

CZ June 2016

$$\textcircled{1} \text{ (i) Area} = \frac{1}{2} ab \sin C$$
$$\frac{1}{2} \times 8 \times c \times \sin 30 = 20 \Rightarrow c = 10 \text{ cm}$$

$$\text{(ii) } a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 30 = 25.43593539$$

25.4 cm 35F

$$\textcircled{2} \text{ (i) } 360 = 2\pi$$
$$1 = \frac{\pi}{180}$$
$$54 = \frac{54\pi}{180} = \frac{3\pi}{10}$$

$$\text{(ii) } r\theta + 2r = 60$$

$$\frac{3\pi r}{10} + 2r = 60$$

$$r \left(\frac{3\pi}{10} + 2 \right) = 60$$

$$r = \frac{60}{\left(\frac{3\pi}{10} + 2 \right)} = 20.390978$$

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$$3(i) (a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n$$

$$3^3 + {}_3C_1 3^2(kx) + {}_3C_2 3(kx)^2 + (kx)^3$$
$$27 + 27kx + 9k^2x^2 + k^3x^3$$

$$(ii) 9k^2 = 27 \quad k = \sqrt{3}$$

$$4(i) 2 \log_3 x - \log_3 (x+4) = \log_3 \left(\frac{x^2}{x+4} \right)$$

$$(ii) 3^2 = 3 \log_3 \left(\frac{x^2}{x+4} \right)$$

$$9 = \frac{x^2}{x+4} \Rightarrow 9x + 36 = x^2 \Rightarrow x^2 - 9x - 36 = 0$$
$$(x+3)(x-12) = 0$$

~~$x = -3$~~ $x = 12$
not possible

$$5(a) \int 2x^3 - 3x^2 + 4x - 6 dx = \frac{x^4}{2} - x^3 + 2x^2 - 6x + k$$

$$(b) \int_1^a (6x^{-2} - 4x^{-3}) dx = [-6x^{-1} + 2x^{-2}]_1^a = \left[\frac{-6}{a} + \frac{2}{a^2} \right] - (-6 + 2)$$

$$= 4 \frac{-6}{a} + \frac{2}{a^2}$$

$$(ii) \int_1^{\infty} (6x^{-2} - 4x^{-3}) dx = 4 \frac{-6}{\infty} + \frac{2}{\infty^2} = \underline{\underline{4}}$$

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(6) $u_1 = 5$

$$u_{n+1} = u_n + 1.5 \quad n \geq 1$$

(i) $u_k = 140$

$$u_n = a + (n-1)d$$

$$140 = 5 + (k-1) \times 1.5 \Rightarrow k = 91$$

$$w_n = 120 (0.9)^{n-1} \quad n \geq 1$$

$$(ii) S_n = \frac{a(1-r^n)}{1-r} = \frac{120(1-0.9^{15})}{1-0.9} = 977.6375773$$

978

$$(iii) S_n = \frac{1}{2} n \{2a + (n-1)d\} > \frac{a}{1-r}$$

$$\frac{1}{2} N \{2 \times 5 + (N-1)1.5\} > \frac{120}{1-0.9}$$

$$10N + 1.5N^2 - 1.5N > 2400$$

$$1.5N^2 + 8.5N - 2400 > 0$$

$$3N^2 + 17N - 4800 > 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-17 \pm \sqrt{17^2 - 4 \times 3 \times (-4800)}}{2 \times 3}$$

$$\frac{-17 \pm 240.601}{6}$$

37.2669 or -42.9336

So need N to be 38 to exceed this

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(7) $f(x) = x^3 - 3x^2 - x + 3$

(i)
$$\begin{array}{r} x^2 - 4x + 3 \\ x + 1 \overline{) x^3 - 3x^2 - x + 3} \\ \underline{-(x^3 + x^2)} \\ -4x^2 - x + 3 \\ \underline{-(-4x^2 - 4x)} \\ 3x + 3 \\ \underline{-(3x + 3)} \\ 0 \end{array}$$

quotient $x^2 - 4x + 3$ remainder 0

(ii) $(x - 1)(x - 3) = 0$

roots 1, 3 and -1.

(iii) $y = x^4 - 4x^3 - 2x^2 + 12x + 9$

$$\frac{dy}{dx} = 4x^3 - 12x^2 - 4x + 12$$

stationary points $\frac{dy}{dx} = 0$

$$0 = 4x^3 - 12x^2 - 4x + 12$$

$\div 4$

$$0 = x^3 - 3x^2 - x + 3$$

(iv) $\int_{-1}^3 x^4 - 4x^3 - 2x^2 + 12x + 9 dx = \left[\frac{x^5}{5} - x^4 - \frac{2x^3}{3} + 6x^2 + 9x \right]_{-1}^3$

$$= \left[\frac{3^5}{5} - 3^4 - \frac{2 \times 3^3}{3} + 6 \times 3^2 + 9 \times 3 \right] - \left[\frac{(-1)^5}{5} - (-1)^4 - \frac{2(-1)^3}{3} + 6(-1)^2 + 9(-1) \right]$$
$$= \frac{153}{5} + \frac{53}{5} = \frac{512}{5}$$

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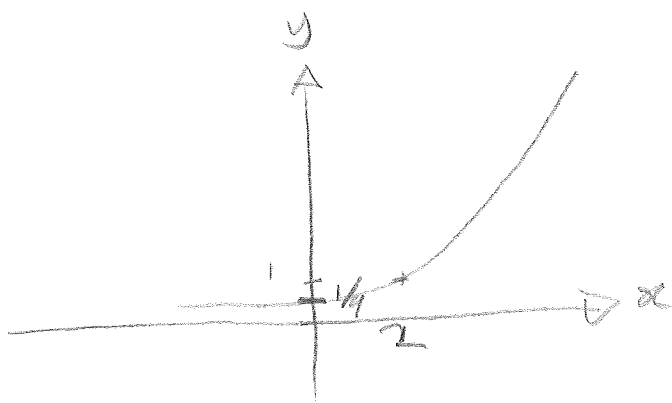
(8)(i) $y = 3^x$ $y = 3^{x-2}$

Translation: 2 units in the x direction

(ii) $y = 3^{x-2} = 3^x 3^{-2} \Rightarrow \frac{y}{3^{-2}} = 3^x$

stretch SF $1/9$ in the y direction

(iii)



if $x=0$ $y = 3^{-2} = 1/9$
crosses y axis at $1/9$

(iv)

$y = 3^{x-2} \Rightarrow 180 = 3^{x-2}$
 $\log 180 = (x-2) \log 3$

$x = \frac{\log 180}{\log 3} + 2 = 6.726883028$
 $= \underline{\underline{6.73}}$

(v) $\int_1^4 3^{x-2} dx$

x	y
1	3^{-1}
2.5	$3^{1/2}$
4	3^2

$\frac{1}{2} h ((y_0 + y_n) + 2(y_1 + \dots + y_{n-1}))$

$\frac{1}{2} \times \frac{3}{2} ((1/3 + 9) + 2 \times \sqrt{3}) = \frac{14 + 3\sqrt{3}}{2}$

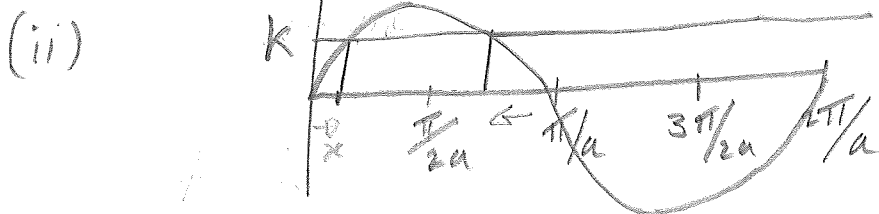
9.59

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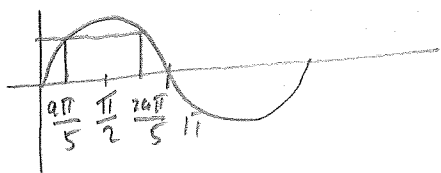
(9) (i) $y = \sin(ax)$ $y = \sin(x/1a)$

graph stretched scale factor $1/a$ in the x direction $\Rightarrow 2\pi \times \frac{1}{a} = \frac{2\pi}{a}$

$y = \sin(ax)$



so $\sin\left(\frac{a\pi}{5}\right) = \sin\left(\frac{2a\pi}{5}\right)$



$$\pi - \frac{2a\pi}{5} = \frac{a\pi}{5}$$

$$\text{so } \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} = k$$

$$5\pi - 2a\pi = a\pi$$

$$5 = 3a$$

$$a = \frac{5}{3}$$

(iii) let $z = ax$

$$\sin z = \sqrt{3} \cos z$$

$$\Rightarrow \tan z = \sqrt{3}$$

$$\Rightarrow z = \frac{\pi}{3}$$

$$z = \frac{\pi}{3} \text{ or } \frac{4\pi}{3}$$

so

$$ax = \frac{\pi}{3}$$

$$\text{or } \frac{4\pi}{3}$$

$$x = \frac{\pi}{3a}$$

$$\text{or } \frac{4\pi}{3a}$$