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YEAR 10 MATHS – GEOMETRY & ALGEBRA

TARGET GRADE 7 OR 8

SUMMER LEARNING PROGRAMME

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**Volumes of Solids**

**1.** A child’s toy is in the shape of a cone on top of a hemisphere.
The diameter of the hemisphere is 15 cm and the overall height of the toy is 26 cm.



 Calculate the volume of this toy.

Answer ................................................................. cm3

(5 marks)

**2.** *OQP* is a sector of a circle of radius 15 cm.

 The angle of the sector is 120°.



 (a) Show that the length of the arc *PQ* is 10 cm.

**(2 marks)**

The sector is folded to form a cone.



 (b) Calculate the radius of the base of the cone.

Answer ......................................................... cm

(2 marks)

**3.** The first diagram shows a cone of base radius 12 cm and perpendicular height 10 cm.

 A small cone of base radius 6 cm and perpendicular height 5 cm is cut off the bottom to leave a frustum.

 The frustum has a lower radius of 6 cm, an upper radius of 12 cm and a perpendicular height of 5 cm (see second diagram).



Find the volume of the frustum, giving your answer in terms of .

Answer ....................................... cm3

(4 marks)

**4.** A hemispherical bowl of radius 6 cm has the same volume as a cone of perpendicular height 27 cm.

Not drawn accurately



 Calculate the base radius, *r,* of the cone.

Answer ................................................................... cm

(4 marks)

**5.** Two spheres of radius 5 cm just fit inside a tube.



Calculate the volume inside the tube not filled by the spheres.

Answer …………………………………………….. cm2

(5 marks)

**Geometry Problems involving Quadratic Equations**

**1.** The diagrams show a rectangle and an L shape
All the angles are right angles.
All lengths are in centimetres.
The shapes are equal in area.



 Calculate the value of *y*.

Answer .................................................................. cm

(6 marks)

**2.** The perimeter of a rectangle is 25 cm.
The length of the rectangle is *x* cm.

|  |
| --- |
|  |

 *x* cm Not to scale

(a) Write down an expression for the width of the rectangle in terms of *x*.

Answer ................................................................... cm

(1 mark)

 (b) The area of the rectangle is 38 cm2.
Show that 2*x*2 – 25*x* + 76 = 0

 (2 marks)

**3.** A rectangle has length (*x* + 5) cm and width (*x* – 2) cm.
A triangle has base (*x* *+* 8) cm and height *x* cm.



 The area of the rectangle is equal to the area of the triangle.

 Show that *x*2 – 2*x* – 20 = 0 (You are **not** required to solve this equation.)

 (4 marks)

**4.**

****

**3D Pythagoras and Trigonometry**

**1.** The diagram shows a cuboid.
*AB* = 3 cm, *AE* = 4 cm, *BC* = 12 cm.



Not drawn accurately

(a) Find the length of *BH*.

Answer ................................................................ cm

(2 marks)

(b) The angle between *BH* and *BD* is *x* and the angle between *BH* and *BC* is *y*.

 Which angle is bigger, *x* or *y*?
You **must** show your working.

Answer ......................................................................

(3 marks)

**2.** *ABCDEFGH* is a cuboid with sides of 5 cm, 5 cm and l2 cm as shown.



 Calculate angle *DFH*.

Answer ............................................... degrees

(5 marks)

**3*.*** *VABCD* is a right pyramid on a square base.
*V* is vertically above the centre of the square.
*VA = VB = VC = VD =* 20 cm
*AB =* 15 cm



 Calculate the angle between the edge *VA* and the base *ABCD.*

Answer ....................................................... degrees

(5 marks)

**4.** The diagram shows a door-wedge with a rectangular horizontal base *PQRS.*
The sloping face *PQTU* is also rectangular.
*PQ* = 3.8 cm and angle *TQR* = 6°
The height *TR* is 2.5 cm.



 Calculate the length of the diagonal *PT*.

Answer ................................................................. cm

(5 marks)

**5*.*** *VABCD* is a right pyramid on a rectangular base.

 *VA* = *VB* = *VC* = *VD* = 16 cm.

 *AB* = 20 cm and *BC* = 14 cm.



 Calculate the angle between the edge *VC* and the base *ABCD*.

Answer ........................................................... degrees

(5 marks)

**Circle Theorems (Alternate Segment Theorem)**

**1.** (a) *O* is the centre of the circle.
A, *B* and *C* are points on the circumference.

Write down the value of angle *x.*



Answer *x* = ........................................................degrees

(1 mark)

 (b) *P, Q* and *R* are points on the circumference of the circle.
*NPT* is the tangent to the circle at *P.*



 Calculate the value of *z*.
Give a reason for each step of your working.

Answer .......................................................degrees

(3 marks)

**2.** *ABCD* is a cyclic quadrilateral.
*PAQ* is a tangent to the circle at *A.
BC* = *CD*.
*AD* is parallel to *BC*.
Angle *BAQ* = 32°.



 Find the size of angle *BAD*.

 You **must** show all your working.

Answer Angle *BAD* = ............................... degrees

(5 marks)

**3.** (a) Points *P*, *Q*, *R* and *S* lie on a circle.

 *PQ* = *QR*

Angle *PQR* = 116°



 Explain why angle *QSR* = 32°.

 (2 marks)

(b) The diagram shows a circle, centre *O*.
*TA* is a tangent to the circle at *A*.
Angle *BAC* = 58° and angle *BAT* = 74°.



(i) Calculate angle *BOC*.

Answer Angle *BOC* = ............................... degrees

(1 mark)

 (ii) Calculate angle *OCA*.

Answer Angle *OCA* = ............................... degrees

(3 marks)

**4.** *ABCD* is a cyclic quadrilateral.
*PAQ* is a tangent to the circle at *A*.
*BC* = *CD*Angle *QAB* = 38° and angle *BAD* = 76°

Not drawn accurately



 Show that *AD* is parallel to *BC.*Give reasons to justify any values you write down or calculate.

 (4 marks)

**5.** *ABC* are three points on the circumference of a circle centre *O*.
*SCT* is a tangent to the circle.
 *SCA* = 56°  *COB* = 130°

Not drawn accurately



 Find the size of angle *OBA*.

Answer Angle *OBA* = ............................... degrees

(3 marks)

**6.** *CD* is a tangent to the circle at *C*.



 Calculate the value of *c*.

 Give reasons for your answer.

Answer ......................................................... degrees

(3 marks)

**7.** *P, Q,* and *R* are points on the circumference of the circle.
*NPT* is the tangent to the circle at *P.*



 Calculate the value of *z.*Give a reason for each step of your working.

Answer ...................................................................... degrees

(3 marks)

**8**. *ABCD* is a cyclic quadrilateral within a circle centre *O*.

 *XY* is the tangent to the circle at *A*.

 Angle *XAB* = 58°

 Angle *BAD* = 78°

 Angle *DBC* = 34°

*C*

Not drawn

accurately

34°

*B*

*D*

58°

78°

*X*

*Y*

*A*

 Prove that *AB* is parallel to *CD*.

**(5 marks)**

**9.** *PQRS* is a cyclic quadrilateral as shown.

 

3 *x*+ 10°

4 *x*+ 30°

7 *x*− 30°

Not drawn

accurately

 Prove that *PQ* is parallel to *SR*.

**(5 marks)**

**Areas and Volumes of Similar Shapes**

**1.** (a) Explain why the volume of a cube increases by a factor of 8 when the side length is doubled.

 (2 marks)

(b) June recently bought a small toy in the local shop.

|  |  |
| --- | --- |
| **ALIEN**Place inwater andit becomes6 timesbigger! |  |

 It was originally 8 cm tall.
After she placed it in water it grew to a similarly shaped alien.
The height was then 14.5 cm.
Is the claim on the pack justified?

 (3 marks)

**2.** A child’s rugby ball is 10 cm long and has a volume of 200 cm3.
It is similar in shape to a full-size rugby ball.
A full-size rugby ball is 22 cm long.



 Find the volume of the full-size ball.

Answer .......................................................... cm3

(2 marks)

 **3.** Two similar bottles are shown below.

 The smaller bottle is 20 cm tall and holds 480 ml of water.

 The larger bottle is 30 cm tall.



 How much water does the larger bottle hold?

Answer ................................................................. ml

(2 marks)

**4.** A square-based pyramid with a base of side 2 cm has a volume of 2.75 cm3.



Not to scale

 What is the volume of a similar square-based pyramid with a base of side 6 cm?

Answer ...................................................................cm3

(2 marks)

**5.** Gnomes ‘R’Us makes garden gnomes in two sizes.
The gnomes are similar in shape.
The smaller gnome is 28 cm high and the larger one is 35 cm high.



It takes 7936 cm3 of plaster to make a small gnome.

How much plaster is needed to make a large gnome?

Answer........................................................................................cm3

(3 marks)

**6.** *A* and *B* are two similar cylinders.



 The height of cylinder *A* is 10 cm and its volume is 450 cm3.The volume of cylinder *B* is 3600 cm2.

 Calculate the height of cylinder *B*.

Answer .................................................... cm

(3 marks)

**7.** A large tub holds 500 grams of butter.

 

Not drawn accurately

The height of the 250 gram size tub is 50 mm.

Work out the height of the 500 gram tub.

Answer ................................. mm

**(3 marks)**

Sine and Cosine Rule

**1.** (a) *ABC* is a triangle.
*AC =* 19 cm, *BC =* 17 cm and angle *BAC =* 60°



Not to scale

 Calculate the size of angle *ABC.*

Answer ........................................................... degrees

(3 marks)

 (b) *PQR* is a triangle.
*PR =* 23 cm, *PQ =* 22 cm and angle *QPR =* 48°



Not to scale

 Calculate the length of *QR.*Give your answer to an appropriate degree of accuracy.

Answer ........................................................... cm

(4 marks)

 **2.** *ABC* is a triangle.

Not drawn accurately



(a) Calculate the length of side *BC*.

Answer ................................................................ cm

(3 marks)

(b) Find the size of angle *BCA*.

Answer ......................................................... degrees

(3 marks)

***3.*** *ABCD* is a quadrilateral.
*AB* = 7 cm, *AD* = 6 cm and *BC* = 9 cm.
Angle *ABC* = 75° and angle *ADC* = 90°



 Calculate the perimeter of *ABCD*.

Answer ........................................................... cm

(5 marks)

**4.** In triangle *ABC, AB* = 5 cm, *BC =* 8 cm and *AC* = 9 cm.



Use the cosine rule to show that triangle *ABC* does **not** contain an obtuse angle.

 (3 marks)

**5.** Two ships, *A* and *B*, leave port at 13 00 hours.
Ship *A* travels at a constant speed of 18 km per hour on a bearing of 070°.
Ship *B* travels at a constant speed of 25 km per hour on a bearing of 152°.



 Calculate the distance between *A* and *B* at 14 00 hours.

Answer ........................................................... km

(4 marks)

**6.**



 Find the area of triangle *ABC*.

Answer ..................................... cm2

(5 marks)

Transformations of Graphs

**1.** The sketch below is of the graph of *y = x*2



 On the axes provided, sketch the following graphs.
The graph of *y = x*2 is shown dotted on each set of axes to act as a guide.

(a) *y* = *x*2 + 2



(1 mark)

(b) *y* = (*x –* 2)2



(1 mark)

(c) 



(1)

**2.** The diagram shows the graph of *y* = *x*2 for 2  *x*  2.



 Each of the graphs below is a transformation of this graph.
Write down the equation of each graph.



(c)



Answer (a) *y* = ....................................................

Answer (b) *y* = ....................................................

Answer (c) *y* = ....................................................

(3 marks)

**3.** This is the graph of *y* = sin *x* for 0°  *x*  360°



 Draw the graphs indicated for 0°  *x*  360°

 In each case the graph of *y =* sinx is shown to help you.

(a) *y =* 2 sin*x*



(1 mark)

(b) *y* = – sin*x*



(1 mark)

(c) *y* = sin 2*x*



(1 mark)

**4.** This is the graph of *y* = cos *x* for 0°  *x*  360°



 Write the equation of each of the transformed graphs.
In each case the graph of *y* = cos *x* is shown dotted to help you.

(a)



Equation *y* = ..............................................................

(1 mark)

(b)



Equation *y* = ..............................................................

(1 mark)

(c)



Equation *y* = ..............................................................

(1 mark)

(d)



Equation *y* = ..............................................................

(1 mark)

**5.** This is the graph of *y =* cos *x* for 0°  *x*  360°



(a) On the axes below draw the graph of *y =* cos(*x* – 90) for 0°  *x*  360°



(b) Write down a possible equation of the following graph.

Answer ……………………………………………

(3 marks)

**6.** The graph of *y* = sin *x* for 0°  *x*  360° is shown on the grid below.
The point *P*(90, 1) lies on the curve.



 On both of the grids that follow, sketch the graph of the transformed function.
In both cases write down the coordinates of the transformed point *P*.

(a) *y* = sin (*x* – 45)



*P* (......................., ......................)

(2 marks)

(b) *y* = 2sin*x*



*P* (......................., ......................)

(2 marks)

**Other Graphs**

**1.** Below are three graphs.

 Match each graph with one of the following equations.

 Equation A: *y* = 3*x* – *p*
Equation B: *y* = *x*2 + *p*
Equation C: 3*x* + 4*y* = *p*
Equation D: *y* = *px*3

 In each case *p* is a **positive** number.



Answer Graph (i) Equation ................................................

Graph (ii) Equation ................................................

Graph (iii) Equation ................................................

(3 marks)

**2.** Each of the graphs represents one of the following equations.

|  |  |  |
| --- | --- | --- |
| A *y* = 3*x* + 4 |  | B 2*x* + 3*y* = 12  |
|  |  |  |
| C *y* = *x*2 – 2 |  | D *y* = *x*3 |

 Write down the letter of the equation represented by each graph

Equation ................................

(1 mark)

Equation ................................

(1 mark)

Equation ................................

(1 mark)

**3.** (a) Four graphs are sketched.



 Complete the following statements.

*y* = 2*x* + 4 matches graph ………………….

*y* = *x*2 + 4 matches graph ………………….

*y* + 2*x* = 4 matches graph ………………….

(3 marks)

(b) Sketch the graph of *y* = *x*3 on the axes below.



(2 marks)

**4.** Match each of the sketch graphs to one of these equations.

**A** *y* = 2 – 2*x* **B**  *y* = 2*x* + 2**C** *y* = 3 – *x*2 **D** *y* = *x*3 + 4 **E**  *y* = 



Graph **1** represents equation ……................

Graph **2** represents equation ……................

Graph **3** represents equation ……................

Graph **4** represents equation ……................

(4 marks)

**5.** The graph shows the function 



(a) Write down the coordinates of the point where the graph intersects with the *y*-axis.

Answer ( ................................... , ................................... )

(1 mark)

b) Find the value of *a*.

Answer ……………………………………………………

(2 marks)

**6.** (a) Complete the table of values for *y* = (0.8)x

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *x* | 0 | 1 | 2 | 3 | 4 |
| *y* | 1 | 0.8 | 0.64 |  | 0.41 |

(1 mark)

 (b) On the grid below, draw the graph of *y* = (0.8)*x* for values of *x* from 0 to 4.



(2 marks)

(c) Use your graph to solve the equation (0.8)*x* = 0.76

Answer ..................................................................

(1 mark)

Method of Intersections

**1.** The grid shows the graph of *y* = *x*2 + 2*x* – 5



By drawing an appropriate straight line, solve the equation *x*2 + 2*x* – 5 = *x* – 1

Answer ......................................................................

(3 marks)

**2.** The grid below shows the graph of *y* = *x*2 + 3*x* – 2



(a) By drawing an appropriate straight line on the graph solve the equation

*x*2 + 3*x* – 3 = 0

Answer ………………………………..

(2 marks)

(b) By drawing an appropriate straight line on the graph solve the equation

*x*2 + 2*x* – 1 = 0

Answer ………………….……………..

(3 marks)

**3.** The graph *y* = *x*2 – 2*x* – 4 is drawn below for values of *x* between –3 and +4.



(a) Using the graph, find the solutions of *x*2 – 2*x* – 4 = 0, giving your answers to 1 decimal place.

Answer ............................................

(1 mark)

 (b) By drawing an appropriate linear graph, write down the solutions of

*x*2 – 3*x* – 2 = 0

Answer ............................................

(3 marks)

**4.** (a) Complete the table of values for *y* = *x*2 – 4*x* – 2

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | –2 | –1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| *y* | 10 | 3 | –2 | –5 |  | –5 | –2 | 3 | 10 |

 (1 mark)

(b) On the grid below, draw the graph *y* = *x*2 – 4*x* – 2 for values of *x* between –2 and 6.



(2 marks)

 (c) Use your graph to write down the solutions of the equation *x*2 – 4*x* – 2 = 0

Answer .............................................................

(2 marks)

(d) By drawing an appropriate linear graph, write down the solutions of

*x*2 – 5*x* –3 = 0

Answer .............................................................

(3 marks)

**Direct and Inverse Proportion**

**1.** *W* and *P* are both positive quantities.
*W* is directly proportional to the square root of *P*.
When *W* = 12, *P* = 16.

(a) Express *W* in terms of *P*.

Answer ..........................................................

(3 marks)

(b) What is the value of *W* when *P* = 25?

Answer ..........................................................

(1 mark)

(c) What is the value of *P* when *W* = 21?

Answer ..........................................................

(2 marks)

**2.** *y* is directly proportional to the square of *x.*When *y* = 5, *x* = 4.
Find the value of *y* when *x =* 8.

Answer ………………….……………..

(3 marks)

 **3.** *A* is directly proportional to 

When *A* = 50, *B* = 10

(a) Find an equation connecting *A* and *B*.

Answer …………………………………………………..

(3 marks)

(b) Find the value of *B* when *A* = 72

Answer …………………………………………………..

(2 marks)

**4.** *y* is inversely proportional to the square of *x*.
When *y* = 3, *x* = 2
Find the value of *y* when *x* = 4

Answer *y* = ..........................................................

(3 marks)

**5.** *M* and *G* are positive quantities.
*M* is inversely proportional to *G*.
When *M* = 90, *G* = 40.

 Find the value of *M* when *G =* *M*.

Answer *M*= ................................................................

(4 marks)

**Vectors**

**1.**  = –4**a** + 5**b** and  = 5**a** – **b**.



 *R* is a point on  such that*PR* : *RQ* = 1 : 2.

Express  in terms of **a** and **b**.

Answer ....................................................

(3 marks)

**2*.*** *OPQR* is a parallelogram.
*M* is the midpoint of the diagonal *OQ.
 =* 2*p* and  *=* 2*r*



Express in terms of *p* and *r*.

Answer .......................................................

(1 mark)

**3.** *ABCDEF* is a regular hexagon with centre *O*.

 ** = **a** and  = **b**



 Find expressions, in terms of **a** and **b**, for

(i) **

Answer ..................................................................

(1 mark)

(ii) **

Answer ..................................................................

(1 mark)

(iii) **

Answer ..................................................................

(1 mark)

**4.** In the diagram *OACD, OADB* and *ODEB* are parallelograms.





 Express, in terms of **a** and **b**, the following vectors.
Give your answers in their simplest form.

(i) 

Answer...............................................................................

(1 mark)

 (ii) 

Answer...............................................................................

(1 mark)

 (iii) 

Answer...............................................................................

(1 mark)

**5.** In the diagram  *=* 4**a**, = **a**,  *=* 5**b**,  *=* 3**b** and  *=*  



Not drawn accurately

Find, in terms of **a** and **b**, simplifying your answers,

(i) 

Answer ……………………………………………

(1 mark)

(ii) 

Answer ……………………………………………

(2 marks)

**6.** *PQRS* is a trapezium as shown.



*R*

*Q*

*P*

**a**

*S*

Not drawn accurately

**a** –2**b**

*SR* is parallel to *PQ*.

*SR* = 2*PQ*.

 **(a)** Write down in terms of **a** and **b** vector *SR*.

 Answer .................................

**(1 mark)**

 **(b)** Work out in terms of **a** and **b** vector *QR*.

Give your answer as simply as possible.

Answer .................................

**(2 marks)**

**7.**

****

**Volume of prism** = area of cross-section × length

**Area of trapezium** = (*a* + *b*)*h*

*h*

*a*

*b*

length

cross-

section

**Formulae Sheet: Higher Tier**

**Volume of sphere** = π*r* 3

**Surface area of sphere** = 4π*r* 2

*r*

*r*

*h*

*l*

**Volume of cone** = π*r* 2 *h*

**Curved surface area of cone** = π*r* *l*

**In any triangle** *ABC*

**Area of triangle** = *ab* sin *C*

**Sine rule**  =  = 

**Cosine rule** *a* 2 = *b* 2 + *c* 2 – 2*bc* cos *A*

**The Quadratic Equation**

The solutions of *ax* 2 + *bx* + *c* = 0, where *a* ≠ 0, are given by

 *x* = 

*A*

*B*

*C*

*a*

*b*

*c*