Exam Solutions

~~22/10/13~~ C1

May 2013.

1. i) $4\sqrt{15} \times \sqrt{3} = 4\sqrt{45}$
$= 4\sqrt{9 \times 5}$
$= 12\sqrt{5}$

ii) $\frac{20}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{20\sqrt{5}}{5} = 4\sqrt{5}$

iii) $\sqrt{5^3} = \sqrt{125}$
$= \sqrt{25 \times 5}$
$= 5\sqrt{5}$

2. let $y^2 = x^6$

$y = x^3$

$8y^2 + 7y - 1 = 0$

$(y+1)(8y-1) = 0$

$y = -1 \quad y = \frac{1}{8}$

$x^3 = -1 \quad x^3 = \frac{1}{8}$

$x = -1 \quad x = \frac{1}{2}$

3. $f(x) = 6x^{-2} + 2x$

i) $f'(x) = -12x^{-3} + 2$

ii) $f''(x) = 36x^{-4}$

4. i) $3x^2 + 9x + 10 = 3(x^2 + 3x) + 10$
$= 3\left(\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}\right) + 10$
$= 3\left(x + \frac{3}{2}\right)^2 - \frac{27}{4} + \frac{40}{4}$
$= 3\left(x + \frac{3}{2}\right)^2 + \frac{13}{4}$

ii) $\left(-\frac{3}{2}, \frac{13}{4}\right)$

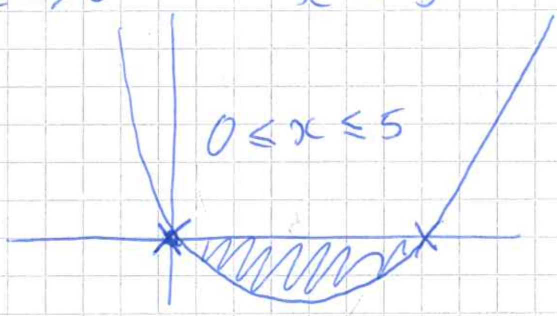
iii) $b^2 - 4ac = 9^2 - (4 \times 3 \times 10)$
$= -39$

7. i) $3 - 8x > 4$
$-8x > 1$
$-x > \frac{1}{8}$
$x < -\frac{1}{8}$

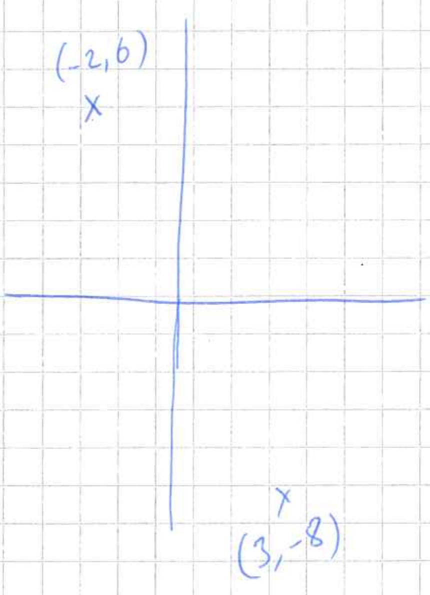
ii) $2x^2 - 10x + 12 \leq 12$
$2x^2 - 10x \leq 0$
$2x(x - 5) \leq 0$
$\cancel{2x \leq 0}$
$\cancel{x}$
$x \to 0$

$x - 5 = 0$
$x = 5$

$0 \leq x \leq 5$



8.



Midpoint AB $= \left(\frac{-2+3}{2}, \frac{6+-8}{2}\right)$

$= \underline{\underline{\left(\frac{1}{2}, -1\right)}}$

m_{line} $x - 3y + 15 = 0$
$3y = x + 15$
$y = \frac{1}{3}x + 5$

gradient $= \frac{1}{3}$

Hence $m_l = \underline{\underline{-3}}$
$m_1 \times m_2 = -1$

sub in values:
$-1 = -3\left(\frac{1}{2}\right) + c$
$-1 = \frac{-3}{2} + c$
$c = \frac{1}{2}$

$2y + 6x - 1 = 0$ $y = -3x + \frac{1}{2}$
$a = 6, \ b = 2, \ c = 1$

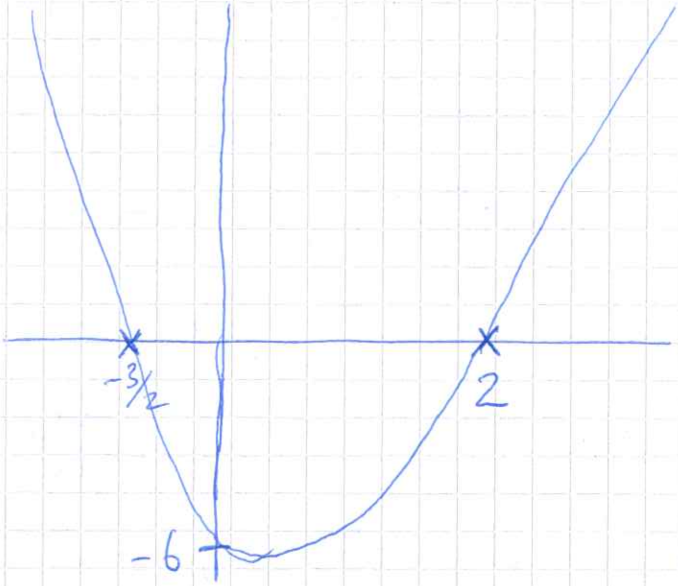
9 i) $y = 2x^2 - x - 6$

$y = (2x+3)(x-2)$

$2x+3=0$

$2x = -3$

$x = -3/2$



ii) $y = 2x^2 - x - 6$

$\frac{dy}{dx} = 4x - 1$ @ vertex $4x-1=0$

$x = \frac{1}{4}$

decreasing function when $x < \frac{1}{4}$

iii) $y = 4$

$y = 2x^2 - x - 6$

$\Rightarrow 4 = 2x^2 - x - 6$

$0 = 2x^2 - x - 10$

$= (2x-5)(x+2)$

$2x = 5$ $\quad x = -2$

$x = \frac{5}{2}$

$PQ = \frac{5}{2} - -2$

$= 4\frac{1}{2}$

10. i) $y = (1-x)(x^2+4x+k)$

$= \cancel{x^2} \cancel{+4x} + k \cancel{-x^3} \cancel{-4x^2} \cancel{-kx}$

$= \cancel{-3x^2} \quad -x^3 - 3x^2 + (4-k)x \quad + k$

$\frac{dy}{dx} = -3x^2 - 6x + (4-k)$

when $x = -3$ $\quad \frac{dy}{dx} = 0$

so $\quad 0 = -3(-3)^2 - 6(-3) + (4-k)$

$= -27 + 18 + 4 - k$

$= -5 - k$

$\cancel{5 = -k}$ $\quad \underline{\underline{k = -5}}$

$\cancel{k = 5}$

27 $\quad$ 27 − 18 = 9

−9 + 4 = −5

ii) $\frac{dy}{dx} = -3x^2 - 6x + 9$

$\frac{d^2y}{dx^2} = -6x - 6$

when $x = -3$ $\quad \frac{d^2y}{dx^2} = -6(-3) - 6$

$= 18 - 6$

$= 12 > 0$ hence minimum point.

iii) $y = 9x - 9$

gradient $= 9$

hence $\frac{dy}{dx} = 9 = -3x^2 - 6x + 9$

$-3x^2 - 6x = 0$

$-3x(x+2) = 0$

$\underline{x=0} \quad \underline{x=-2}$

~~when $x=0$ $y=-9$~~

~~when $x=-2$ $y=$~~

~~$y = 9(-2) - 9$~~

~~$= -18 - 9$~~

~~$y = -27$~~

iii) continued . . .

when $x=0$ $y=-9$ line

$y=(1)(-5)=-5$ curve.

when $x=-2$ $y=9(-2)-9$
$=-27$ line

$y=(3)(4-8-5)$
$=-27$ curve

Hence point A must be $(-2,-27)$