

$$i) \quad i) \quad y = x^3 e^{2x}$$

$$\frac{dy}{dx} = 3x^2 e^{2x} + 2x^3 e^{2x}$$

$$ii) \quad y = \ln(3+2x^2)$$

$$\frac{dy}{dx} = \frac{1}{3+2x^2} \times 4x = \frac{4x}{3+2x^2}$$

$$iii) \quad y = \frac{x}{2x+1}$$

$$\frac{dy}{dx} = \frac{2x+1 - 2x}{(2x+1)^2} = \frac{1}{(2x+1)^2}$$

$$2i) \quad y = \ln x \rightarrow y = -\ln(x) \rightarrow y = -\ln(x-4)$$

$$ii) \quad T, S, S$$

$$3) \quad \operatorname{cosec} \theta (3 \cos 2\theta + 7) + 11 = 0$$

$$\frac{3 \cos 2\theta}{\sin \theta} + \frac{7}{\sin \theta} + 11 = 0$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 1 - 2\sin^2 \theta$$

$$3(1 - 2\sin^2 \theta) + 7 + 11\sin \theta = 0 \quad \begin{array}{ll} -6 & 11 \\ -15 & +4 \end{array}$$

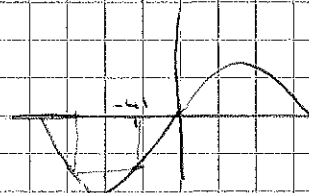
$$3 - 6\sin^2 \theta + 11\sin \theta + 7 = 0$$

$$6\sin^2 \theta - 11\sin \theta - 10 = 0$$

$$iii) \quad (3\sin \theta + 2)(2\sin \theta - 5) = 0$$

$$\sin \theta = -\frac{2}{3}$$

$$\theta = -41.8, -138.2$$



$$4) \quad y = \frac{k}{x}$$

$$\int_2^6 \frac{k}{x} dx = \ln 81$$

$$\left[ k \ln x \right]_2^6 = k(\ln 6 - \ln 2) = \ln 81$$

$$= k \ln 3$$

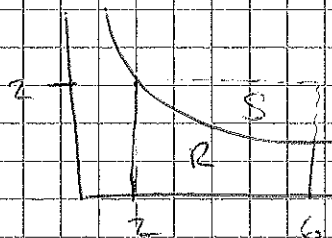
$$\ln 3^k = \ln 81$$

$$3^k = 81$$

$$\underline{k = 4}$$

$$ii) \quad y = \frac{4}{x}$$

$$\text{At } x=2 \quad y=2$$



$$\pi \int_2^6 \left( 2^2 - \frac{4^2}{x^2} \right) dx$$

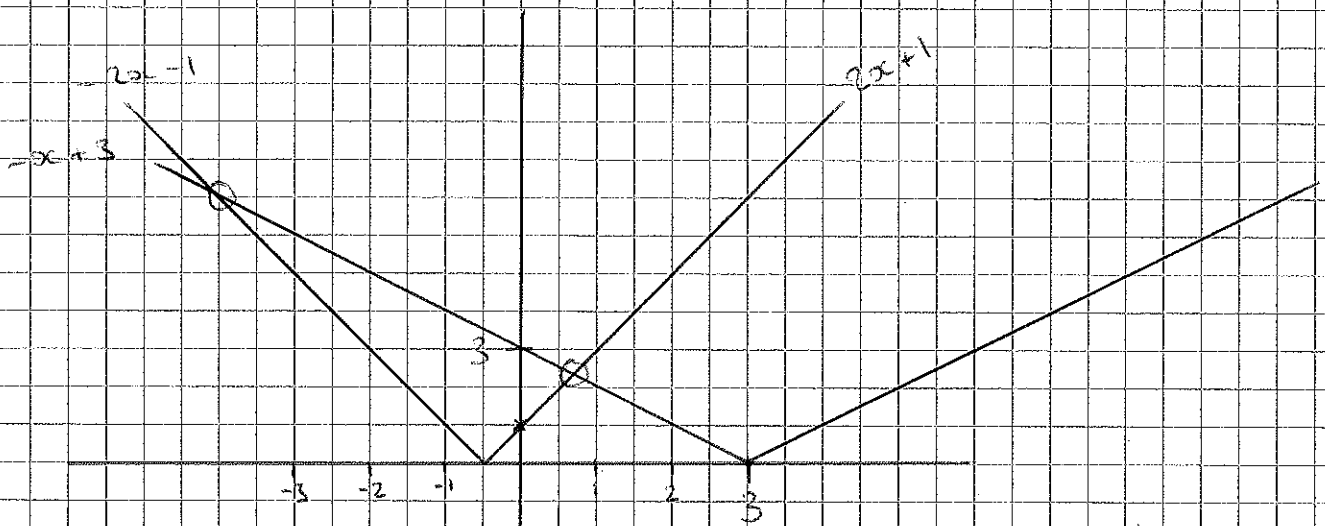
$$= \pi \left( 4x + \frac{16}{x} \right) \Big|_2^6$$

$$= \pi \left( 24 + \frac{16}{6} \right) - \left( 8 + 8 \right)$$

$$= \pi \left( 10 \frac{2}{3} \right)$$

$$= \frac{32\pi}{3}$$

5)  $|2x+1| \leq |x-3|$



$$2x+1 = -x+3$$

$$3x = 2$$

$$x = \frac{2}{3}$$

$$-x+3 = -2x+1$$

$$x = -4$$

$$-4 \leq x \leq \frac{2}{3}$$

ii)  $\Rightarrow$  when  $x = \frac{2}{3}$

$$|x+2| = \frac{2\sqrt{3}}{3}$$

i)  $\tan^2 x - x - 2 = 0$

At  $x = 1.0$

$$\tan^2 1 - 1 - 2 = -0.57$$

At  $x = 1.1$

$$\tan^2 1.1 - 1.1 - 2 = 0.76$$

Sign changed

ii)  $x_1 = 1$

$x_2 = 1.04719$

$x_3 = 1.050571$

$x_4 = 1.050809$

$x_5 = 1.050826$

$x_6 = 1.0508272$

$x = 1.05083$  (5dp)

$$\text{iii) } \tan^2 \theta + 1 = \sec^2 \theta$$

$$\sec^2 \theta - 1 = \tan^2 \theta$$

$$\sec^2 \theta - 1 - \theta - 2 = 0$$

$$\sec^2 \theta - \theta - 3 = 0$$

$$\text{when } \theta = 2x$$

$$2x = 1.05083$$

$$x = 0.5254 \text{ (3sf)}$$

$$7) \quad y = (3x-1)^4$$

$$\frac{dy}{dx} = 12(3x-1)^3$$

$$\text{At } x=1$$

$$\frac{dy}{dx} = 12(3-1)^3 = 96$$

$$y-16 = 96(x-1)$$

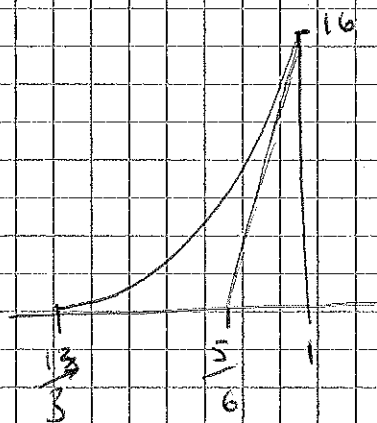
$$y = 96x - 80$$

$$\frac{80}{96} = x = \frac{5}{6}$$

$$\int_{\frac{1}{3}}^1 (3x-1)^4 dx = \frac{4}{3}$$

$$= \left[ \frac{(3x-1)^5}{15} \right]_{\frac{1}{3}}^1 = \frac{4}{3}$$

$$= \left( \frac{2^5}{15} - 0 \right) - \frac{4}{3} = \underline{\underline{0.8}}$$



$$g) \quad 3 \cos x + 3 \sin x$$

$$R = \sqrt{18}$$

$$\alpha = \tan^{-1} 1 = \frac{\pi}{4}$$

$$3 \cos x + 3 \sin x = \sqrt{18} \cos\left(x - \frac{\pi}{4}\right)$$

$$ii) \quad \frac{8}{\sqrt{18} \cos\left(x - \frac{\pi}{4}\right)} \quad \text{not defined when}$$

$$\cos\left(x - \frac{\pi}{4}\right) = 0$$

$$x - \frac{\pi}{4} = \frac{\pi}{2}$$

$$x = \frac{3\pi}{4}$$

$$b) \quad T(3x) = \frac{8}{\sqrt{18} \cos\left(3x - \frac{\pi}{4}\right)} = \frac{8}{9} \sqrt{6}$$

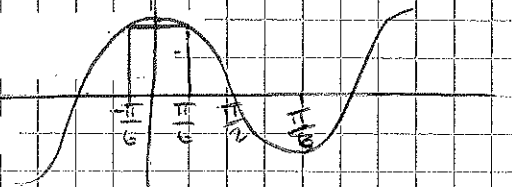
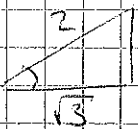
$$9 = \sqrt{18} \times \sqrt{6} \cos\left(3x - \frac{\pi}{4}\right)$$

$$\frac{9}{\sqrt{18}} = \cos\left(3x - \frac{\pi}{4}\right)$$

$$\frac{\sqrt{3}}{2} = \cos\left(3x - \frac{\pi}{4}\right)$$

$$\left(3x - \frac{\pi}{4}\right) = \frac{\pi}{6}$$

$$\frac{-\frac{\pi}{6} + \frac{\pi}{4}}{3} = \underline{\underline{\frac{\pi}{36}}}$$



9)  $f(x) = 4x^2 - 12x$        $g(x) = ax + b$

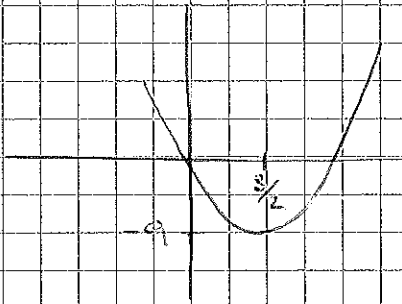
i)  ~~$f(x) = 4x(x-3)$~~

$$\begin{aligned} f(x) &= 4(x^2 - 3x) \\ &= 4\left(x - \frac{3}{2}\right)^2 - \frac{9}{1} \\ &= 4\left(x - \frac{3}{2}\right)^2 - 9 \end{aligned}$$

⇒ range  $f(x) \geq -9$

ii) because it's not a one-one function.

iii)



ii)  $y = ax + b$

$$\frac{y-b}{a} = x$$

$$g^{-1}(x) = \frac{x-b}{a}$$

$$\frac{x-b}{a} = ax + b$$

$$x - b = a^2x + ab$$

~~$$x(1 - a^2) = b + ab$$~~

$$1 = a^2 \quad \text{and} \quad -b = ab$$

So  $a = -1$

iv) \*

$$gf(x) < 5$$

$$g(x) = -x + b$$

$$gf(x) = -(4x^2 - 12x) + b$$

$$-4x^2 + 12x + b < 5$$

$$4x^2 - 12x + b > -5 \quad b > -5$$

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$$4x^2 - 12x - b + 5 > 0$$

$$\sqrt{144 - 16b}$$

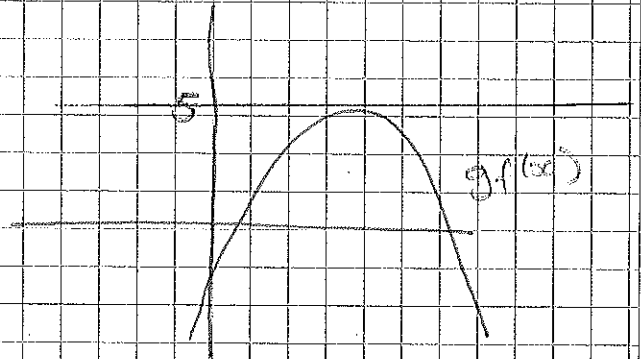
$$4\left(x - \frac{3}{2}\right)^2 - 9 > b - 5$$

$$4\left(x - \frac{3}{2}\right)^2 > b + 4$$

$$4\left(x - \frac{3}{2}\right)^2 - 4 > b$$

$a > 0$

$$\underline{b < -4}$$



$$-4x^2 + 12x + b$$

$$-4\left(x^2 - 3x\right) + b$$

$$-4\left(\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right) + b$$

$$-4\left(x - \frac{3}{2}\right)^2 + 9 + b$$

needs to be  $< 5$

$$* 9 + b < 5$$

$$b < -4$$

