Core 1: Jan 2009 - Miss Watson's solutions.

(1) $\sqrt{45} + \frac{20}{\sqrt{5}} \longrightarrow \frac{20}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{20\sqrt{5}}{5} = 4\sqrt{5}$ (1)

$\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$ (1)

$3\sqrt{5} + 4\sqrt{5} = \underline{\underline{7\sqrt{5}}}$ (1) $\boxed{3}$

(2) (i) $(\sqrt[3]{x})^6 = (x^{1/3})^6 = x^{6/3} = \underline{\underline{x^2}}$ (1) $\boxed{1}$

(ii) $\frac{3y^4 \times (10y)^3}{2y^5} = \frac{3y^4 \times 1000y^3}{2y^5}$ (1)

$= \frac{3000y^7}{2y^5} = \underline{1500y^2}$ (1) (1) $\boxed{3}$

(3) $3x^{2/3} + x^{1/3} - 2 = 0$ substitution $t = x^{1/3}$ (1)

$3t^2 + t - 2 = 0$ (1)

$(3t - 2)(t + 1) = 0$ (1)

$3t - 2 = 0$ $\quad t + 1 = 0$

$\underline{t = 2/3}$ $\quad \underline{t = -1}$ (1)

$t = x^{1/3}$ $\quad t = x^{1/3}$

$\frac{2}{3} = x^{1/3}$ $\quad -1 = x^{1/3}$

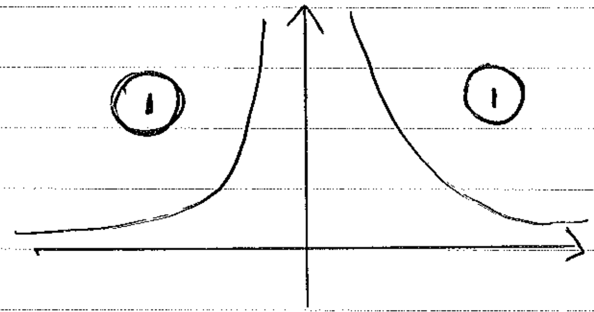
$\left(\frac{2}{3}\right)^3 = x$ $\quad (-1)^3 = x$

$\frac{8}{27} = x$ $\quad -1 = x$

$\underline{\underline{x = \frac{8}{27}}}$, $\underline{\underline{x = -1}}$ (1) $\boxed{5}$

4 (i) $y = \frac{1}{x^2}$

1 1



$\boxed{2}$

(ii) $y = \frac{1}{(x+3)^2}$ 2

$\boxed{2}$

(iii) $(1, 4)$

1 1

$\boxed{2}$

5 (i) $y = 10x^{-5}$ 1

$\frac{dy}{dx} = -50x^{-6}$

1

$\boxed{2}$

(ii) $y = x^{1/4}$ 1

$\frac{dy}{dx} = \frac{1}{4}x^{-3/4}$ 1

1

$\boxed{3}$

(iii) $y = x(x+3)(1-5x)$

$y = x(-5x^2 - 14x + 3)$ 1

$y = -5x^3 - 14x^2 + 3x$ 1

	$x+3$
1	$x+3$
$-5x$	$-5x^2 - 15x$

$\frac{dy}{dx} = -15x^2 - 28x + 3$ 2

$\boxed{4}$

6 (i) $5x^2 + 20x - 8$

$5(x^2 + 4x) - 8$ 1
$5((x+2)^2 - 4) - 8$ 1
$5(x+2)^2 - 20 - 8$ 1
$5(x+2)^2 - 28$ 1 $\boxed{4}$

(ii) vertex $(-2, -28)$
so equation of line of symmetry $\Rightarrow$ $x = -2$ 1

$\boxed{1}$

(iii) $a = 5$ $b = 20$ $c = -8$

$b^2 - 4ac$ 1
$20^2 - 4 \times 5 \times -8$
$400 + 160 = 560$ 1 $\boxed{2}$

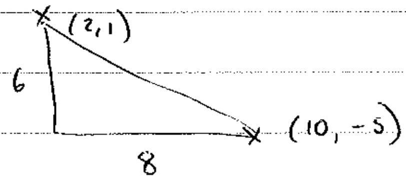
(iv) Two real roots 1 $\boxed{1}$

7 (i) $3x + 4y - 10 = 0$
$4y = 10 - 3x$
$y = \frac{10 - 3x}{4}$ $(10, k)$
$y = \frac{10 - 3 \times 10}{4}$ 1 for sbstitution
$y = \frac{-20}{4} = -5$

$k = -5$ 1 $(10, -5)$ $\boxed{2}$

(ii) $A(2,1)$ $B(10,-5)$

$$AB = \sqrt{6^2+8^2} \quad 1$$
$$= \sqrt{100}$$
$$= \underline{\underline{10}} \quad 1$$



$\boxed{2}$

(iii) centre $= (6,-2)$ 1

radius $= 5$ 1

$\boxed{2}$

(iv) Length of AB is 10 which is $2 \times$ radius 1

Midpoint of AB is the centre $\left(\frac{2+10}{2}, \frac{1+-5}{2}\right) = (6,-2)$

1 A and B lie on the circumference.

$$(2-6)^2+(1+2)^2 = 25$$
$$(-4)^2 + (3)^2 = 25$$
$$16 + 9 = 25 \checkmark$$

$$(10-6)^2 + (-5+2)^2 = 25$$
$$(4)^2 + (-3)^2 = 25$$
$$16 + 9 = 25 \checkmark$$

centre $(6,-2)$ lies of the line

$$3x + 4y - 10 = 0$$
$$3 \times 6 + 4 \times -2 - 10 = 0$$
$$18 - 8 - 10 = 0$$
$$0 = 0 \checkmark$$

$\boxed{2}$

8 (i) $5 - 8x - x^2 = 0$

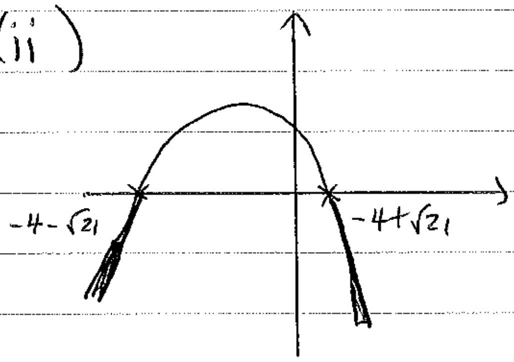
$a = -1 \quad b = -8 \quad c = 5$

$x = \dfrac{8 \pm \sqrt{(-8)^2 - 4 \times -1 \times 5}}{2 \times -1}$ 1

$x = \dfrac{8 \pm \sqrt{84}}{-2} = \dfrac{8 \pm 2\sqrt{21}}{-2} = -4 \mp \sqrt{21}$ 1

$x = -4 - \sqrt{21}, \; -4 + \sqrt{21}$ 1 $\boxed{3}$

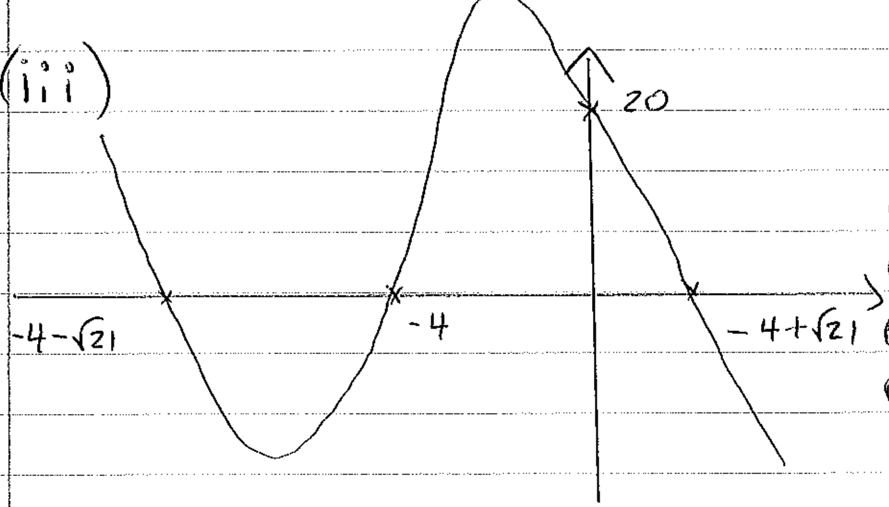
(ii)



$x \leqslant -4 - \sqrt{21}$ 1 $\quad x \geqslant -4 + \sqrt{21}$ 1

$\boxed{2}$

(iii)



1 shape
1 $(-4, 0)$ root
1 $(0, 20)$ y-intercept
1 3 roots
1 completely correct graph.

$y = (5 - 8x - x^2)(x + 4)$ $\boxed{5}$

(9) $y = x^3 + px^2 + 2$

$\frac{dy}{dx} = 3x^2 + 2px$ (1) attempt to differentiate (1) correct answer

(1) $0 = 3x^2 + 2px$ sub in $x = 4$ (1)

$0 = 3 \times 16 + 8p$

$0 = 48 + 8p$

$\underline{p = -6}$ (1)

$y = x^3 -6x^2 + 2$

$\frac{dy}{dx} = 3x^2 - 12x$

$\frac{d^2y}{dx^2} = 6x - 12$ (1) sub in $x = 4$

$= 24 - 12$

$= 12$ = positive = ∪ happy face = $\underline{\text{minimum}}$ (1)

$\boxed{7}$

10 (i) $y = x^2 + x$

$\frac{dy}{dx} = 2x + 1$ 1 at $x = 2$

$= 5$ 1

$\boxed{2}$

(ii) gradient of normal $= -\frac{1}{5}$ 1

$y = -\frac{1}{5}x + c$ when $x = 2$ $y = 2^2 + 2$

$y = 6$ 1

1 $6 = -\frac{2}{5} + c$ sub in $(2, 6)$

$30 = -2 + 5c$

$\frac{32}{5} = c$

$y = -\frac{1}{5}x + \frac{32}{5}$ $(\times 5)$

$5y = -x + 32$

$x + 5y - 32 = 0$ 1

$\boxed{4}$

(iii) $y = kx - 4$ $y = x^2 + x$

1 $kx - 4 = x^2 + x$

$0 = x^2 + x - kx + 4$

1 $0 = x^2 + (1-k)x + 4$

$a = 1$ $b = 1 - k$ $c = 4$

Discriminant $b^2 - 4ac$

1 $(1-k)^2 - 4 \times 1 \times 4$

$1 - 2k + k^2 - 16$

$k^2 - 2k - 15$

need $= 0$ for tangent 1

1 $(k+3)(k-5) = 0$

$k = -3$ or $k = 5$

1

$\boxed{6}$