# June 2009 (C2)

**1**

a)


$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 - b^2 - c^2 = -2bc\cos A$$

$$\frac{-a^2+b^2+c^2}{2bc} = \cos A$$

$$A = \cos^{-1}\left(\frac{-a^2+b^2+c^2}{2bc}\right)$$

$$= 114.9036598 \text{ (c)}$$

$$= \underline{\underline{114.9^\circ}}$$

b) $A = \frac{1}{2}(6.4)(7)\sin(114.9)$

$= 20.31718\ldots$

$= \underline{\underline{20.3\text{cm}^2}}$

**2** i) $U_{10} = a + (n-1)d$

$= a + 9d$

$U_4 = a + 3d$

$2a + 6d = a + 9d$

$a + 6d = 9d$

$a - 3d = 0$ (1)

$U_{20} = a + 19d$

$a + 19d = 44$ (2)

$$\begin{aligned} \cancel{a} + 19d &= 44 \\ \cancel{a} - 3d &= 0 \\ \hline 22d &= 44 \\ \underline{d} &\underline{= 2} \end{aligned}$$

$a = 44 - 19d$

$= 44 - 38$

$\underline{a = 6}$

ii) $S_{50} = \frac{50}{2}(2a + 49d)$

$= 25(\cancel{6}^{12} + 98)$

$= \cancel{2???} \; 2750$

③ $7^x = 2^{x+1}$

$x \log 7 = (x+1)\log 2$

$x \log 7 = x \log 2 + \log 2$

$x \log 7 - x \log 2 = \log 2$

$x(\log 7 - \log 2) = \log 2$

$x = \frac{\log 2}{\log 7 - \log 2}$

$x = 0.5532947550$

$\underline{x = 0.55}$ (2dp).

④ i) $(x^2-5)^3 = {}^3C_0 x^6 + {}^3C_1 x^4(-5) + {}^3C_2 x^2(-5)^2 + {}^3C_3(-5)^3$

$= x^6 \cancel{+} -15x^4 + 75x^2 - 125$

ii) $\int (x^2-5)^3 \, dx = \int x^6 - 15x^4 + 75x^2 - 125 \, dx$

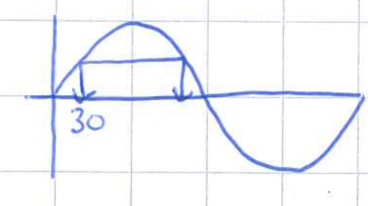
$= \frac{1}{7}x^7 - 3x^5 + 25x^3 - 125x + C$

⑤ $0 \leq x \leq 180°$ $\qquad$ $0 \leq 2x \leq 360°$

i) $\sin 2x = 0.5$ $\qquad$ $2x = \sin^{-1}(0.5)$

$2x = 30, 150$

$\therefore$ $\underline{x = 15°, 75°}$




ii) $2\sin^2 x = 2 - \sqrt{3}\cos x$

$* \sin^2 x + \cos^2 x = 1$

$$2(1 - \cos^2 x) = 2 - \sqrt{3}\cos x$$
$$2 - 2\cos^2 x = 2 - \sqrt{3}\cos x$$
$$0 = 2\cos^2 x - \sqrt{3}\cos x$$
$$\cos x(2\cos x - \sqrt{3}) = 0$$

$\cos x = 0$ $\qquad$ and $\qquad$ $2\cos x - \sqrt{3} = 0$

$\underline{x = 90°}$

$\cos x = \frac{\sqrt{3}}{2}$


$x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\underline{x = 30°}$

⑥ $\frac{dy}{dx} = 3x^2 + a$

$y = x^3 + ax + c$

$(-1, 2)$ $2 = (-1)^3 + \cancel{a}(-1)a + c$

$2 = -1 - a + c$

$3 = -a + c$ ①

$(2, 17)$ $17 = (2)^3 + 2a + c$

$9 = 2a + c$ ②

$9 = 2a + c$ $-$

$3 = -a + c$

$6 = 3a$

$\underline{\underline{a = 2}}$

$3 = -2 + c$

$c = 5$

$\Rightarrow$ eqn of curve:

$y = x^3 + 2x + 5$

⑦ $f(x) = 2x^3 + 9x^2 + 11x - 8$

i) $f(-2) = 2(-2)^3 + 9(-2)^2 + 11(-2) - 8$

$= \cancel{16} -16 + 36 - 22 - 8$

$= -10$

ii) $(2x-1)$ is a factor so $f(0.5) = 0$

$f(0.5) = 2(0.5)^3 + 9(0.5)^2 + 11(0.5) - 8$

$= 0.25 + 2.25 + 5.5 - 8$

$= 0$ Hence $(2x-1)$ is a factor.

iii) $f(x) = 2x^3 + 9x^2 + 11x - 8$

$= (2x-1)(\cancel{a}x^2 + bx + 8)$

$$\begin{array}{r} -\cancel{a}x^2 \\ +2bx^2 \\ \hline (-\cancel{1}+2b)x^2 \end{array}$$

$-\cancel{1}+2b = 9$

$2b = 10$

$b = \cancel{5}\ 5$

$f(x) = (2x-1)(x^2 + 5x + 8)$

iv) Using quadratic discriminant:

$b^2 - 4ac = 5^2 - 4(1)(8)$

$= 25 - 32$

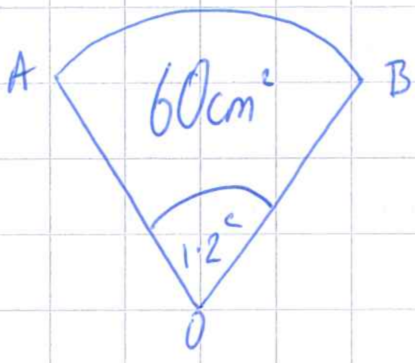
$= -7$

$-7 < 0$ Hence quadratic has no solutions.

$f(x)$ has 1 real root from $2x-1$

$(x = \frac{1}{2})$

(8)




$$A = \frac{\theta}{2} r^2$$

$$60 = \frac{1.2}{2} r^2$$

$$\sqrt{\frac{120}{1.2}} = r$$

$$r = 10\text{cm}$$

Arc length $= r\theta$

$= 10 \times 1.2$

$= 12\text{cm}$

$\therefore$ perimeter $= 2 \times 10 + 12$

$= 32\text{cm}$

ii) a) $5^{th}$ pattern $= 60 \times \left(\frac{3}{5}\right)^4$

$= 7.776\text{cm}^2$

b) $a = 60 \quad r = 3/5$

$$S_{10} = \frac{a(1 - 3/5^n)}{1 - 3/5} = \frac{60\left(1 - \left(\frac{3}{5}\right)^{10}\right)}{1 - 3/5}$$

$= 149.0930074$

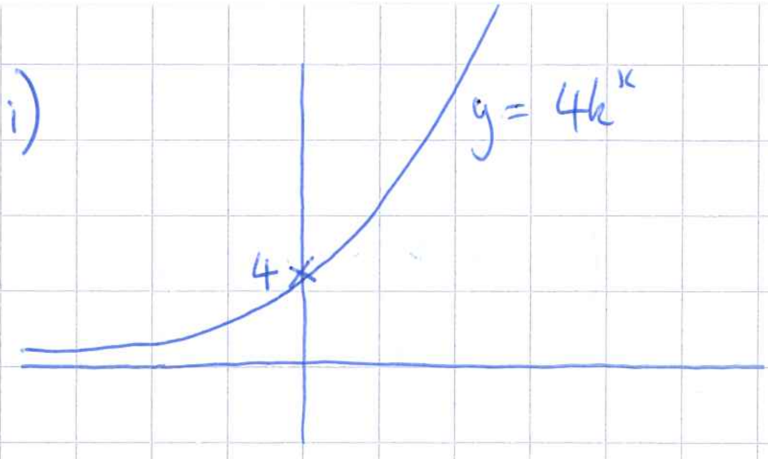
$= 149.1\text{cm}^2$

c) $r < 1$ Hence series will converge to a limit

$$S_\infty = \frac{a}{1-r} = \frac{60}{1 - 3/5}$$

$$\underline{\underline{= 150}}$$

9 i)




ii) $y = 4k^x$

$20k^2 = 4k^x$

$5k^2 = k^x$

$\cancel{2(\log_k(5k)) = x\log_k k}$ $\quad \log_k 5k^2 = \log_k k^x$

$\cancel{2(\log_k 5 + \log_k k) = x\log_k k}$ $\quad \log_k 5 + 2\log_k k = x\log_k k$

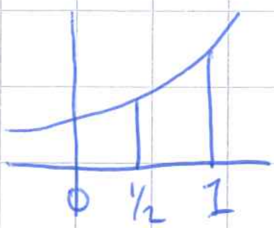
$\log_k 5 + 2 = x$

$2 + \log_k 5 = x$

iii) $\int_0^1 4k^x \, dx =$

Trapezium rule Area $\approx \frac{1}{2}\left(\frac{1}{2}\right)\left(4 + 4k + 8k^{\frac{1}{2}}\right)$

$= \frac{1}{4}\left(4 + 4k + 8k^{1/2}\right)$




$= 1 + k + 2k^{1/2}$

b) $2k^{1/2} + k + 1 = 16$

PTO

Let $k = x^2$  Hence $x^2 + 2x + 1 = 16$

$$(x+1)^2 = 16$$

$$x+1 = \pm 4$$

$$x = 3$$

$$k^{1/2} = 3$$

$$\sqrt{k} = 3$$

$$\underline{\underline{k = 9}}$$