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| **Pearson Edexcel Level 3** |
| **GCE Mathematics** **Advanced Subsidiary** **Paper 1: Pure Mathematics** |
| **Sample assessment material for first teaching September 2017** **Time: 2 hours** | **Paper Reference(s)** |
| **8MA0/01** |
| **You must have:** **Mathematical Formulae and Statistical Tables, calculator** |

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

**Instructions**

• Use black ink or ball-point pen.

• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).

• Fill in the boxes at the top of this page with your name, centre number and candidate number.

• Answer all the questions and ensure that your answers to parts of questions are clearly labelled.

• Answer the questions in the spaces provided – there may be more space than you need.

• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

• Inexact answers should be given to three significant figures unless otherwise stated.

**Information**

• A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

• There are 17 questions in this question paper. The total mark for this paper is 100.

• The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

**Advice**

• Read each question carefully before you start to answer it.

• Try to answer every question.

• Check your answers if you have time at the end.

 • If you change your mind about an answer, cross it out and put your new answer and any working underneath.

**Answer ALL questions.**

**1.** The line *l* passes through the points *A* (3, 1) and *B* (4, − 2).

Find an equation for *l*.

**(3)**

**(Total for Question 1 is 3 marks)**

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**2.** The curve C has equation

*y* = 2*x*2 − 12 + 16.

Find the gradient of the curve at the point *P*(5, 6).

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(4)**

**(Total for Question 2 is 4 marks)**

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**3.** Given that the point *A* has position vector 3**i** – 7**j** and the point *B* has position vector 8**i** + 3**j**,

(*a*) find the vector .

**(2)**

(*b*) Find . Give your answer as a simplified surd.

**(2)**

**(Total for Question 3 is 4 marks)**

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**4.** f(*x*) = 4*x*3 – 12*x*2 + 2*x* – 6

(*a*) Use the factor theorem to show that (*x* – 3) is a factor of f(*x*).

 **(2)**

(*b*) Hence show that 3 is the only real root of the equation f(*x*) = 0.

**(4)**

**(Total for Question 4 is 6 marks)**

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**5.** Given that f(*x*) = 2*x* + 3 + , *x* > 0,

 show that  = 16 + 3√2.

**(5)**

**(Total for Question 5 is 5 marks)**

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**6.** Prove, from first principles, that the derivative of 3*x*2 is 6*x*.

**(4)**

**(Total for Question 6 is 4 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**7.** (*a*) Find the first 3 terms, in ascending powers of *x*, of the binomial expansion of

, giving each term in its simplest form.

**(4)**

(*b*) Explain how you would use your expansion to give an estimate for the value of 1.9957.

**(1)**

**(Total for Question 7 is 5 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**8.**

Not to scale

*C*

*B*

*A*

70°

60°

30 m

**Figure 1**

A triangular lawn is modelled by the triangle *ABC*, shown in Figure 1. The length *AB* is to be 30 m long.

Given that angle *BAC* = 70° and angle *ABC* = 60°,

(*a*) calculate the area of the lawn to 3 significant figures.

**(4)**

(*b*) Why is your answer unlikely to be accurate to the nearest square metre?

**(1)**

**(Total for Question 8 is 5 marks)**

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**9.** Solve, for 360° ≤ *x* < 540°,

 12 sin2 *x* + 7 cos *x* − 13 = 0.

 Give your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(5)**

**(Total for Question 9 is 5 marks)**

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**10.** The equation *kx*2 + 4*kx* + 3 = 0, where *k* is a constant, has no real roots.

 Prove that 0 ≤ *k* < 

**(4)**

**(Total for Question 10 is 4 marks)**

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**11.** (*a*) Prove that for all positive values of *x* and *y*,

  ≤ 

**(2)**

(*b*) Prove by counterexample that this is not true when *x* and *y* are both negative.

**(1)**

**(Total for Question 11 is 3 marks)**

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**12.** A student was asked to give the exact solution to the equation

22*x* + 4 – 9(2*x*) = 0

 The student’s attempt is shown below:

 22*x* + 4 – 9(2*x*) = 0

 22*x*+ 24 – 9(2*x*) = 0

 Let 2*x* = *y*

 *y*2 – 9*y* + 8 = 0

(*y* – 8)(*y* – 1) = 0

 *y* = 8 or *y* = 1

 So *x* = 3 or *x* = 0

(*a*) Identify the two errors made by the student.

**(2)**

(*b*) Find the exact solution to the equation.

**(2)**

**(Total for Question 12 is 4 marks)**

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**13.** (*a*) Factorise completely *x*3 + 10*x*2 + 25*x*

**(2)**

 (*b*) Sketch the curve with equation

*y* = *x*3 + 10*x*2 + 25*x*

 showing the coordinates of the points at which the curve cuts or touches the *x*-axis.

**(2)**

The point with coordinates (−3, 0) lies on the curve with equation

*y* = (*x* + *a*)3 + 10(*x* + *a*)2 + 25(*x* + *a*),

where *a* is a constant.

(*c*) Find the two possible values of *a*.

**(3)**

**(Total for Question 13 is 7 marks)**

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**14.**

log10 *P*

*l*

(0, 5)

*O*

*t*

**Figure 2**

 A town’s population, *P*, is modelled by the equation *P* = *abt*, where *a* and *b* are constants and *t* is the number of years since the population was first recorded. The line *l* shown in Figure 2 illustrates the linear relationship between *t* and log10 *P* for the population over a period of 100 years.

 The line *l* meets the vertical axis at (0, 5) as shown. The gradient of *l* is .

(*a*) Write down an equation for *l*.

**(2)**

(*b*) Find the value of *a* and the value of *b*.

**(4)**

(*c*) With reference to the model, interpret

(i) the value of the constant *a*,

 (ii) the value of the constant *b*.

**(2)**

(*d*) Find

 (i) the population predicted by the model when *t* = 100, giving your answer to the nearest hundred thousand,

 (ii) the number of years it takes the population to reach 200 000, according to the model.

**(3)**

(*e*) State two reasons why this may not be a realistic population model.

**(2)**

**(Total for Question 14 is 13 marks)**

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**15.**

Diagram not drawn to scale

*C*1

*C*2

*y*

*x*

*O*

**Figure 3**

 The curve *C*1, shown in Figure 3, has equation *y* = 4*x*2 − 6*x* + 4.

 The point P lies on *C*1.

 The curve *C*2, also shown in Figure 3, has equation *y* = *x* + ln (2*x*).

The normal to *C*1 at the point *P* meets *C*2 at the point *Q*.

Find the exact coordinates of *Q*.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(8)**

**(Total for Question 15 is 8 marks)**

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**16.**

*C*

*E*

*A*

2*x* m

*y* m

*B*

*D*

 **Figure 4**

 Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to
semi-circular section *BCD* as shown in Figure 4.

 Given that *AE* = 2*x* metres, *ED* = *y* metres and the area of the pool is 250 m2,

1. show that the perimeter, *P* metres, of the pool is given by

*P* = 2*x* +  + 

 **(4)**

(*b*) Explain why 0 < *x* < 

**(2)**

(*c*) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

**(4)**

 **(Total for Question 16 is 10 marks)**

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**17.** A circle *C* with centre at (−2, 6) passes through the point (10, 11).

(*a*) Show that the circle *C* also passes through the point (10, 1).

**(3)**

The tangent to the circle C at the point (10, 11) meets the *y*-axis at the point *P*
and the tangent to the circle *C* at the point (10, 1) meets the *y*-axis at the point *Q*.

(*b*) Show that the distance *PQ* is 58, explaining your method clearly.

**(7)**

**(Total for Question 17 is 10 marks)**

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**TOTAL FOR PAPER IS 100 MARKS**