June 2012 C2

1 i) $(3+2x)^5 = {}^5C_0 3^5 + {}^5C_1 3^4(2x) + {}^5C_2 3^3(2x)^2 + {}^5C_3 3^2(2x)^3$

$+ {}^5C_4 3(2x)^4 + {}^5C_5 (2x)^5$

$= 243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5$

ii) $\ 243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5$

$+\ 243 - 810x + 1080x^2 - 720x^3 + 240x^4 - 32x^5$

$\quad 486$

$= \cancel{243} + 2160x^2 + 480x^4$

2 i) $\int x^2 - 2x + 5 \, dx = \frac{1}{3}x^3 - 1x^2 + 5x + c$

ii) $y = \frac{1}{3}x^3 - x^2 + 5x + c$

$y = 11$

$x = 3$

$11 = \frac{1}{3}(3)^3 - (3)^2 + 5(3) + c$

$11 = 9 - 9 + 15 + c$

$11 = 15 + c$

$c = -4$

$y = \frac{1}{3}x^3 - x^2 + 5x - 4$

(3) i) $72^\circ = \frac{2\pi}{360} \times 72$

$= \frac{2\pi}{5}$ radians.

ii) 

Area of sector $= \frac{1}{2} r^2 \theta$

$45\pi = \frac{2\pi}{10} r^2$

$225 = r^2$

$r = 15cm$

iii) Area of triangle $= \frac{1}{2} ab \sin C$

$= \frac{1}{2}(15)(15) \sin\left(\frac{2\pi}{5}\right)$

$= 106.9938581$

A of segment $= 45\pi - 106.9938581..$

$= \underline{34.4 cm^2}$

(4) $\cos^2 x = 1 - \sin^2 x$ $\therefore$ $4(1-\sin^2 x) + 7\sin x - 7 = 0$

$4 - 4\sin^2 x + 7\sin x - 7 = 0$

$-4\sin^2 x + 7\sin x - 3 = 0$

$4\sin^2 x - 7\sin x + 3 = 0$

$(\sin x - 1)(4\sin x - 3) = 0$

(3)

(4) (continued)

$$(\sin x - 1) = 0$$

$$\sin x = 1$$
$$x = 90^\circ$$

$$4\sin x - 3 = 0$$
$$\sin x = 3/4$$
$$x = 48.59^\circ,\ 131.41^\circ$$

(5) a) i) $U_1 = 4$ ii) alternating sequence.

$U_2 = \frac{1}{2}$

$U_3 = 4$

b) $U_9 = a + 8d$ $a + 8d = 18$ ①

$$S_9 = \frac{9}{2}(2a + 8d)$$

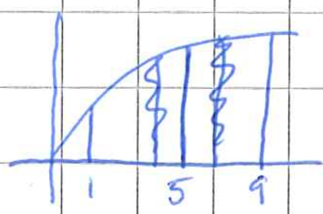
$$72 = \frac{9}{2}(2a + 8d)$$
$$16 = 2a + 8d \quad ②$$

② − ①

$$16 = 2a + 8d$$
$$18 = a + 8d$$
$$-2 = a$$

$$-2 + 8d = 18$$
$$8d = 20$$
$$d = 5/2$$

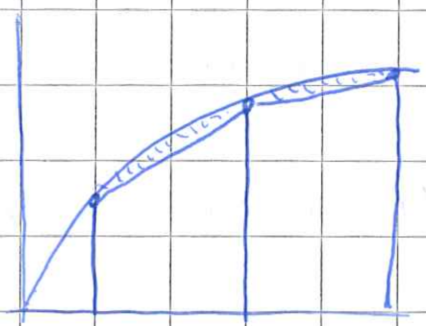
⑥ $\displaystyle\int_1^9 4\sqrt{x}\,dx = 32 + 16\sqrt{5}$



$A \approx \frac{1}{2}(4)\left(4 + 12 + 2(4\sqrt{5})\right)$

$= 2(16 + 8\sqrt{5})$

$= 32 + 16\sqrt{5}$

ii) 

The trapezium rule is an underestimate of the actual area. The shaded parts are not included.

iii) $\displaystyle\int_1^9 4\sqrt{x}\,dx = \int_1^9 4x^{1/2}\,dx = \left[\frac{8}{3}x^{3/2}\right]_1^9$

$= \frac{8}{3}(9)^{3/2} - \frac{8}{3}(1)^{3/2}$

$= 72 - \frac{8}{3}$

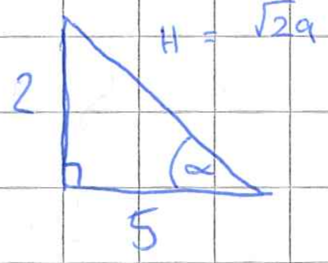
$= \underline{69\frac{1}{3}}$

7 a) i) $\tan\alpha = \frac{2}{5}$

$~~\alpha = 21.8~~$

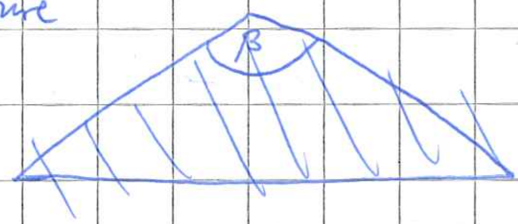
~~cos α ≤~~ $\cancel{\tan\alpha = \frac{\sin\alpha}{\cos\alpha}}$

$\cancel{\frac{2}{5} = \frac{\sin\alpha}{\cos\alpha}}$



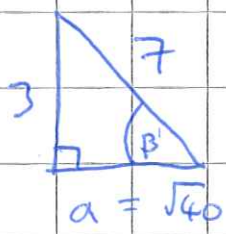
$\cos\alpha = 5/\sqrt{29}$

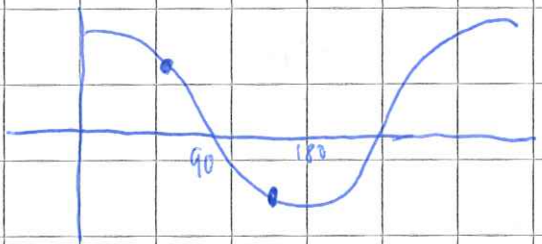
ii) $\sin\beta = 3/7$ β is obtuse



$\cancel{\sin\beta = \frac{O}{H} = \frac{3}{7}}$

find ~~cos β~~ acute version

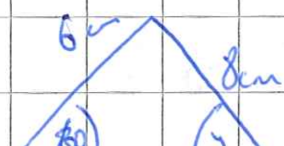




$\cos\beta' = \frac{\sqrt{40}}{7}$

$\cos\beta = -\frac{\sqrt{40}}{7}$

b)



$\frac{\sin 60}{8} = \frac{\sin y}{6}$

$\frac{\frac{6\sqrt{3}}{2}}{8} = \sin y$

$\sin y = \frac{3\sqrt{3}}{8}$

(8) if $(x-2)$ is a factor $f(2)$ and $g(2)=0$

$f(2) = 2^3 + 2(a-3) + 2b$ $\quad$ $g(2) = 3(2)^3 + 2^2 + 10a + 4b$

$0 = 8 + 2a - 6 + 2b$ $\quad$ $0 = 24 + 4 + 10a + 4b$

$0 = 2 + 2a + 2b$ $\quad$ $0 = 28 + 10a + 4b$

$2a + 2b = -2$ $\quad$ $10a + 4b = -28$

$a + b = -1$ ① $\quad$ $2.5a + b = -7$ ②

② − ①

$$\begin{array}{rcl} 2.5a + b &=& -7 \\ a + b &=& -1 \\ \hline 1.5a &=& -6 \\ a &=& -4 \end{array}$$

$-4 + b = -1$

$b = 3$

ii) $f(x) = x^3 - 7x + 6 = (x-2)(x^2 + bx - 3)$

$$\begin{array}{r} bx^2 \\ -\ 2x^2 \\ \hline b - 2 = 0 \\ b = 2 \end{array}$$

$\Rightarrow x^3 - 7x + 6 = (x-2)(x^2 + 2x - 3)$

$g(x) = 3x^3 + x^2 - 20x + 12 = (x-2)(3x^2 + bx - 6)$

$$\begin{array}{r} bx^2 \\ -\ 6x^2 \\ \hline b - 6 = 1 \\ b = 7 \end{array}$$

$= (x-2)(3x^2 + 7x - 6)$

8 ii) (continued)

$$f(x) = (x-2)(x^2+2x-3)$$

$$= (x-2)(x+3)(x-1)$$

$$g(x) = (x-2)(3x^2+7x-6)$$

$$= (x-2)(3x-2)(x+3)$$

$(x+3)$ is the second common factor of $g(x)$ and $f(x)$

9 a) i) $a = \log_2 27$ $d = \log_2 x$

$$U_4 = \log_2 27 + 3\log_2 x$$
$$= \log_2 27 + \log_2 x^3$$
$$= \log_2 (27x^3)$$

ii) $\cancel{6 = \log_2 27x^3}$

$\cancel{6 = \log_2 27 + 3\log x}$

$\cancel{6 - \log_2 27 = 3\log x}$

$$6 = \log_2 64$$

$$\log_2 64 = \log_2 27x^3$$

$$\therefore 64 = 27x^3$$

$$x = \sqrt[3]{\frac{64}{27}}$$

$$x = \frac{4}{3}$$

$$\log_2 y = 6$$
$$y = 2^6$$
$$y = 64$$

b) i) $a = \log_2 27$ $\quad r = \log_2 y$

$$S_\infty = \frac{a}{1-r} \qquad -1 < r < 1$$

$$\therefore \quad -1 < \log_2 y < 1$$

$$\frac{1}{2} < y < 2$$

$$\log_2 y < 1$$
$$y < 2^1$$
$$y < 2$$

$$\log_2 y > -1$$
$$y > 2^{-1}$$
$$y > 1/2$$

ii) $$S_\infty = \frac{\log_2 27}{1 - \log_2 y}$$

$$3(1 - \log_2 y) = \log_2 27$$
$$3 - 3\log_2 y = \log_2 27$$
$$3 - \log_2 y^3 = \log_2 27$$
$$3 = \log_2(27y^3)$$

$$\log_2 8 = 3$$

$$\log_2 8 = \log_2(27y^3)$$

$$8 = 27y^3$$
$$y = \cancel{\sqrt[3]{\frac{27}{8}}} \; \sqrt[3]{\frac{8}{27}}$$
$$y = \cancel{\frac{3}{2}} \; \frac{2}{3}$$