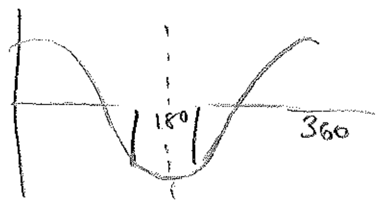
AS Practice Paper B

1\. b) $2\cos 2x + \sqrt{3} = 0$

$2\cos 2x = -\sqrt{3}$

$\cos 2x = -\dfrac{\sqrt{3}}{2}$



$\theta = 2x, \quad \cos\theta = -\dfrac{\sqrt{3}}{2}$

$\theta = 150°, 210°$

$2x = 150°, 210°$

$x = 75°, 105°$

a) Student did not correctly find all values of $2x$.
Student should have subtracted 150° from 360° first, and then divided by 2.

2\. $a = 4i - 7j$

$|a| = \sqrt{4^2 + (-7)^2}$

$|a| = \sqrt{65}$

$\dfrac{1}{\sqrt{65}}(4i - 7j)$

3\. $\dfrac{6\sqrt{3} - 4}{8 - \sqrt{3}} \times \dfrac{(8+\sqrt{3})}{(8+\sqrt{3})} = \dfrac{(6\sqrt{3}-4)(8+\sqrt{3})}{64-3}$

$= \dfrac{48\sqrt{3} + 18 - 32 - 4\sqrt{3}}{61}$

$= \dfrac{44\sqrt{3}}{61} - \dfrac{14}{61}$

4a) $1 + 3x^2 + x^3 < (1+x)^3$ then $x > 0$ ②.

$$1 + 3x^2 + x^3 < (1+x)(1+x)(1+x).$$

$$1 + 3x^2 + x^3 < (1+x)(1+2x+x^2)$$

$$1 + 3x^2 + x^3 < (1 + 2x + x^2 + x + 2x^2 + x^3).$$

$$1 + \cancel{3x^2} + \cancel{x^3} < \cancel{3x^2} + \cancel{x^3} + 3x + 1.$$

$$0 < 3x.$$

$$0 < x$$

$$\underline{x > 0}.$$

b) $x = -1$, $1 + 3(-1)^2 + (-1)^3 < (1+(-1))^3$

$$1 + 3 - 1 < 0$$

$$3 < 0$$

Inequality does not hold.

5a) $y = h(x)$ $(4, 19)$

$$h'(x) = 15x\sqrt{x} - \frac{40}{\sqrt{x}}$$

$$h'(x) = 15x'(x)^{\frac{1}{2}} - 40x^{-\frac{1}{2}}$$

$$h'(x) = 15x^{\frac{3}{2}} - 40x^{-\frac{1}{2}}.$$

$$h(x) = \frac{2}{5} . 15x^{5/2} - 2.40x^{\frac{1}{2}} + c$$

$$h(x) = 6x^{5/2} - 80x^{1/2} + c.$$

$(4, 19)$, $19 = 6(4)^{5/2} - 80(4)^{1/2} + c.$

$$19 = 6(32) - 80(2) + c.$$

$$19 = 192 - 160 + c.$$

$$-13 = c$$

$$h(x) = 6x^{5/2} - 80x^{1/2} - 13.$$

6. $8 - 7\cos x = 6\sin^2 x$ $\qquad \sin^2 x + \cos^2 x = 1$ ③

$8 - 7\cos x = 6(1-\cos^2 x)$ $\qquad \sin^2 x = 1 - \cos^2 x$

$8 - 7\cos x = 6 - 6\cos^2 x$

$6\cos^2 x - 7\cos x + 2 = 0$

$\theta = \cos x,$ $\quad 6\theta^2 - 7\theta + 2 = 0$

$(3\theta - 2)(2\theta - 1) = 0$

$3\theta = 2 \quad 2\theta = 1$

$\theta = \frac{2}{3} \quad \theta = \frac{1}{2}$

$\cos x = \frac{2}{3} \quad \cos x = \frac{1}{2}$

$0 \leq x \leq 360°$

$x = 48.2°, 60°, 311.8°, 300°$

7. a) $(1+3x)^8$

$= 1^8 + \binom{8}{1} 1^7 (3x) + \binom{8}{2} 1^6 (3x)^2 + \binom{8}{3} 1^5 (3x)^3$

$= 1 + 24x + 252x^2 + 1512x^3$

b) $1 + 3x = 1.03$

$3x = 0.03$

$x = 0.01$

$= 1 + 24(0.01) + 252(0.01)^2 + 1512(0.01)^3$

$= 1.2667$ (5sf)

8. a) $y = \log_9(x+a) \quad a>0, \ x > -a$ ④

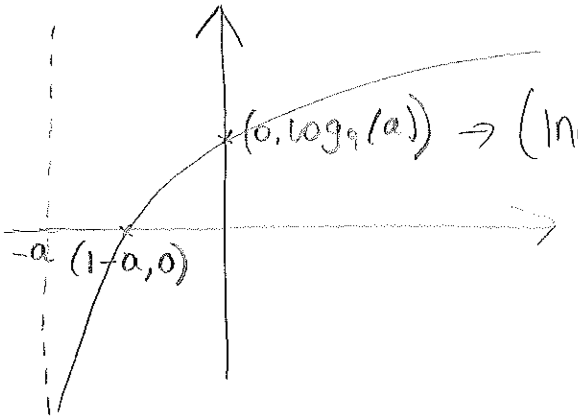
$y=0, \ 0 = \log_9(x+a)$

$1 = x + a$

$x = 1 - a$

$\boxed{3^2 = 9 \quad \log_3 9 = 2}$

$x = 0, \ y = \log_9 a$

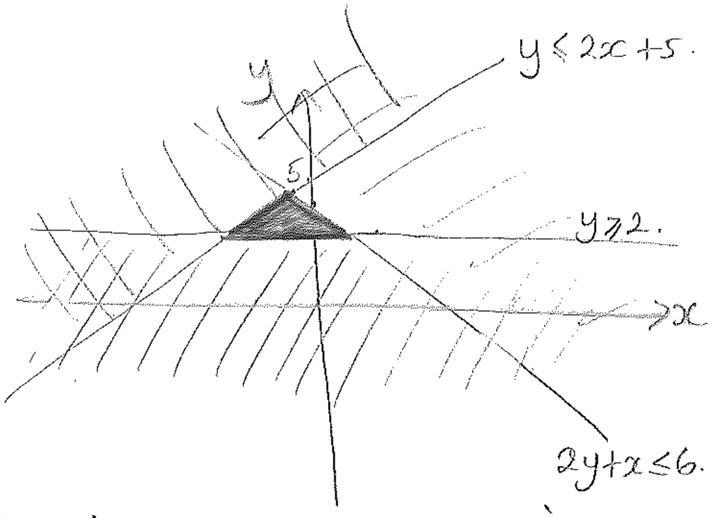


→ (incorrect on markscheme)

b) $x > -a \qquad \log_9(x+a)^2 = 2\log_9(x+a)$

↑ stretch parallel to y-axis, stretch factor 2 of the graph $y = \log_9(x+a)$

9. a)



$y = 2x+5$

x	-2	0	2	4
y	1	5	9	13

$2y + x = 6$

x	-2	0	2	4
y	4	3	2	1

(shade out the regions you do not want)

b)

9b) $y = 2x + 5, \quad 2y + x = 6.$

$2(2x+5) + x = 6.$

$4x + 10 + x = 6.$

$5x = -4$

$x = -\frac{4}{5}, \quad y = 2\left(-\frac{4}{5}\right) + 5$

$y = 3.4.$

$A = \frac{1}{2} \times b \times h.$

$A = \frac{1}{2} \times 3.5 \times (3.4 - 2)$

$A = \underline{2.45 \text{ units}^2}$

Base:

$y = 2, \quad 2 = 2x + 5$

$x = -\frac{3}{2}.$

$y = 2, \quad 4 + x = 6.$

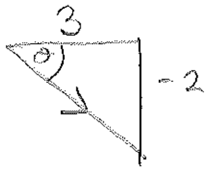
$x = 2.$

$\text{Base} = 2 + \frac{3}{2} = 3.5$

10. a) $F_1 = (8i - 10j)N \qquad m = 6kg.$

$F_2 = (pi + qj)N$

$a = (3i - 2j)ms^{-2}$



$\tan\theta = \frac{-2}{3}$

$\underline{\theta = -33.7°}$

b) $F = (8+p)i + (-10+q)j$

$F = ma$

$F = 6(3i - 2j) = 18i - 12j$

$(8+p)i + (-10+q)j = 18i - 12j$

$8 + p = 18 \qquad -10 + q = -12.$

$p = 10 \qquad q = -2.$

10 c) $R = 18i - 12j$

$|R| = \sqrt{18^2 + (-12)^2}$

$|R| = \sqrt{468}$

$|R| = \underline{6\sqrt{13}}$

11 a) $f(x) = x^3 - 7x^2 - 24x + 18$

$f'(x) = 3x^2 - 14x - 24$

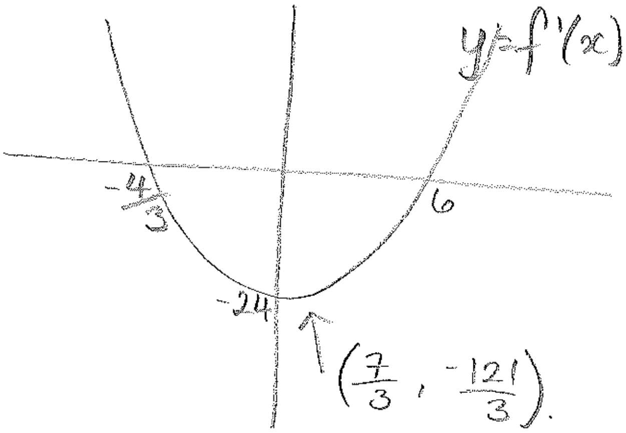
b)

$y=0, \; 3x^2 - 14x - 24 = 0$

$(3x+4)(x-6) = 0$

$x = -\frac{4}{3} \quad x = 6$

$x = 0, \; y = -24$



c) Min point: $f''(x) = 0$

$f''(x) = 6x - 14$

$6x - 14 = 0$

$6x = 14$

$x = \frac{14}{6}, \frac{7}{3}$

$y = -\frac{121}{3}$

12. a) $y = x^2 - 8x + 20$

$y = x + 6$

$x^2 - 8x + 20 = x + 6$

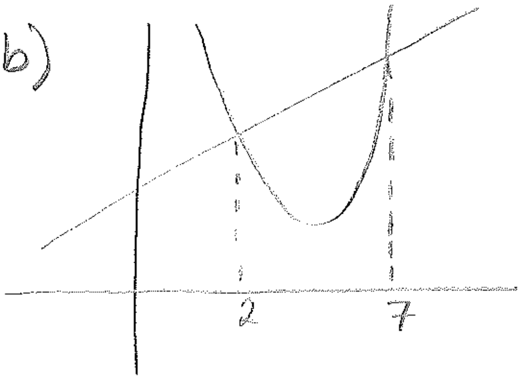
$x^2 - 9x + 14 = 0$

$(x-2)(x-7) = 0$

$x = 2, \; x = 7. \quad A(2, 8)$

$y = 8, \; y = 13. \quad B(7, 13)$

12 b)



Area of trapezium

$= \frac{1}{2}(a+b) \times h.$

$= \frac{1}{2}(8+13) \times 5.$

$= \underline{52.5 \text{units}^2}$

c) Area under $y = x^2 - 8x + 20$

$$\int_2^7 x^2 - 8x + 20 \, dx.$$

$$= \left[\frac{x^3}{3} - \frac{8x^2}{2} + 20x\right]_2^7$$

$$= \left[\frac{x^3}{3} - 4x^2 + 20x\right]_2^7$$

$$= \left[\left(\frac{7^3}{3} - 4(7)^2 + 20(7)\right) - \left(\frac{2^3}{3} - 4(2)^2 + 20(2)\right)\right]$$

$$= \left(\frac{175}{3}\right) - \left(\frac{80}{3}\right) = \frac{95}{3} \text{ units}^2$$

d) Shaded area $= 52.5 - \frac{95}{3} = \underline{20.8 \text{ units}^2}$

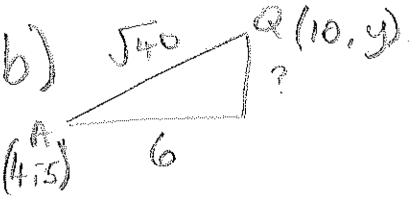
13. a) $x^2 \quad 8x + y^2 + 10y + 1 = 0.$

$(x-4)^2 - 16 + (y+5)^2 - 25 + 1 = 0.$

$(x-4)^2 + (y+5)^2 = 40.$

Centre $(4, -5)$ Radius $= \sqrt{40} = \sqrt{4}\sqrt{10} = 2\sqrt{10}.$

b)



$a^2 + b^2 = c^2$

$6^2 + b^2 = 40.$

$36 + b^2 = 40.$

$b^2 = 4$

$b = 2.$

$\underline{Q(10, -3)}.$

13. c) Gradient of $AQ = \frac{2}{6} = \frac{1}{3}$

Gradient of $l_2 = -3$ $\quad Q(10, -3)$

$y - y_1 = m(x - x_1)$

$y - -3 = -3(x - 10)$

$y + 3 = -3x + 30$

$y = -3x + 27$

d) $\vec{AQ} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

$\vec{AP} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$

$P(2, 1)$

Gradient of $l_1 = \frac{1}{3}$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{3}(x - 2)$

$3y - 3 = x - 2$

$3y = x + 1$

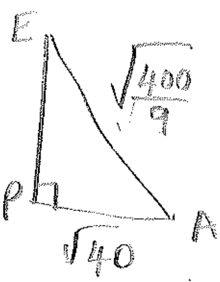
$y = \frac{x}{3} + \frac{1}{3}$

For line l_1,

e) $x = 0, \ y = \frac{1}{3}$ $\quad E\left(0, \frac{1}{3}\right)$. $A(4, 5)$

h of $\Delta = 5\frac{1}{3}$

B of $\Delta = \sqrt{\frac{40}{9}}$



$EP^2 + 40 = \frac{400}{9}$

$EP = \sqrt{\frac{40}{9}}$

$A = \frac{1}{2} \times \sqrt{40} \times \sqrt{\frac{40}{9}} = \frac{20}{3}$