Further A level Statistics 1 2020

(1) a(i) Poisson 2 every 5 mins

$X \sim Po(24)$

$P(X = 26) = 0.0719127 1609$

(ii) $P(X > 20) = 1 - 0.2426386839$
$= 0.7573613161$

b revid 40 mins, 10 customers

2 every 5 mins $X \sim Po(16)$ for 40 mins

$H_0 \quad \lambda = 16$
$H_1 \quad \lambda < 16$

$P(X \leq 10) / X \sim Po(16) = 0.07739601581$

As $0.07739601581 > 5\%$ ~~reject H_0 in favour of the~~ ~~and~~ insufficient evidence to reject the null hypothesis. ~~that there are fewer customers~~ No evidence there are fewer customers.

1c Type II error, the distribution has changed, but we make the error to not reject the previous distribution

20 min period critical value for 2 every 5
8 every 20

$X \sim Po(8)$ $P(X \leq 4) = 0.09963$
$P(X \leq 3) = 0.04238011199$

critical value is 3

So $P(X \geq 4)$ / $X \sim Po(4)$

$= 1 - 0.4334701306 = 0.5665298694$

(2) (a) Requires large n and small p, so not a good approximation

(b) X and Y must be independent

(c) $\text{Var}\,\cancel{X}(W) = 10 \times 0.4 \times 0.6 = 2.4$

$P(X+Y < 2.4)$ ✓ $X+Y \sim Po(7)$

As only whole numbers $P(X+Y \leq 2) = 0.0296361638$

(3) 4 R 1 B

C_1 B = win else Jon go C_2 B = win

(a) qq|qq|qq|w $\left(\frac{4}{5}\right)^6\left(\frac{1}{5}\right) = 0.0524288$

(b) qq|qq|q $\left(\frac{4}{5}\right)^5 = 0.32768$

(c) Geo mean $1/p = 1/\frac{1}{5} = 5$

Geo SD $= \sqrt{\frac{1-p}{p^2}} = \sqrt{\frac{1-\frac{1}{5}}{\left(\frac{1}{5}\right)^2}} =$

$= \sqrt{\frac{4/5}{1/25}} = \sqrt{20} = 2\sqrt{5}$

(d) $w + q^2 w + q^4 w + \ldots$

So G.P. $r = q^2$

$S_\infty = \frac{a}{1-r} = \frac{1/5}{1-\left(\frac{4}{5}\right)^2} = \frac{1/5}{9/25} = \underline{\underline{\frac{5}{9}}}$

4 a $Var(X) = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$= E(X^2) - (E(X))^2$

$= 13 - 2^2 = \underline{\underline{9}}$

y	25	4	-7	10
$P(Y=y)$	$1/12$	$1/6$	$1/4$	$1/2$

b $P(Y < 9) = 1/6 + 1/4 = 5/12$

c $E(XY)$

xy	-125	-8	21	40
$P(XY=xy)$	$1/12$	$1/6$	$1/4$	$1/2$

$E(XY) = 27/2$

(5) $\frac{\text{defective pins}}{\text{Total}} = \frac{0\times19 + 1\times11 + 2\times7 + 3\times2 + 4\times0 + 5\times1 + 6\times0}{40\times6} = 0.15$

$X \sim B(\overset{n}{6}, \overset{p}{0.15})$

10% significance level

Test stat 2.689

Number of Pins	0	1	2	3	4	5	6
Expected frequency	0.~~3~~3771	0.3993	0.1761	0.0414	5.4×10^{-3}	3.8×10^{-4}	1.1×10^{-5}
$\times 40$	15.084	15.972	7.044	1.656	0.216	0.0152	4.4×10^{-4}

These need to be combined as each cell must be greater than 5

So

Pins	0	1	2^+
freq	15.084	15.972	8.71564

degrees of freedom 3 cells − 1 total must add up
− 1 estimated p

$\underline{\underline{1}}$

H_0 : This is a suitable model

H_1 : This is not a suitable model

(5) $\chi^2_{(1.0, 10)} = 2.705$

The value the engineer obtained was 2.689

As the engineer's value was within the allowed tolerance then there is insufficient evidence to reject Ho. The data is consistent with the engineer's model.

(c) The proportion of defective pins has not changed. The changes that were made all fall in the same 2^+ grouped cell, so there will be no changes overall.

⑥ $G_X(t) = \frac{1}{64}(a + bt^2)^2$

$= \frac{1}{64}(a^2 + 2abt^2 + b^2t^4)$

$= \frac{a^2}{64} + \frac{2abt^2}{64} + \frac{b^2t^4}{64}$

(a) $P(X=3) = 0$

(b) (i) $P(X=4) = \frac{b^2}{64} = \frac{25}{64} \Rightarrow b = 5$

$\frac{a^2}{64} + \frac{2a \times 5}{64} + \frac{25}{64} = 1$

$a^2 + 10a + 25 = 64$

$a^2 + 10a - 39 = 0$

$a = 3$ or $a = -13$, just use positive

$P(X=2) = \frac{2ab}{64} = \frac{2 \times 3 \times 5}{64} = 15/32$

(ii) $E(X) = G'_X(1)$

$G'_X(t) = + \frac{4abt}{64} + \frac{4b^2t^3}{64}$

$G'_X(1) = + \frac{4 \times 3 \times 5}{64} + \frac{4 \times 5^2}{64}$

$= \quad = 5/2$

(1) 1, 2, 3, 4, 5, 6

(a) Use ~~a~~ the Central Limit Theorem to enable a normal distribution of the mean

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$\mu = \frac{\Sigma fx}{\Sigma f} = \left(\frac{1+2+3+4+5+6}{6}\right)$$

$$= 3\tfrac{1}{2}$$

$$\sigma^2 = E(X^2) - (E(X))^2 =$$

$$= \left(\frac{1^2+2^2+3^2+4^2+5^2+6^2}{6}\right) - \left(\frac{7}{2}\right)^2 = \frac{35}{12}$$

So $\bar{S} \sim N\left(3\tfrac{1}{2}, \frac{35}{540}\right)$

Z value for 0.05 -1.644853667

$$Z = \frac{X-\mu}{\sigma} = \frac{K - 3\tfrac{1}{2}}{\sqrt{\frac{35}{540}}}$$

$K = 3.081240754$

(b) If n is large enough and looking at the distribution of the mean then the distribution of these means will be normal.

H_0 : The die is fair

H_1 : The die is not fair

$\bar{S} < 3.1$ or $\bar{S} > 3.9$ H_0 will be rejected

True mean 4 variance 3 of S

(c) Power is prob of correctly rejecting the null hypothesis.

$$P(\bar{S} > 3.9) \;/\; \bar{S} \sim N\left(4, \frac{3}{45}\right)$$

~~0.352 3020 1536~~ 0.6507323209

$$P(\bar{S} < 3.1) \;/\; \bar{S} \sim N(4, 3/45) = 2.54 \times 10^{-4}$$

Combining 0.6509777603

(d) If $n \uparrow$ the variance $\downarrow$ so chance of $\bar{S} > 3.9$ increases significantly. There is a reduction in chance $\bar{S} < 3.1$, but this is negligible. Overall power increases.