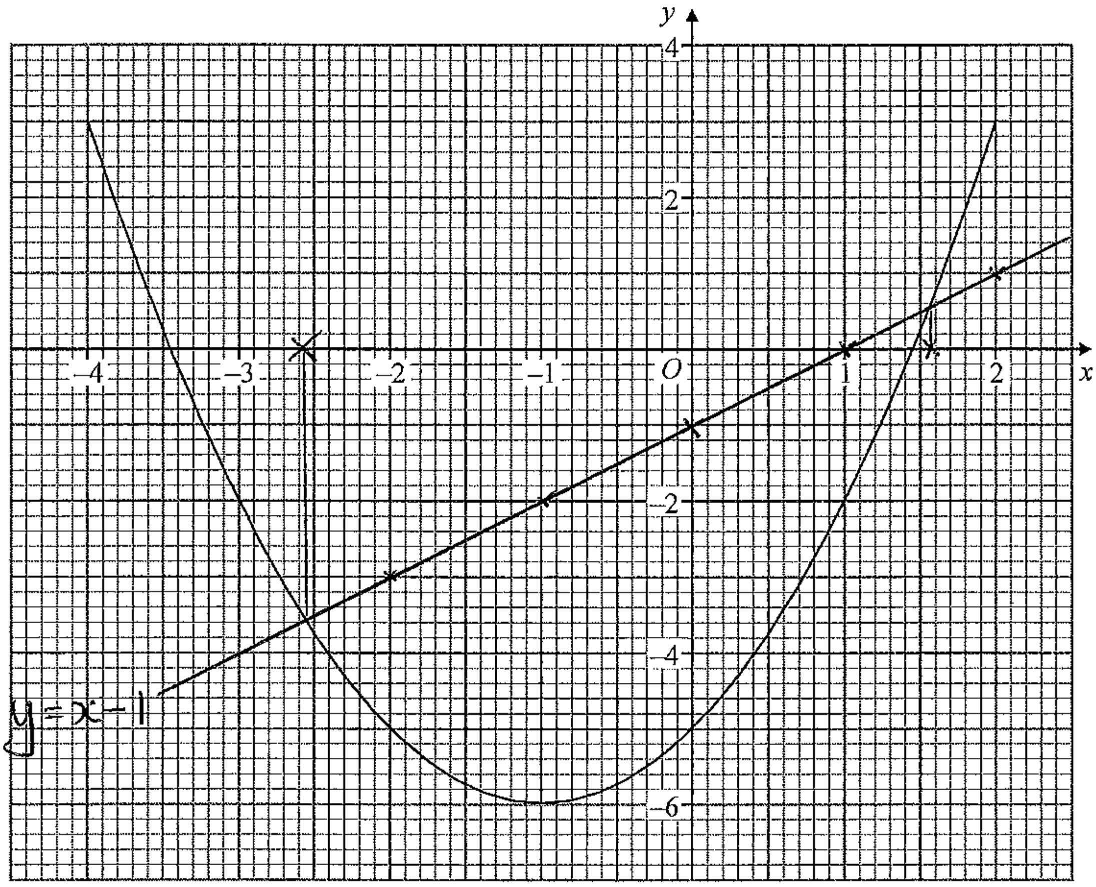
Answer ......1.2......

(1 mark)

draw line $y = 0.76$

Method of Intersections

1. The grid shows the graph of $y = x^2 + 2x - 5$



Note.

x, y axes have different scales.

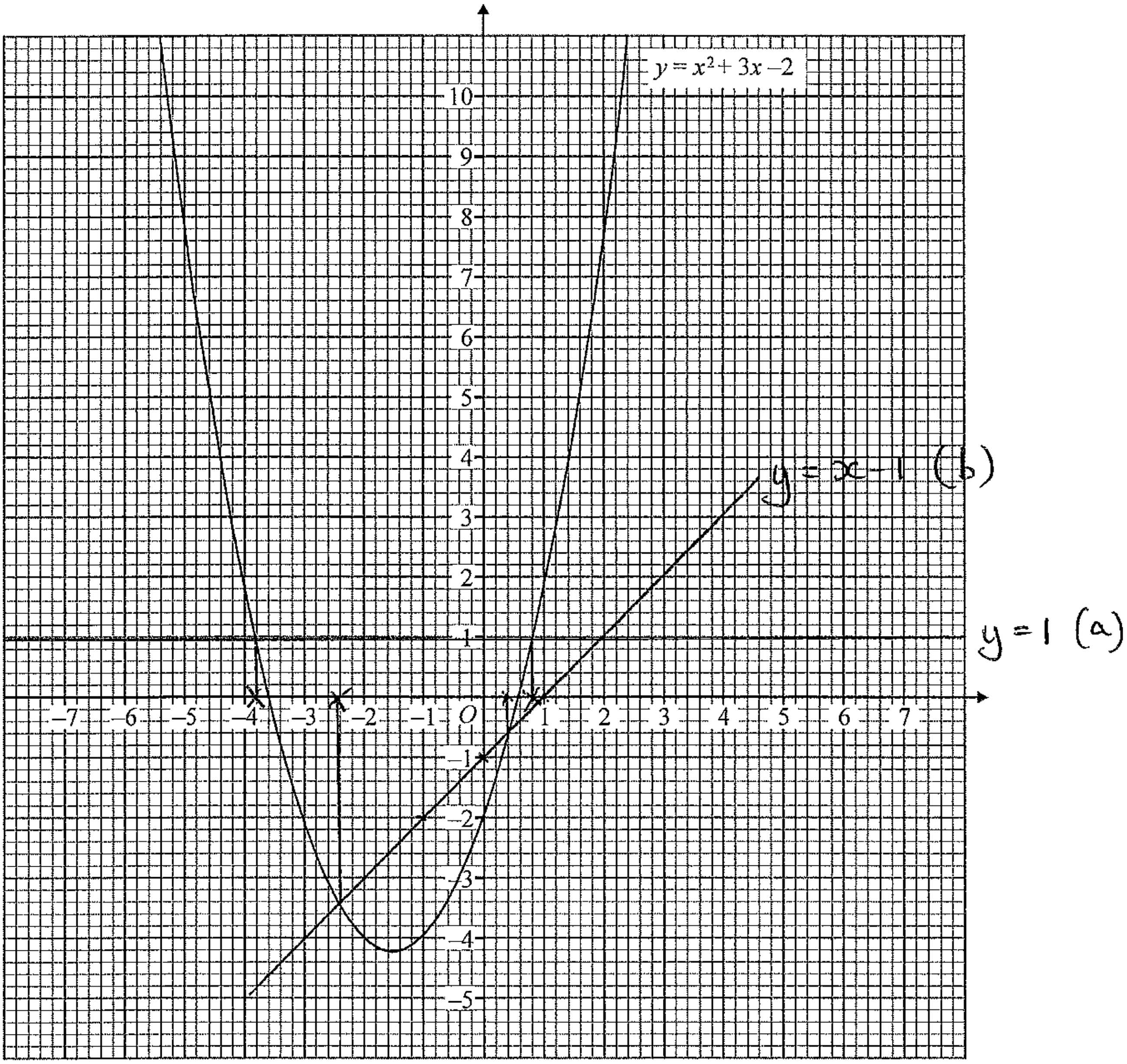
By drawing an appropriate straight line, solve the equation $x^2 + 2x - 5 = x - 1$

draw graph $y = x - 1$ (find intersection)

Answer .... -2.55 and 1.55 ....

(3 marks)

2. The grid below shows the graph of $y = x^2 + 3x - 2$



(a) By drawing an appropriate straight line on the graph solve the equation

Graph $x^2 + 3x - 2$

$$x^2 + 3x - 3 = 0$$

draw $y = 1$.

New $x^2 + 3x - 3$ ⊖

1

Answer $x = -3.8$ and $x = 0.8$

(2 marks)

(b) By drawing an appropriate straight line on the graph solve the equation

$$x^2 + 2x - 1 = 0$$

draw $y = x - 1$.

Graph $x^2 + 3x - 2$

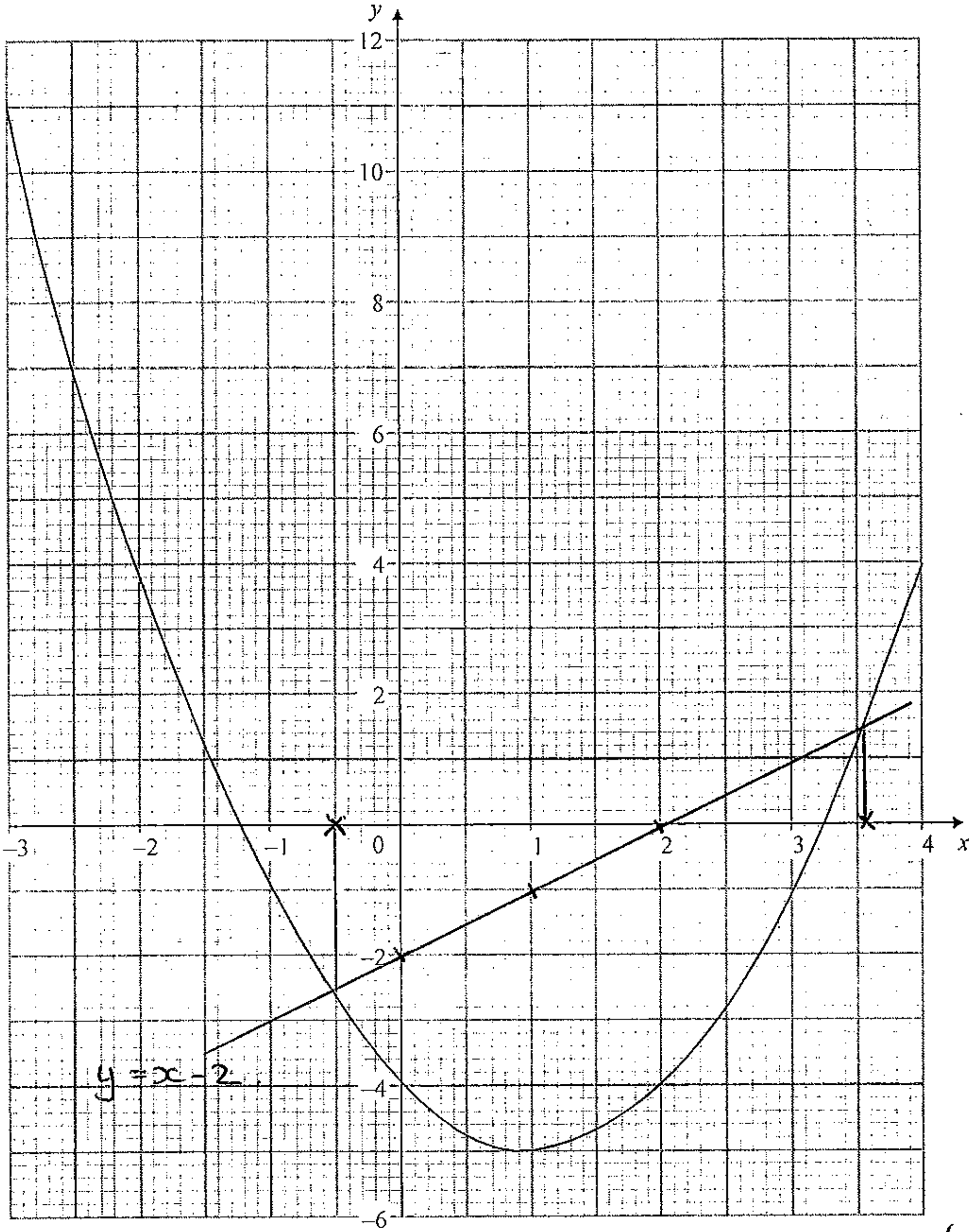
Answer $x = -2.4$ and $x = 0.4$

New $x^2 + 2x - 1$ ⊖

(3 marks)

$x - 1$

3. The graph $y = x^2 - 2x - 4$ is drawn below for values of x between –3 and +4.



Note.
different scales on x and y axes.

where y = 0 (or where does it cross the x-axis!)

(a) Using the graph, find the solutions of $x^2 - 2x - 4 = 0$, giving your answers to 1 decimal place.

Answer $x = -1.2$ or $x = 3.2$

(1 mark)

(b) By drawing an appropriate linear graph, write down the solutions of

$x^2 - 3x - 2 = 0$ draw $y = x - 2$

Graph $x^2 - 2x - 4$
New $x^2 - 3x - 2$ $-$

$x - 2$

Answer $x = -0.5$ and $x = 3.6$.

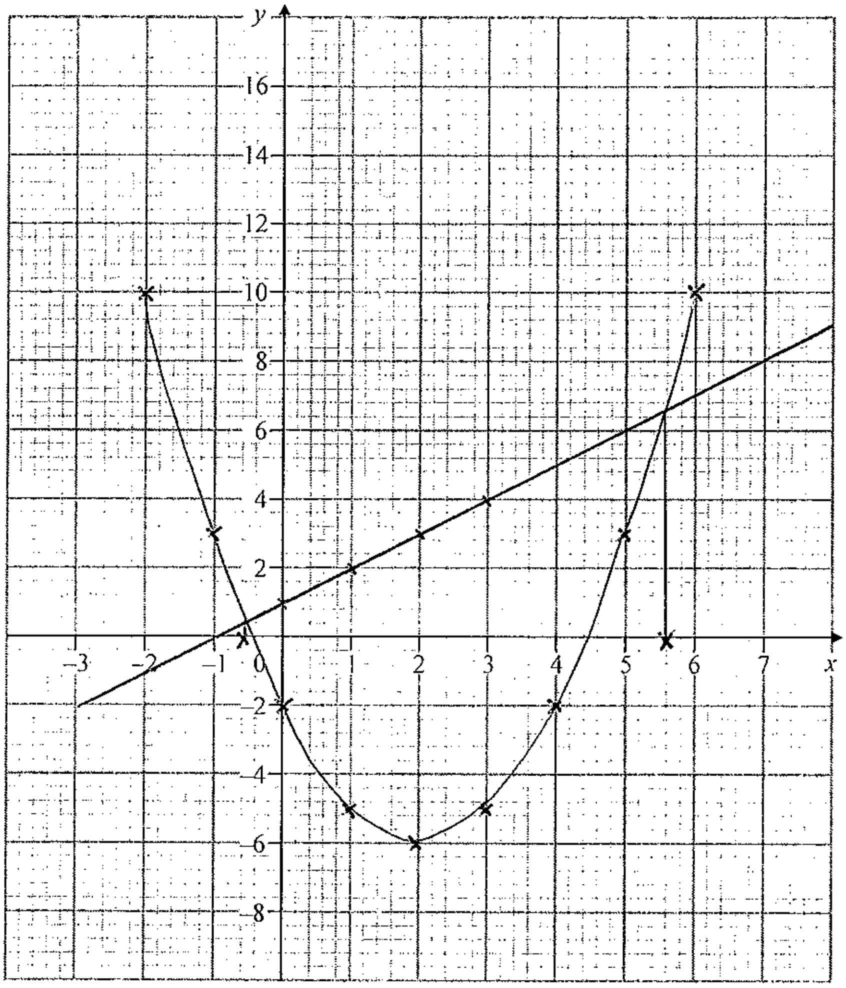
(3 marks)

4. (a) Complete the table of values for $y = x^2 - 4x - 2$

x	-2	-1	0	1	2	3	4	5	6
y	10	3	-2	-5	-6	-5	-2	3	10

(1 mark)

(b) On the grid below, draw the graph $y = x^2 - 4x - 2$ for values of x between -2 and 6.



$y = x + 1$

Note.

different scale on x, y axes.

(2 marks)

(c) Use your graph to write down the solutions of the equation $x^2 - 4x - 2 = 0$ ← when $y = 0$ (crosses x-axis)

Answer $x = -0.4$ and $x = 4.4$

(2 marks)

(d) By drawing an appropriate linear graph, write down the solutions of

$x^2 - 5x - 3 = 0$ draw $y = x + 1$

Graph $x^2 - 4x - 2$

New $x^2 - 5x - 3$ ⊖

$\overline{x + 1}$

Answer $x = -0.6$, $x = 5.6$.

(3 marks)

Direct and Inverse Proportion

1. W and P are both positive quantities.
W is directly proportional to the square root of P.
When $W = 12$, $P = 16$.

$W \propto \sqrt{P}$

$W = k\sqrt{P}$.

(a) Express W in terms of P.

$12 = k \times \sqrt{16}$

$k = 3$

Answer $W = 3\sqrt{P}$.

(3 marks)

(b) What is the value of W when $P = 25$?

$W = 3 \times \sqrt{25}$

$= 3 \times 5$

$= 15$

Answer 15

(1 mark)

(c) What is the value of P when $W = 21$?

$21 = 3\sqrt{P}$

$7 = \sqrt{P}$

$P = 7^2$

Answer 49

(2 marks)

2. y is directly proportional to the square of x.
When $y = 5$, $x = 4$.
Find the value of y when $x = 8$.

$y \propto x^2$

$y = kx^2$

$y = \frac{5}{16}x^2$.

$5 = k \times 4^2$

$16k = 5$

$k = \frac{5}{16}$

Answer 20

(3 marks)

When $x = 8$

$y = \frac{5}{16} \times 8^2$

$= \frac{5}{16} \times 64 = 20$

3. A is directly proportional to B^2
When $A = 50$, $B = 10$

$A \propto B^2$

$A = kB^2$

(a) Find an equation connecting A and B.

$50 = k \times 10^2$

$50 = 100k$

$k = 0.5$

Answer $A = 0.5B^2$

(3 marks)

(b) Find the value of B when $A = 72$

$72 = 0.5B^2$

$(\times 2)$ $144 = B^2$

Answer $B = 12$ (or $B = -12$).

(2 marks)

4. y is inversely proportional to the square of x.
When $y = 3$, $x = 2$
Find the value of y when $x = 4$

$y \propto \frac{1}{x^2}$

$y = \frac{k}{x^2}$

$3 = \frac{k}{2^2}$

$k = 12$.

$\therefore$ $y = \frac{12}{x^2}$

Answer $y =$ $3/4$

(3 marks)

When $x = 4$

$y = \frac{12}{4^2}$

$= \frac{12}{16}$

$= 3/4$

5. M and G are positive quantities.
M is inversely proportional to G.
When $M = 90$, $G = 40$.

Find the value of M when $G = M$.

$$m \propto \frac{1}{G}$$

$$m = \frac{k}{G}$$

$$90 = \frac{k}{40}$$

$$k = 3600$$

Answer M= ....60........................

(4 marks)

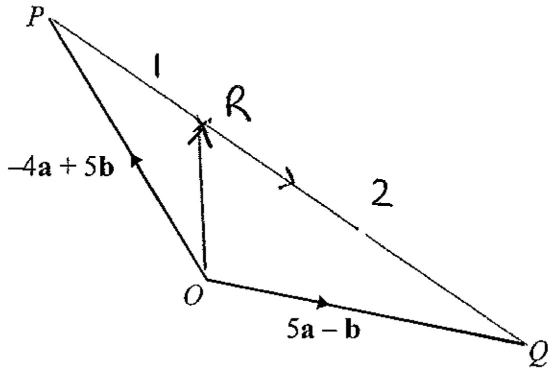
$$m = \frac{3600}{G}$$

When $G = m$,

$$m = \frac{3600}{m}, \quad m^2 = 3600, \quad m = 60$$

Vectors

1. $\overrightarrow{OP} = -4\mathbf{a} + 5\mathbf{b}$ and $\overrightarrow{OQ} = 5\mathbf{a} - \mathbf{b}$.



R is a point on $\overrightarrow{PQ}$ such that $PR : RQ = 1 : 2$.

$$\overrightarrow{PQ} = -(-4a+5b) + 5a - b$$
$$= 4a - 5b + 5a - b$$
$$= 9a - 6b$$

$$\overrightarrow{PR} = \frac{1}{3}(9a - 6b) = 3a - 2b.$$

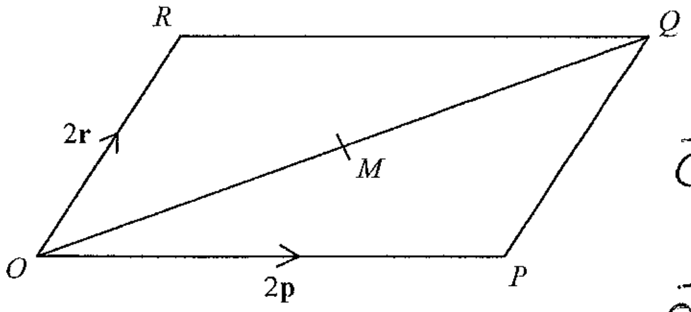
Express $\overrightarrow{OR}$ in terms of **a** and **b**.

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$$
$$= (-4a+5b) + (3a-2b)$$
$$= -a + 3b$$

Answer ......$-a + 3b.$......

(3 marks)

2. $OPQR$ is a parallelogram.
M is the midpoint of the diagonal OQ.
$\overrightarrow{OP} = 2p$ and $\overrightarrow{OR} = 2r$



$$\overrightarrow{OQ} = \overrightarrow{OR} + \overrightarrow{RQ}$$
$$= 2r + 2p.$$
$$\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OQ}$$
$$= r + p$$

Express $\overrightarrow{OM}$ in terms of p and r.

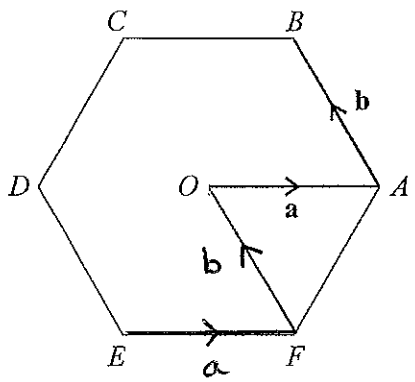
Answer $\overrightarrow{OM}$ ......$r + p$......

(1 mark)

3. *ABCDEF* is a regular hexagon with centre *O*.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{AB} = \mathbf{b}$

Diagram drawn accurately



Find expressions, in terms of **a** and **b**, for

(i) $\overrightarrow{OB}$

Answer $a + b$

(1 mark)

(ii) $\overrightarrow{AC}$

Answer $b + -a = b - a$

(1 mark)

(iii) $\overrightarrow{EC}$

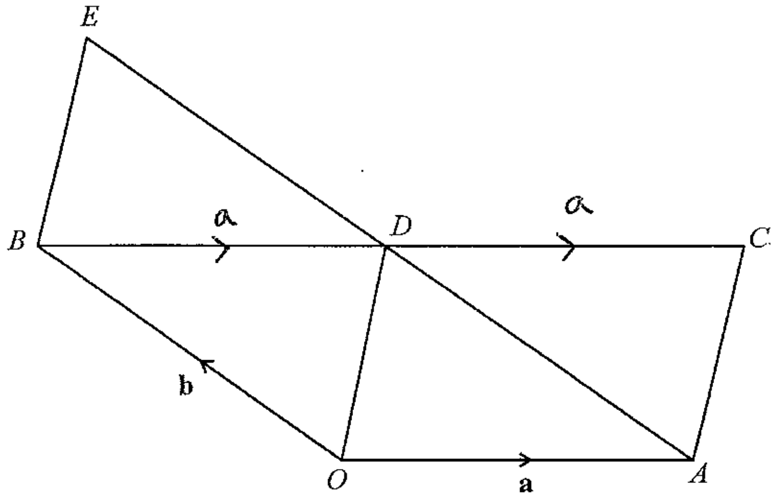
Answer $a + b + a + b - a$

(1 mark)

$a + 2b$

4. In the diagram *OACD, OADB* and *ODEB* are parallelograms.

$\overrightarrow{OA} = \mathrm{a}$ and $\overrightarrow{OB} = \mathrm{b}$



Express, in terms of **a** and **b**, the following vectors.
Give your answers in their simplest form.

(i) $\overrightarrow{OD}$

Answer.......... b + a. ....

(1 mark)

(ii) $\overrightarrow{OC}$

Answer.......... b + 2a ....

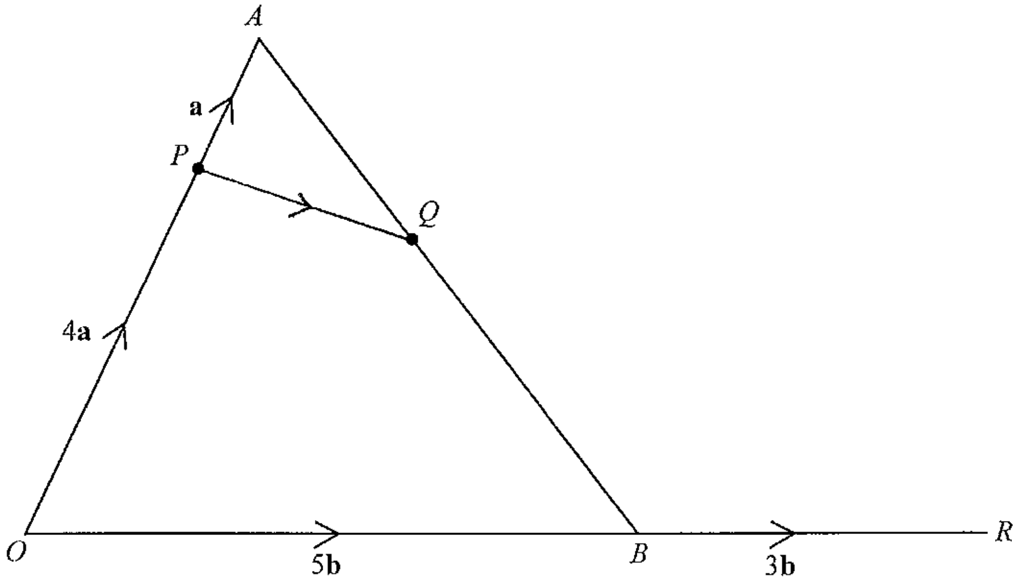
(1 mark)

(iii) $\overrightarrow{AB}$

Answer.......... −a + b = b − a ....

(1 mark)

5. In the diagram $\overrightarrow{OP} = 4\mathbf{a}$, $\overrightarrow{PA} = \mathbf{a}$, $\overrightarrow{OB} = 5\mathbf{b}$, $\overrightarrow{BR} = 3\mathbf{b}$ and $\overrightarrow{AQ} = \frac{2}{5}\ \overrightarrow{AB}$



Not drawn accurately

Find, in terms of **a** and **b**, simplifying your answers,

(i) $\overrightarrow{AB}$ $\overrightarrow{AB} = -5a + 5b$

Answer $5b - 5a$

(1 mark)

(ii) $\overrightarrow{PQ}$

Answer $2b - a$

(2 marks)

$\overrightarrow{AQ} = \frac{2}{5}\overrightarrow{AB}$

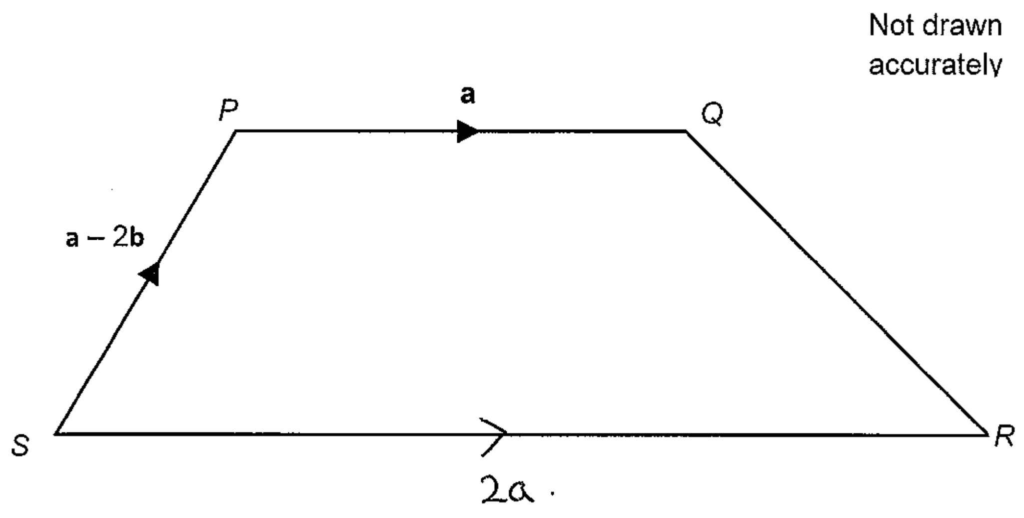
$= \frac{2}{5}(5b - 5a)$

$= 2b - 2a$

$\overrightarrow{PQ} = a + (2b - 2a)$

$= 2b - a$

6. *PQRS* is a trapezium as shown.



SR is parallel to *PQ*.

SR = 2*PQ*.

(a) Write down in terms of **a** and **b** vector *SR*.

Answer ....2a........

(1 mark)

(b) Work out in terms of **a** and **b** vector *QR*.

Give your answer as simply as possible.

Answer ....2b........

(2 marks)

$$\overrightarrow{QR} = -\overrightarrow{PQ} + -\overrightarrow{SP} + \overrightarrow{SR}$$
$$= -a + -(a-2b) + 2a$$
$$= -a - a + 2b + 2a$$
$$= 2b$$

7.

$WXYZ$ is a trapezium.

$\overrightarrow{WX} = \mathbf{s}$

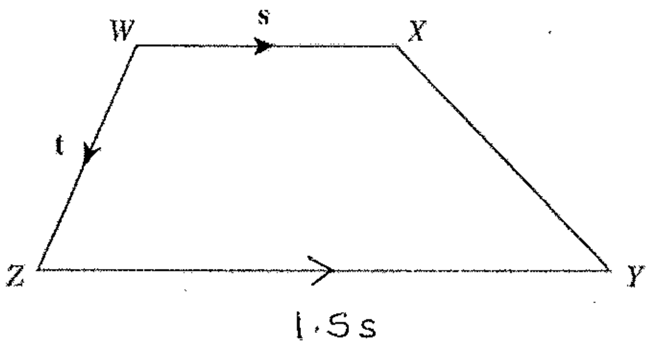
$\overrightarrow{WZ} = \mathbf{t}$

$ZY : WX = 3 : 2$

$$\frac{\overrightarrow{ZY}}{\overrightarrow{WX}} = \frac{3}{2}$$

$$\overrightarrow{ZY} = \frac{3}{2}\overrightarrow{WX}$$

$$= \underline{\underline{1 \cdot 5s}}.$$



Write vector $\overrightarrow{ZY}$ in terms of s

....

Answer .... $1 \cdot 5s.$ .... *(1 mark)*

Work out vector $\overrightarrow{XY}$ in terms of s and t
Give your answer in its simplest form.

$\overrightarrow{XY} = -\overrightarrow{WX} + \overrightarrow{WZ} + \overrightarrow{ZY}$

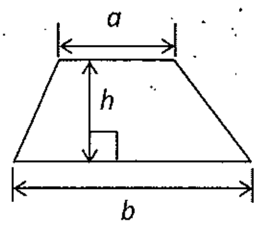
$= -s + t + 1 \cdot 5s$

....

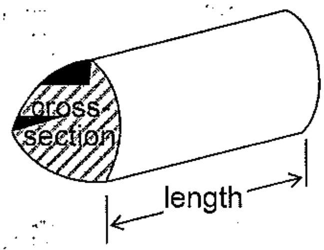
Answer .... $0 \cdot 5s + t$ .... *(2 marks)*

Formulae Sheet: Higher Tier

Area of trapezium = $\frac{1}{2}(a+b)h$

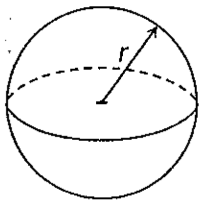


Volume of prism = area of cross-section × length



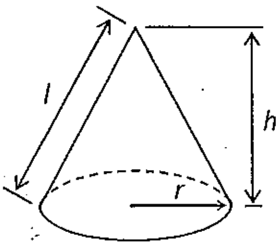
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

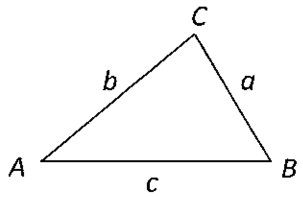


Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$



In any triangle ABC



Area of triangle = $\frac{1}{2}ab\sin C$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$