June 2010 C2

(1) i) $f(2) = 0 \quad \therefore \quad 2^3 + a \times 2^2 - 2a - 14 = 0$

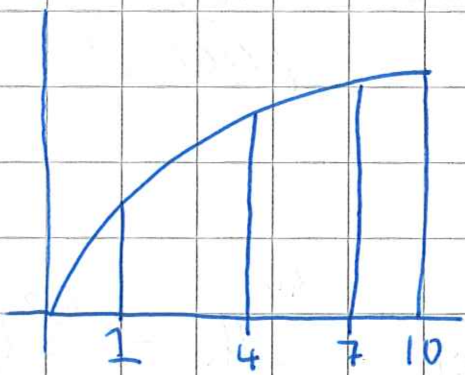
$$8 + 4a - 2a - 14 = 0$$
$$2a = 6$$
$$\underline{\underline{a = 3}}$$

$$f(x) = x^3 + 3x^2 - 3x - 14$$

ii) $f(-1) = -1 + 3 + 3 - 14$

$$= \underline{\underline{-9}}$$

(2) i) $y = \sqrt[3]{7+x}$



$$\text{Area} \approx \frac{3}{2}\left(y_0 + y_3 + 2(y_1 + y_2)\right)$$

$$= \tfrac{3}{2}\left(2 + \sqrt[3]{17} + 2\left(\sqrt[3]{11} + \sqrt[3]{14}\right)\right)$$

$$= 20.7592\ldots$$
$$= 20.8 \quad (\text{to } 3\text{sf})$$

ii) More strips.

3 i) $\left(1+\frac{1}{2}x\right)^{10} = {}^{10}C_0 1^{10} + {}^{10}C_1 1^9\left(\frac{1}{2}x\right) + {}^{10}C_2 1^8\left(\frac{1}{2}x\right)^2 + {}^{10}C_3 1^7\left(\frac{1}{2}x\right)^3 + \dots$

$= 1 + 5x + 11.25x^2 + 15x^3 + \dots$

ii)

$\left(3+4x+2x^2\right)\left(1+5x+11.25x^2+15x^3+\dots\right)$

$\underline{\underline{x^3}}$

$3 \times 15x^3 = 45x^3$

$4x \times 11.25x^2 = 45x^3$

$2x^2 \times 5x = 10x^3$ +

$100x^3$

4 $U_n = 5n+1$

i) $U_1 = 6$

$U_2 = 11$

$U_3 = 16$

ii) $\sum_{n=1}^{40} U_n = \frac{40}{2}\left(2a+(n-1)d\right)$

$= 20\left(12+39\times 5\right)$

$= 4140$

iii) $W_1 = 2$

$W_2 = 11$

$W_3 = 56$

$5n+1 = 56$

$5n = 55$

$\underline{n = 11}$

check $5\times 11 + 1 = 56$ QED

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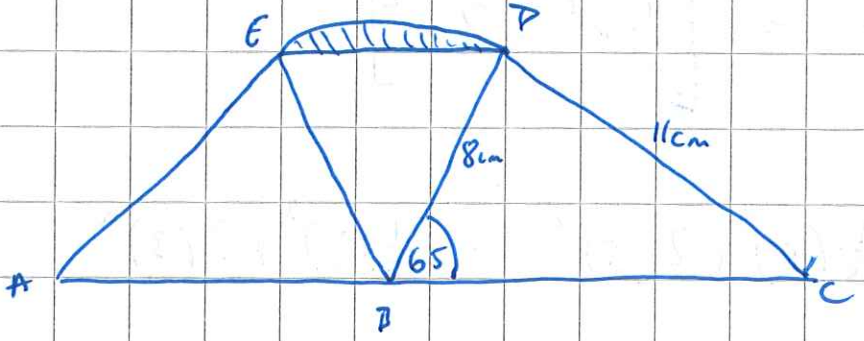


$$\frac{\sin BCD}{8} = \frac{\sin 65}{11}$$

$$B\hat{C}D = \sin^{-1}\left(\frac{8\sin 65}{11}\right)$$

$$= 41.23377\ldots$$

$$= 41.2^\circ \ (3sf)$$

ii) a)

$$E\hat{B}D = 180 - (2 \times 65^\circ)$$

$$= 50^\circ$$

$$50^\circ \equiv \frac{50}{360} \times 2\pi \text{ radians.}$$

$$= 0.872664\ldots$$

$$= 0.873 \text{ radians.}$$

b) Area of shaded region = Area of sector − Area of triangle

$$= \tfrac{1}{2}r^2\theta - \tfrac{1}{2}ab\sin C$$

$$= \tfrac{1}{2} \times 8^2 \times 0.873 - \tfrac{1}{2} 8^2 \sin 50^\circ$$

$$= 27.936 - 24.513\ldots$$

$$= 3.422577\ldots$$

$$= \underline{3.42 \text{ units}^2}$$

⑥ a) $\int_3^5 x^2+4x \, dx = \left[\frac{1}{3}x^3 + 2x^2\right]_3^5$

$= \frac{1}{3}(5)^3 + 2(5)^2 - \frac{1}{3}(3)^3 + 2(3)^2$

$= \frac{275}{3} - 27$

$= \frac{194}{3}$

b) $\int 2 - 6y^{\frac{1}{2}} \, dy = 2\cancel{x}y - 4y^{3/2} + c$

c) $\int_1^\infty \frac{8}{x^3} \, dx = \int_1^\infty 8x^{-3} \, dx = \left[-4x^{-2}\right]_1^\infty$

$= \cancel{\frac{-4}{x^2}} \; \frac{-4}{\infty^2} - \frac{-4}{1^2}$

$= 4$

$\frac{-4}{\infty^2}$ value will tend towards 0, $- \frac{-4}{1} = 4$.

7) i) $$\frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$$

$$= \tan^2 x - 1$$

ii) $\tan^2 x - 1 = 5 - \tan x$

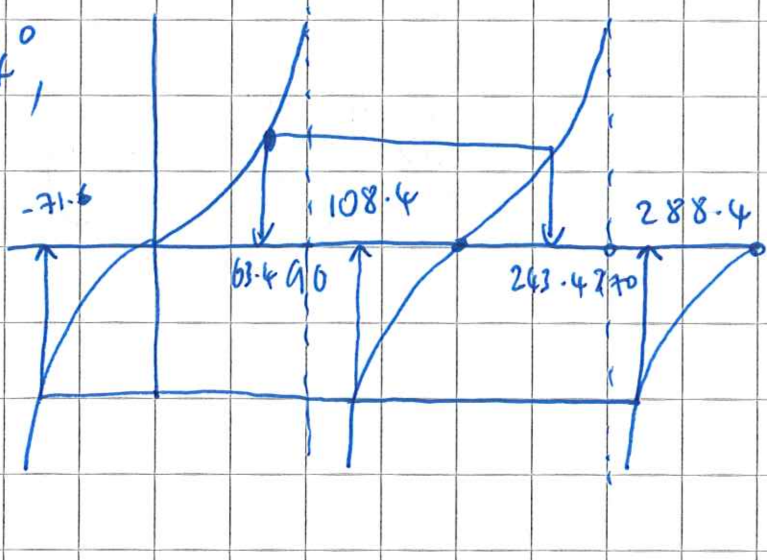
$\tan^2 x + \tan x - 6 = 0$

$(\tan x + 3)(\tan x - 2) = 0$

$\tan x = -3 \qquad \tan x = 2$

$\cancel{\tan}\ x = -71.56 \qquad x = 63.4$

$x = 108.4^\circ, 288.4^\circ,$
$63.4^\circ, 243.6^\circ$



8 a) $5^{3w-1} = 4^{250}$

$$(3w-1)\log 5 = 250\log 4$$

$$3w-1 = \frac{250\log 4}{\log 5}$$

$$w = \left(\frac{250\log 4}{\log 5} + 1\right) \div 3$$

$$w = 72.11275\ldots$$
$$w = 72.1 \ (3\text{ sf})$$

b) $\log_x(5y+1) - \log_x 3 = 4$

$$\log_x\left(\frac{5y+1}{3}\right) = 4$$

$$x^4 = \frac{5y+1}{3}$$

$$\frac{3x^4-1}{5} = y$$

9 i) $a = a$

$ar = a + d$ (1) $d = ar - r$

$ar^4 = a + 2d$ (2)

$\cancel{ar^4 = a + ar - r}$

$\cancel{0 = a + ar - r - ar^3}$

$\cancel{ar^4 = a + r(a-1)}$

$\times$ (1) by 2

$$2ar = 2a + 2d \quad (3)$$
$$ar^4 = a + 2d \quad (2)$$
$$2ar - ar^4 = a$$

$$0 = a - 2ar + ar^3$$

$(\div \text{by } a)$

$$r^3 - 2r + 1 = 0$$

ii) $-1 < r < 1$

$$r^3 - 2r + 1 = (r-1)(r^2 + r - 1)$$

$r-1$ must be a factor!

$$r \neq -1 \qquad (r^2 + r - 1) = 0$$

won't factorise...

$$r = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

Hence $r = \dfrac{-1 + \sqrt{5}}{2}$

iii) $S_\infty = \dfrac{a}{1-r}$

$$\frac{a}{1-r} = 3 + \sqrt{5}$$

$$a = (3 + \sqrt{5})\left(1 - \frac{-1+\sqrt{5}}{2}\right)$$

$$= (3+\sqrt{5})\left(\frac{3-\sqrt{5}}{2}\right)$$

$$a = 2$$