October 2021

AS Pure Maths Paper 1

1

$$y = 2x^3 - 4x + 5$$

$$\frac{dy}{dx} = 6x^2 - 4$$

$$x = 2 \qquad \frac{dy}{dx} = 20 \qquad y = 13$$

$$y = mx + c \qquad m = \frac{y - y_1}{x - x_1}$$

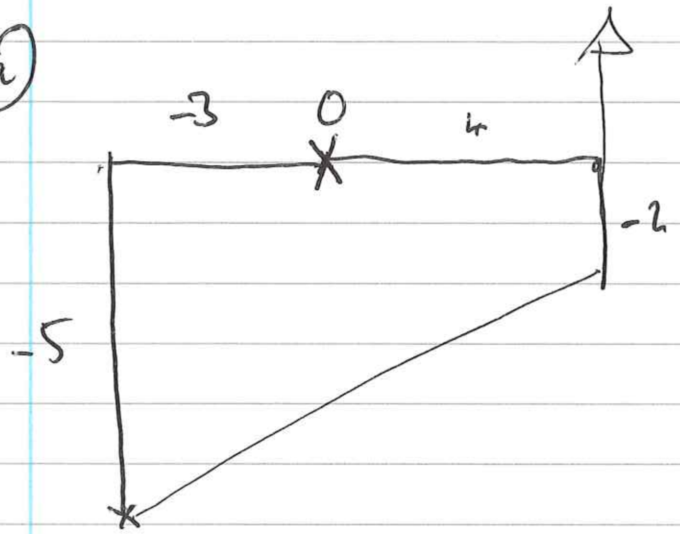
$$20 = \frac{y - 13}{x - 2}$$

$$20x - 40 = y - 13$$

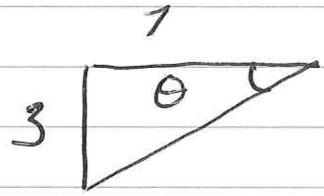
$$y = 20x \mp 27$$

(2)

(a)



so



$\tan\theta = \frac{3}{7} \Rightarrow \theta = 23.19859051$

$270 - 23.19859051$
$= 246.8014095$

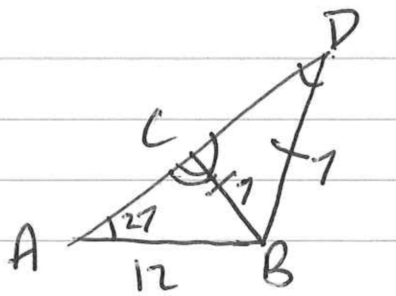
$\underline{\underline{247^\circ}}$ to 3SF

(b) $\sqrt{49+9} = \sqrt{58}$

Time 2hrs 45 min 2.75 hrs

$\frac{\sqrt{58}}{2.75} = \frac{4\sqrt{58}}{11} = 2.77 \text{ Kmh}^{-1}$

5a



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin C}{12} = \frac{\sin 27}{7} \Rightarrow C = 51.10239783$$

But as obtuse $180 - 51.10239783 = 128.8976022$

128.9°

(b) So $B = 180 - 128.9 - 27 = 24.1$

$DCB = 180 - 128.9 = 51.1$

$CDB = 51.1$

$CBD = 180 - 2 \times 51.1 = 77.8$

$$\frac{CD}{\sin 77.8} = \frac{7}{\sin 51.1} \Rightarrow CD = 8.791482807$$

$$\frac{AC}{\sin 24.1} = \frac{7}{\sin 27} \Rightarrow AC = 6.29597585$$

So $AD = 15.08745866$

$12 + 7 + 7 + 16 = 42$

3(i) $x\sqrt{2} - \sqrt{18} = x$

$$\sqrt{18} = x\sqrt{2} - x$$

$$\sqrt{18} = x(\sqrt{2}-1)$$

$$x = \frac{\sqrt{18}}{(\sqrt{2}-1)} \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)} = \frac{6+\sqrt{18}}{2-1}$$

(ii) $4^{3x-2} = \frac{1}{2\sqrt{2}}$

$$(2^2)^{3x-2} \quad 2^{3/2} = 1$$

$$2^{6x-4} \quad 2^{3/2} = 2^0$$

$$2^{6x-4+3/2} = 2^0$$

$$6x - 4 + 3/2 = 0$$

$$x = \frac{4 - 3/2}{6} = 5/12$$

4. (a) $y = mx + c$

emissions A

yr

n yrs after 1997

$$m = \frac{190-169}{-8} = -\frac{21}{8}$$

$$190 = -\frac{21}{8} \times 0 + C \Rightarrow C = 190$$

$$A = -\frac{21}{8}n + 190$$

(b) 2016 $A = 120$

As this is extrapolating a value outside of the data range, then not a suitable model.

6 a $(1+kx)^{10} \approx 1 + \binom{10}{1}kx + \binom{10}{2}k^2x^2$

$$+ \binom{10}{3}k^3x^3 + \cdots$$

$$\approx 1 + 10kx + 45k^2x^2 + 120k^3x^3 + \cdots$$

b $3(10k) = 120k^3$

$$0 = 120k^3 - 30k$$

$$0 = 30k(4k^2 - 1)$$

$$0 = 30k(2k+1)(2k-1)$$

$k = 0$ not allowed as none zero

$\underline{\underline{k = -1/2}}$ or $\underline{\underline{k = 1/2}}$

(7)

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) dx = 4$$

$$\int_1^k \left(\frac{5}{2} x^{-1/2} + 3\right) dx = \left[5 x^{1/2} + 3x\right]_1^k$$

$$= 5\sqrt{k} + 3k - (5+3) = 4$$

$$3k + 5\sqrt{k} - 12 = 0$$

(b) let $y = \sqrt{k}$

$y^2 = k$

$$3y^2 + 5y - 12 = 0$$

$$(3y-4)(y+3) = 0$$

$y = \frac{4}{3}$ $\quad$ $y = -3$

$\sqrt{k} = \frac{4}{3}$ $\quad$ $\sqrt{k} = -3$

so $\quad$ not possible

$$k = \frac{16}{9}$$

(8) $\theta = 18 + 65e^{-t/8}$ t

(a) $t=0$ $\theta = 18+65 = 83$

(b) $35 = 18 + 65e^{-t/8}$

$e^{-t/8} = 17/65$

$$\frac{-t}{8} = \ln(17/65)$$

$t = -8\ln(17/65) = 10.72439141$

$t = 10.7$

(c) As $t \Rightarrow \infty$ $65e^{-t/8} \Rightarrow 0$

So $18 + 65e^{-t/8} \Rightarrow 18$

It will always be above $18°$

$\mu = A + Be^{-t/8}$

So $94 = A + Be^{-0/8}$ $\Rightarrow$ ① $94 = A + B$
$50 = A + Be^{-8/8}$ ② $50 = A + Be^{-1}$

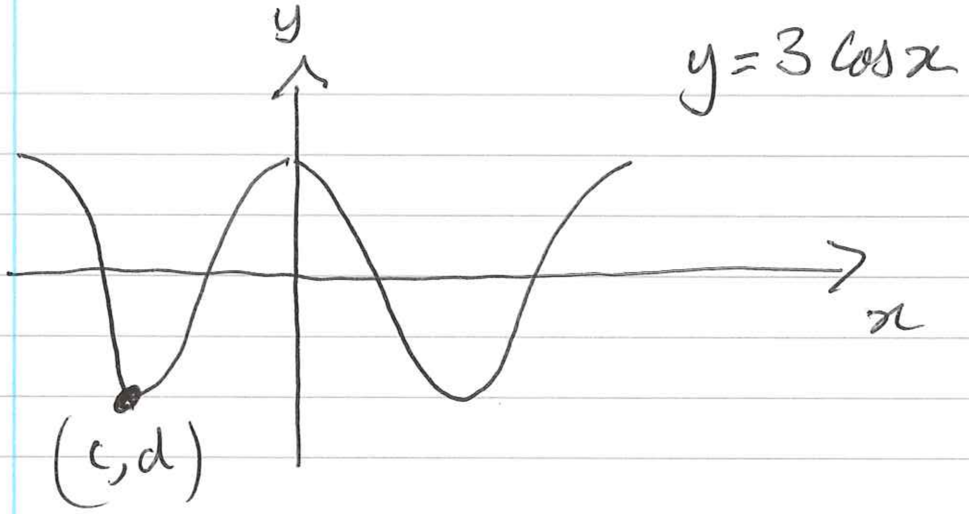
So ① − ② $44 = B(1-e^{-1})$

$$\Rightarrow B = \frac{44}{1-e^{-1}} = 69.6069751$$

$A = 24.3930249$

So asymptote $\mu = 24.4$

9.



(a) $c = -180$ $d = -3$

(b) (i) $y = 3\cos\left(\frac{x}{4}\right)$

$\frac{y}{3} = \cos\left(\frac{x}{4}\right)$

stretch 3 in y direction and 4 in x.

~~so~~ but already 3 stretch in y direction

so $(-720, -3)$

(ii) $y = 3\cos(x - 36)$

translation $36°$ in positive x direction

$(-144, -3)$

(c) $3\cos\theta = \frac{8\sin\theta}{\cos\theta} \Rightarrow 3\cos^2\theta = 8\sin\theta$

$3(1 - \sin^2\theta) = 8\sin\theta$

$3\sin^2\theta + 8\sin\theta - 3 = 0$

9. $(3\sin\theta - 1)(\sin\theta + 3) = 0$

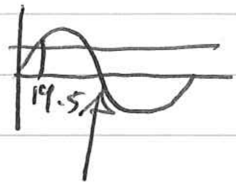
$\sin\theta = 1/3 \qquad \sin\theta = -3$

$\theta = \sin^{-1}(1/3)$ not possible

$= 19.47122063$

19.5 to 3SF

but want $450° \leq \theta < 720$



19.5, 160.5,

379.5, 520.5,

739.5, 880.5

$180 - 19.5 = 160.5$

520.5 only.

(10) $g(x) = 2x^3 + x^2 - 41x - 70$

(a) $2(5)^3 + (5)^2 - 41(5) - 70 = 0$

So 5 is a root
$(x-5)$ is a factor

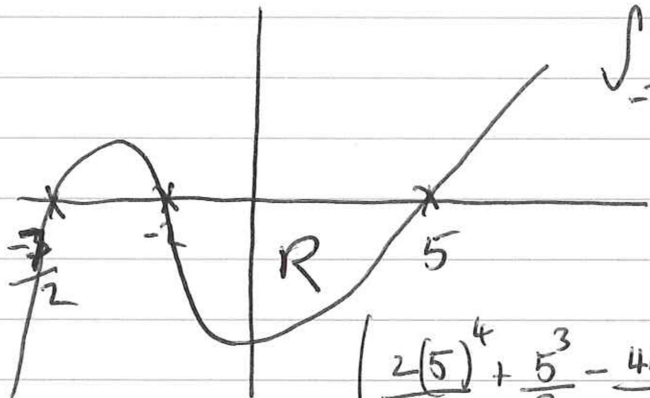
$$
\begin{array}{r l}
 & 2x^2 + 11x + 14 \\
x - 5 & \overline{\big)\, 2x^3 + x^2 - 41x - 70} \\
 & -(2x^3 - 10x^2) \\
 & \quad 11x^2 - 41x - 70 \\
 & \quad -(11x^2 - 55x) \\
 & \qquad 14x - 70 \\
 & \qquad -(14x - 70) \\
 & \qquad\qquad 0
\end{array}
$$

$2x^2 + 11x + 14$

$(x+2)(2x+7) = 0$

$g(x) = (x-5)(x+2)(2x+7)$

(b)



$$\int_{-2}^{5} 2x^3 + x^2 - 41x - 70 \, dx$$

$$\left[\frac{2x^4}{4} + \frac{x^3}{3} - \frac{41x^2}{2} - 70x\right]_{-2}^{5}$$

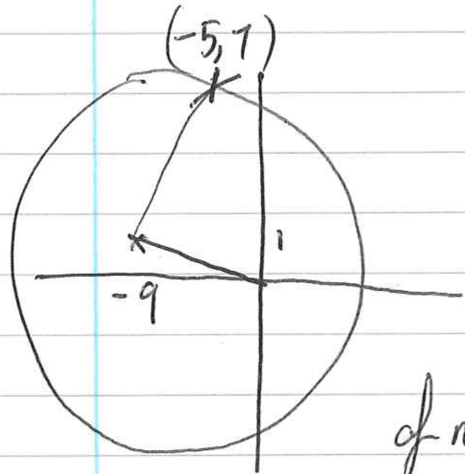
$$\left(\frac{2(5)^4}{4} + \frac{5^3}{3} - \frac{41(5)^2}{2} - 70(5)\right) - \left(\frac{2(-2)^4}{4} + \frac{(-2)^3}{3} - \frac{41(-2)^2}{2} - 70(-2)\right)$$

$= -\frac{1715}{3}$ Area $= 1715/3$

11 (i) $x^2 + 18x + y^2 - 2y + 30 = 0$

$(x+9)^2 - 81 + (y-1)^2 - 1 + 30 = 0$

$(x+9)^2 + (y-1)^2 = (\sqrt{52})^2$



Centre $(-9,1)$

From $(-9,1) \rightarrow (-5,7)$

gradient of normal $= \dfrac{\Delta y}{\Delta x} = \dfrac{-6}{-4} = \dfrac{3}{2}$

Gradient of tangent $-\dfrac{2}{3}$ point $(-5,7)$

$m = \dfrac{y - y_1}{x - x_1}$ $\quad -\dfrac{2}{3} = \dfrac{y-7}{x+5}$

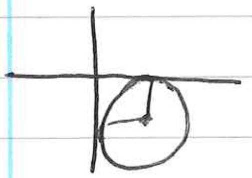
$y = -\frac{2}{3}x - \frac{10}{3} + 7$

$y = -\frac{2}{3}x + \frac{11}{3}$

$3y + 2x - 11 = 0$

(ii) $(x-4)^2 - 16 + (y+6)^2 - 36 + k = 0$

$(x-4)^2 + (y+6)^2 = (\sqrt{52-k})^2$



$4 > (\sqrt{52-k})$

$16 > 52 - k$

$k > 36$

12. $\log_{10} V = 0.072t + 2.379$ $\quad 1 \le t \le 30$

$t \in \mathbb{N}$

(a)

$$10^{\log_{10} V} = 10^{0.072t + 2.379}$$

$$V = \underbrace{\left(10^{2.379}\right)}_{a}\underbrace{\left(10^{0.072}\right)}_{b}^{t}$$

$a = 239.3315756$ $\quad a = 239$ $\quad$ 3SF

$b = 1.180320636$ $\quad b = 1.18$ $\quad$ 3SF

(b) $ab = ab^1$ = the number of views after 1 day.

$V = 239 \; 1.18^{20} = 6547$

6500

13 (a) $$\frac{4a}{b} + \frac{b}{a} \geq 4$$

$$\frac{4a^2 + b^2}{ab} \geq 4$$

Note multiplying by ~~the inequality~~ a and b does not "reverse" the inequality as both a and b are >0.

$$4a^2 + b^2 \geq 4ab$$

$$4a^2 - 4ab + b^2 \geq 0$$

$$(2a-b)(2a-b) \geq 0$$

$$(2a-b)^2 \geq 0$$

So as squaring always ≥ 0

(b) let $a = -1$ and $b = 1$

then $$\frac{4a}{b} + \frac{b}{a} = \frac{-4}{1} + \frac{1}{-1} = -5 \geq 4 \times$$

So as this is not true, the result is not true for all values of a and b.

14 (a) $g(x) = ax^3 + bx^2 + ax + c$

if $x = 0$ $\quad g(x) = 0$ $\quad$ so $\quad c = 0$

$g'(x) = 3ax^2 + 2bx + a$

if $x = 2$ $\quad g'(2) = 0$ $\quad$ so $\quad 0 = 12a + 4b + a$

$0 = 13a + 4b$

$x = 2$ $\quad g(2) = 9$ $\quad 9 = 8a + 4b + 2a$

so (1) − (2)

$$\begin{aligned} 0 &= 13a + 4b \\ 9 &= 10a + 4b \\ \hline -9 &= 3a \\ a &= -3 \end{aligned}$$

$0 = 13 \times -3 + 4b \Rightarrow b = \frac{39}{4}$

$g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$

(b) $g''(x) = 6ax + 2b$

if $x = 2$ $\quad -36 + \frac{39}{2} < 0$

so maximum