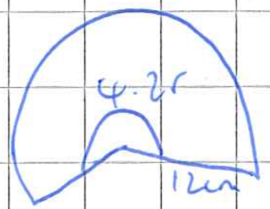
Jan 2012 C2

1

$C = \pi d$

i)



fraction of circle $= \frac{4.2}{2\pi}$

$\text{arc length} = \frac{4.2}{2\pi} \times \pi \times 24$

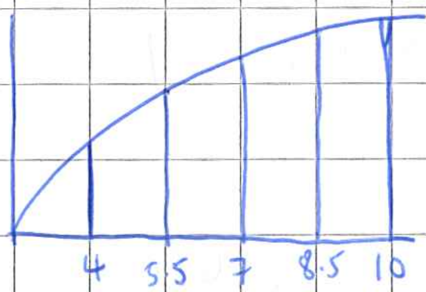
$= 50.4$

$\text{arc length} = r\theta$
$= 50.4$

$\text{Perimeter} = 50.4 + 2 \times 12$
$= 74.4$

ii) $\text{area} = \frac{1}{2} r^2 \theta$
$= 302.4 \text{cm}^2$

2 i)



$A \approx \frac{1}{2}(1.5)\left(\log_{10} 9 + \log_{10} 21 + 2\left(\log_{10} 12 + \log_{10} 15 + \log_{10} 18\right)\right)$

$= 6.97 \quad (3sf)$

ii)

each trapezium is under the curve
$\therefore$ a small amount of area is lost.

③ $(4+ax)^6$

i) for x^3: ${}^6C_3(4)^3(ax)^3$

$= 1280a^3x^3$

$1280a^3 = 160$

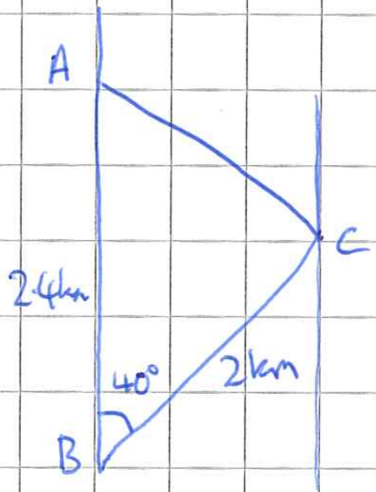
$a^3 = 1/8$

$a = 1/2$

ii) $(4+\frac{1}{2}x)^6 = {}^6C_0(4)^6 + {}^6C_1(4)^5(\frac{1}{2}x) + {}^6C_2(4)^4(\frac{1}{2}x)^2 + \dots$

$= 4096 + 3072x + \dots$

④



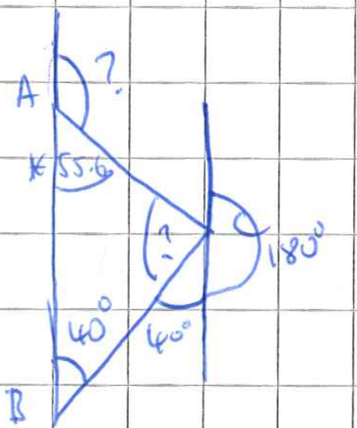
Cosine Rule:

i) $a^2 = b^2 + c^2 - 2bc\cos A$

$AC^2 = 2.4^2 + 2^2 - 2(2)(2.4)\cos 40$

$AC = 1.55$ (3sf)

ii)



$\frac{\sin C}{2.4} = \frac{\sin 40}{1.55}$

$C = \sin^{-1}\left(\frac{2.4\sin 40}{1.55}\right)$

$C = 84.4$ (3sf).

~~∴ Bearing ⇒~~ $\cancel{180+40+84.4 = 304.4^\circ}$

$180 - (84.4+40)$

$A = 55.6$ ∴ Bearing $= \underline{124.4^\circ}$

$\boxed{\text{* Last Part on next page}}$

(5) $f(x) = 2x^3 + 3x^2 - 17x + 6$

divided by ~~fr~~ $(x-3)$

$f(3) = 2(3)^3 + 3(3)^2 - 17(3) + 6$

$= 54 + 27 - 51 + 6$

$= 36$

ii) $f(2) = 0 \quad \therefore \quad (x-2)$ is a factor

$(x-2)(2x^2 + bx - 3)$

$$\begin{array}{l} bx^2 \\ -4x^2 \\ \hline (b-4)x^2 \end{array} \qquad \begin{aligned} b - 4 &= 3 \\ b &= 7 \end{aligned}$$

$\Rightarrow (x-2)(2x^2 + 7x - 3)$

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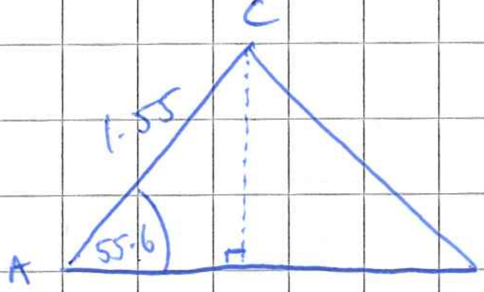
(4) iii)



$\sin\theta = \frac{O}{H}$

$Opp = 1.55 \sin 55.6$

$= 1.28\text{ km}$

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iii) $b^2 - 4ac = 7^2 - 4(2)(-3)$

$= 73$

$73 > 0$ Hence the quadratic has 2 real roots

$\Rightarrow$ 3 real roots in total.

* $(x-2)$ is already a root *

⑥ $U_1 = 85 - 5$
$= 80$

$U_2 = 85 - 10$
$= 75$

$a = 80$
$d = -5$

$U_3 = 85 - 15$
$= 70$

ii) $\sum_{n=1}^{20} U_n = \frac{20}{2}(2a + (n-1)d)$
$= 10(160 + 19 \times -5)$
$= 650$

iii) $U_1 = 80$
$U_5 = 60$
$U_p = 45$

$r = \frac{a_{n+1}}{a_n} = \frac{60}{80}$

$r = 0.75$

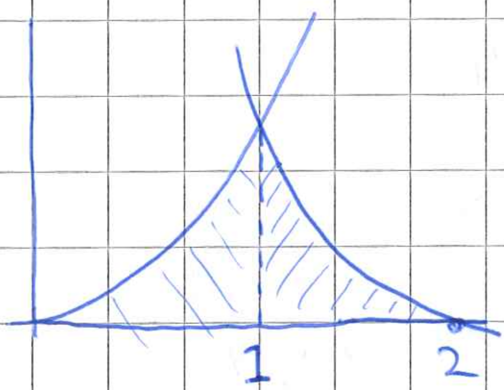
iv) $S_\infty = \frac{a}{1-r} = \frac{80}{0.25}$
$= \underline{320}$

(7) a) Find $\int (x^2+4)(x-6)\,dx$

$$= \int x^3 - 6x^2 + 4x - 24\,dx$$

$$= \frac{1}{4}x^4 - 2x^3 + 2x^2 - 24x + c$$

b) intersect @ (1, 6)



$y=0$

$$y = \frac{8}{x^2} - 2$$

$$2 = \frac{8}{x^2}$$

$$2x^2 = 8$$

$$x^2 = 4$$

$\underline{x = 2}$ (from diagram)

$$\frac{8}{x^2} - 2 = 8x^{-2} - 2$$

$$\text{Area} = \int_0^1 6x^{3/2}\,dx + \int_1^2 \frac{8}{x^2} - 2\,dx$$

$$= \left[\frac{12}{5}x^{5/2}\right]_0^1 + \left[-8x^{-1} - 2x\right]_1^2$$

$$= \left(\frac{12}{5}(1)^{5/2} - 0\right) + \left(-8(2)^{-1} - 2(2) - -8(1)^{-1} - 2(1)\right)$$

$$= \frac{12}{5} + 2$$

$$= \frac{22}{5}$$

8 a) $7^{w-3} - 4 = 180$

$(w-3)\log 7 - \log 4 = 180$

$w\log 7 - 3\log 7 - \log 4 = 180$

$(w-3)\log 7 = 184$

$w\log 7 - 3\log 7 = 184$

$w\log 7 = 184 - 3\log 7$

$w = \dfrac{184 - 3\log 7}{\log 7}$

$w =$

8 a) $(w-3)\log 7 = \cancel{184}\ \log 184$

$w - 3 = \dfrac{\log 184}{\log 7}$

$w = \dfrac{\log 184}{\log 7} + 3$

$w = 5.68$ (3sf).

b) $\log_{10} x + \log_{10} y = \log_{10} 3$

$\log_{10} xy = \log_{10} 3$

$xy = 3$ $\quad x = 3/y$ ①

$\log_{10}(3x + y) = 1$

sub ① into ②

$3x + y = 10$ ②

$3(3/y) + y = 10$

$\frac{9}{y} + y = 10$

$9 + y^2 = 10y$

$y^2 - 10y + 9 = 0$

$(y-1)(y-9) = 0$

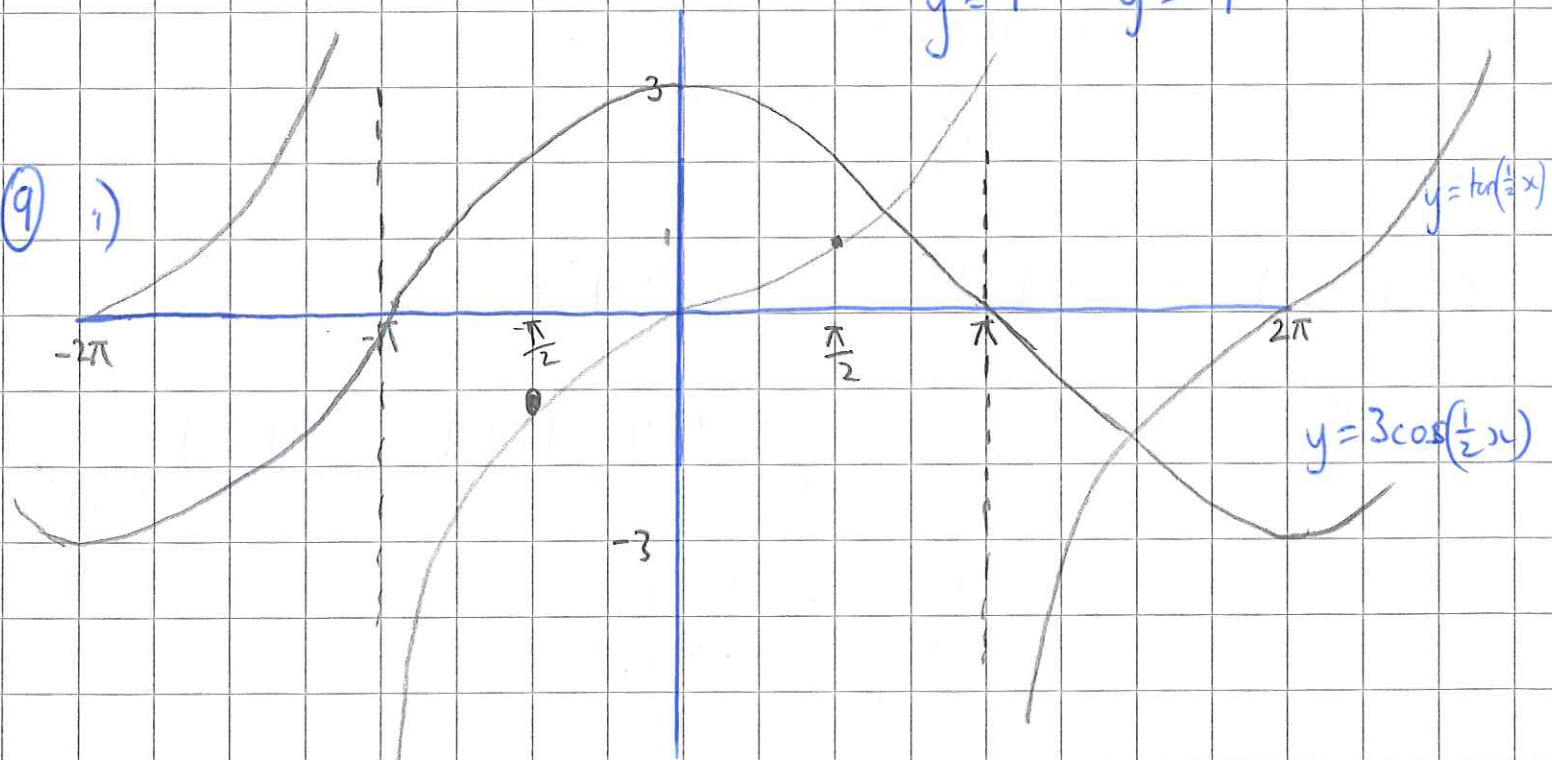
$y = 1 \quad y = 9$

$y = 1$ $\quad$ $y = 9$

$x = 3$ $\quad$ $9x = 3$

$x = 1/3$

9 i)



ii) $\tan\left(\frac{1}{2}x\right) = 3\cos\left(\frac{1}{2}x\right)$

$$\frac{\sin\frac{1}{2}x}{\cos\frac{1}{2}x} = 3\cos\left(\frac{1}{2}x\right)$$

$* \cos^2 x = 1 - \sin^2 x *$

$$\sin\left(\frac{1}{2}x\right) = 3\cos^2\left(\frac{1}{2}x\right)$$

$$\sin\left(\frac{1}{2}x\right) = 3\left(1 - \sin^2\left(\frac{1}{2}x\right)\right)$$

$$\sin\left(\frac{1}{2}x\right) = 3 - 3\sin^2\left(\frac{1}{2}x\right)$$

$$3\sin^2\left(\frac{1}{2}x\right) + \sin\left(\frac{1}{2}x\right) - 3 = 0$$

$$\left(3\sin\left(\frac{1}{2}x\right) \quad\right)\left(\sin\left(\frac{1}{2}x\right) \quad\right)$$

does not factorise !!!

$$\sin\left(\frac{1}{2}x\right) = \frac{-1 \pm \sqrt{1^2 - 4(3)(-3)}}{6}$$

$$\sin\left(\frac{1}{2}x\right) = -1.18,\ 0.847$$

$$\left(\frac{1}{2}x\right) = \quad 57.9,\ 122.1$$

$$x = 115.8^\circ,\ 244.2$$

$$x = 2.02^c,\ 4.26^c$$