Edexcel GCE October 2020

Maths 8MA0/01

Paper 1 Pure Maths

1. $y = 2x^3 - 4x + 5$ $\quad P(2, 13)$

$\frac{dy}{dx} = 6x^2 - 4$ $\quad$ at $x = 2$ $\frac{dy}{dx} = 20$

$m = \frac{y - y_1}{x - x_1}$ $\quad 20 = \frac{y - 13}{x - 2}$

$20(x-2) = y - 13$

$20x - 40 = y - 13$

$y = 20x - 27$

(2) $\underset{\sim}{i} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ East $\quad \underset{\sim}{j} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ North

(a)

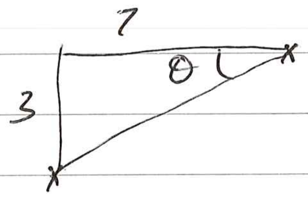
12:45

$(-3\underset{\sim}{i} - 5\underset{\sim}{j})$

10:00

$4\underset{\sim}{i} - 2\underset{\sim}{j}$

direction of boat $\begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -7 \\ -3 \end{pmatrix}$



$\tan^{-1}\left(\frac{3}{7}\right)$

$270 - \tan^{-1}(3/7) = 246.8014095$

246.8° bearing

(b) $\sqrt{7^2 + 3^2} = 7.615773106$

in 2hrs 45 mins

$7.615773106 \div 2.75 = 2.769372038$

2.77 km h^{-1}

3 (i) $x\sqrt{2} - \sqrt{18} = x$

$-\cancel{\sqrt{18}x}$

$x\sqrt{2} - x = \sqrt{18}$

$x(\sqrt{2} - 1) = \sqrt{18}$

$x = \frac{\sqrt{18}}{(\sqrt{2}-1)} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)} = \frac{6+\sqrt{18}}{2-1} =$

$= \underline{\underline{6 + 3\sqrt{2}}}$

(ii) $4^{3x-2} = \sqrt{2}^{-3}$

$(2^2)^{3x-2} = (2^{1/2})^{-3}$

$2^{6x-4} = 2^{-3/2}$

$6x - 4 = -3/2$

$6x = 5/2$

$\underline{\underline{x = 5/12}}$

4) a)

1997 190
2005 169 8 yrs after
A n yrs after

so $y = mx + c$

if $x = 0$ $y = 190$ so $c = 190$
$x = 8$ $y = 169$

$169 = m \times 8 + 190$

$\Rightarrow M = \frac{-21}{8}$

$y = \frac{-21}{8} x + 190$

$A = \frac{-21}{8} n + 190$

b) $2016 - 1997 = 19$

$\frac{-21}{8} \times 19 + 190 = 140.125$

140 compared with 120 significantly different so model not reliable.

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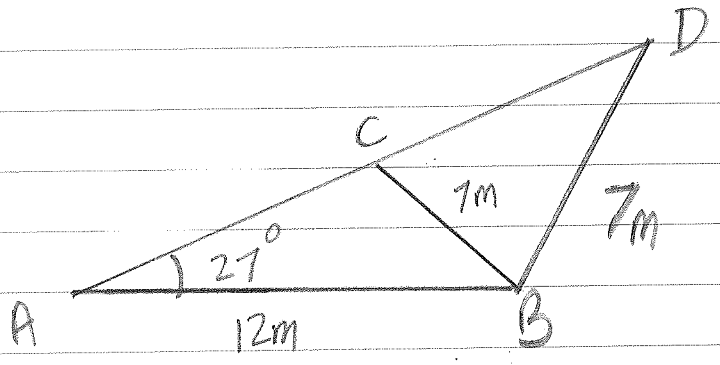


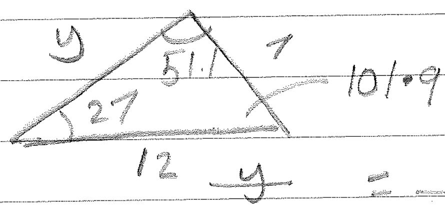
(a) $\frac{\sin C}{12} = \frac{\sin 27}{7}$

$\sin C = \frac{12 \sin 27}{7}$

$C = \sin^{-1}\left(\frac{12 \sin 27}{7}\right) = 51.10239783$

But the angle is obtuse so $180 - 51.10239783$
$= 128.8976022$
$= 128.9°$ to 1 dp

Total Steel length



$\frac{y}{\sin 101.9} = \frac{7}{\sin 27}$

$y = 15.08745866$

$12 + 7 + 7 + 15.08745866$

$= 41.08745866$

must round up, so 42 m

6 (a) $(1+kx)^{10}$

$1^{10} + {}_{10}C_1\, 1^9 (kx)^1 + {}_{10}C_2\, 1^8 (kx)^2 + {}_{10}C_3\, 1^7 (kx)^3 + \dots$

$1 + 10kx + 45k^2x^2 + 120k^3x^3 + \dots$

(b) $120k^3 = 3 \times 10k$

$120k^3 = 30k$

$120k^3 - 30k = 0$

$k(120k^2 - 30) = 0$

$k = 0$ or $120k^2 - 30 = 0$

$k^2 = \frac{3}{12}$

$k = \pm\sqrt{\frac{3}{12}}$

Ignore case when $k=0$, so possible solutions $k = \sqrt{3/12}$ or $-\sqrt{3/12}$

$\sqrt{1/4}$ $\quad -\sqrt{1/4}$

$\underline{\underline{1/2}}$ or $\underline{\underline{-1/2}}$

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$$\int_1^K \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 4$$

$$\int_1^K \frac{5}{2} x^{-1/2} + 3 \, dx = \left[5x^{1/2} + 3x \right]_1^k$$

$$= \left(5\sqrt{k} + 3k \right) - \left(5\sqrt{1} + 3 \right)$$

$$= 3k + 5\sqrt{k} - 8 = 4$$

$$\Rightarrow 3k + 5\sqrt{k} - 12 = 0$$

let $y = k^{1/2}$ $\quad 3y^2 + 5y - 12 = 0$

$$(3y - 4)(y + 3) = 0$$

$$y = 4/3 \quad \text{or} \quad y = -3$$

$$K^{1/2} = 4/3 \quad \text{or} \quad -3$$

only valid solution $K^{1/2} = 4/3 \Rightarrow K = 16/9$

(8) $\theta = 18 + 65e^{-t/8}$ $\quad t \geq 0$

(a) $t = 0 \quad \theta = 18°$

(b) $35 = 18 + 65e^{-t/8}$

$17/65 = e^{-t/8}$

$8 \ln 17/65 = -t$

$t = 10.72939144$

$t = 10.7$

(c) $e^{-t/8} \geq 0$

so $65e^{-t/8} \geq 0$

so $18 + 65e^{-t/8} \geq 18$

always above 15

(d) $\mu = A + B e^{-t/8}$

(1) $94 = A + B$

(2) $50 = A + B e^{-1}$

(1) − (2) $44 = B - Be^{-1}$

$44 = B(1 - e^{-1}) \Rightarrow B = 44/(1 - e^{-1})$

$B = 69.6069751 \Rightarrow A = 24.3930249$

asymptote $\mu = 24.4$

(9) $y = 3\cos x$

(a) $c = -180$
$d = -3$

(i) stretch SF 4 in x direction $(-720, -3)$

(ii) $(-180 + 36, \ -3)$

$(-144, -3)$

(c) $3\cos\theta = 8\tan\theta$

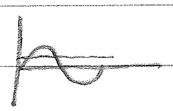
$$3\cos\theta = \frac{8\sin\theta}{\cos\theta}$$

$$3\cos^2\theta = 8\sin\theta$$

$$3(1 - \sin^2\theta) = 8\sin\theta$$

$$0 = 3\sin^2\theta + 8\sin\theta - 3$$
$$0 = (3\sin\theta - 1)(\sin\theta + 3)$$

$\sin\theta = 1/3$ no solution



$\theta = \sin^{-1}(1/3)$

19.47122063 or $180 - 19.47122063$

160.5287794

None in range $\underline{520.5^\circ}$

(10) (a) $2(5)^3 + (5)^2 - 41(5) - 70 = 0$

So 5 a root $x - 5$ a factor

(b)

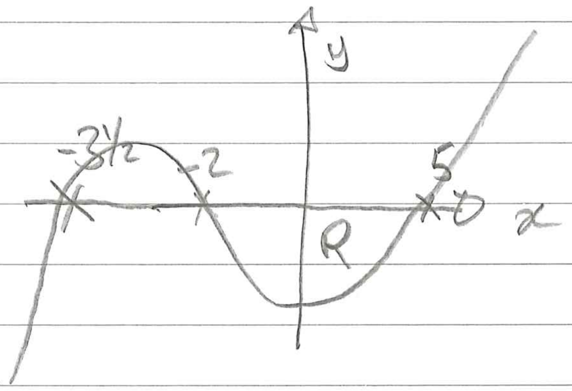
$$\begin{array}{r} 2x^2 + 11x + 14 \\ x - 5 \,\big)\, 2x^3 + x^2 - 41x - 70 \\ -(2x^3 - 10x^2) \\ \hline 11x^2 - 41x \\ -(11x^2 - 55x) \\ \hline 14x - 70 \\ -(14x - 70 \\ \hline 0 \end{array}$$

$2x^2 + 11x + 14$

$(2x + 7)(x + 2)$

$g(x) = (x-5)(2x+7)(x+2)$

(c)



$$\int_{-2}^{5} 2x^3 + x^2 - 41x - 70 \, dx = \left[\frac{x^4}{2} + \frac{x^3}{3} - \frac{41x^2}{2} - 70x \right]_{-2}^{5}$$

$$\left(\frac{-1525}{3} \right) - \left(\frac{190}{3} \right) = \frac{-1715}{3} \quad \text{Area } 571\tfrac{2}{3}$$

11 (i) $x^2+18x+y^2-2y+30=0$

$(x+9)^2-81+(y-1)^2-1+30=0$

$(x+9)^2+(y-1)^2=(\sqrt{52})^2$

centre $(-9,1)$ radius $\sqrt{52}$

$(-9,1) \Rightarrow (-5,7)$ $m=\dfrac{1-7}{-9-5}=\dfrac{-6}{-4}=\dfrac{3}{2}$

$m_{\perp}=-2/3$ $m=\dfrac{y-y_1}{x-x_1} \Rightarrow$

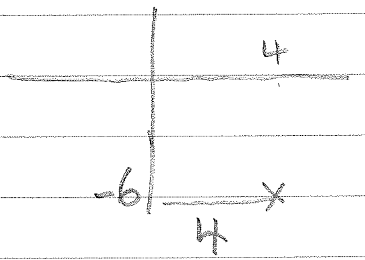
$-\dfrac{2}{3}=\dfrac{y-7}{x+5} \Rightarrow -2=\dfrac{3y-21}{x+5}$

$-2x-10=3y-21 \Rightarrow 0=3y+2x-11$

(ii) $\begin{array}{c|c} 2 & 1 \\ \hline 3 & 4 \end{array}$ $x^2-8x+y^2+12y+k=0$

$(x-4)^2-16+(y+6)^2-36+k=0$

$(x-4)^2+(y+6)^2=(\sqrt{52-k})^2$



Radius must be less than 4

$4>(\sqrt{52-k})^2$

$4>\sqrt{52-k}$

$k>52-16$

$k>36$

(12) (a) $\log_{10} V = 0.072t + 2.379 \qquad 1 \le t \le 30$

$$10^{\log_{10} V} = 10^{0.072t + 2.379}$$

$$V = \left(10^{0.072}\right)^t \quad 10^{2.379}$$

$$V = 1.180320636^t \qquad 239.3315756$$

$a = 239.3315756 \qquad 239 \quad 3SF$

$b = 1.180320636 \qquad 1.18 \quad 3SF$

(b) $ab = ab^1$ this is number of views after 1 day.

(c) $V = ab^{20} = 6546.93527 \qquad 6500 \quad 2SF$

13 a $\frac{4a}{b} + \frac{b}{a} \geq 4$

This is only true if a and b ≥ 0 if a or b ≤ 0 this would turn the inequality sign around.

$4a^2 + b^2 \geq 4ab$

$4a^2 + b^2 - 4ab \geq 0$

$(2a - b)(2a - b) \geq 0$

$(2a - b)^2 \geq 0$

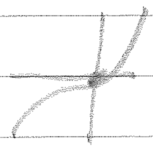
As positive or negative squared always greater than or equal to zero

b So consider $a \geq 0$ and $b \leq 0$

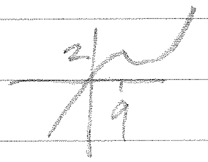
eg $a = 5$ and $b = -1$

$\frac{4 \times 5}{-1} + \frac{-1}{5} \leq 0$ so not true

14 $g(x) = ax^3 + bx^2 + ax + c$



$c = 0$



(a) $\frac{dy}{dx} = 3ax^2 + 2bx + a$

$\frac{d^2y}{dx^2} = 6ax + 2b$

If $x = 2$ $\quad g(2) = 8a + 4b + 2a = 10a + 4b = 9$

$g'(2) = 12a + 4b + a = 13a + 4b = 0$

$g'(2) - g(2) \Rightarrow 3a = -9 \Rightarrow a = -3$

$10 \times -3 + 4b = 9 \Rightarrow b = 39/4$

$g''(2) = 6 \times -3 \times 2 + 2 \times 39/4 = -33/2$

as negative a maximum