End of Year Practice
Mechanics Practice Questions

A ball is released from rest at a point P, which is 20 m above the ground.

The ball falls vertically towards the ground under the influence of gravity.

(a) Calculate the speed of the ball at the instant it hits the ground.

When the ball hits the ground, it rebounds and bounces off the ground.

The ball bounces off the ground with a speed that is 30% of the speed it hit the ground.

(b) Show that the rebound speed of the ball is about 5.9 m s^{-1}.

(c) Calculate the **total** time taken for the ball to hit the ground a second time after being relased from P.

The particle P moves along the x-axis. The velocity v m s^{-1} of P at time t s is given by

$$v = t^2 - k,\ t \geq 0$$

where k is a positive constant.

Given that the **magnitude** of the initial velocity of P is 4 m s^{-1},

(a) write down the value of k.

(b) Calculate the acceleration of the particle P at $t = 2$.

(c) Calculate the total distance travelled by the particle in the first five seconds of its motion.

[In this question, **i** and **j** are perpendicular unit vectors directed due east and due north respectively.]

The particle P has mass 2 kg and is set into motion with an initial velocity of $(6\mathbf{i} + 4\mathbf{j})$ m s^{-1}. The particle moves under the influence of a constant force $\mathbf{F}$ N where $\mathbf{F} = 12\mathbf{i} - 8\mathbf{j}$.

(a) Find the magnitude and direction of the acceleration of P. Give your direction as a bearing.

(b) Find the speed of the particle P three seconds from being set into motion.

Blocks A and B are connected by a light inextensible string that passes over a small smooth pulley, which is fixed to the edge of the table. Block A has a mass of 5 kg and the block B has a mass of 8 kg. The magnitude of the frictional force between the table and the block A is kR N, where k is a constant and R is the magnitude of the normal reaction force extered by the table on A.

The system is released from rest.

(a) (i) Show that the magnitude of the acceleration of the blocks A and B, a m s^{-2}, is given by

$$a = \frac{8g - 5kg}{13}, \quad k < \frac{8}{5}$$

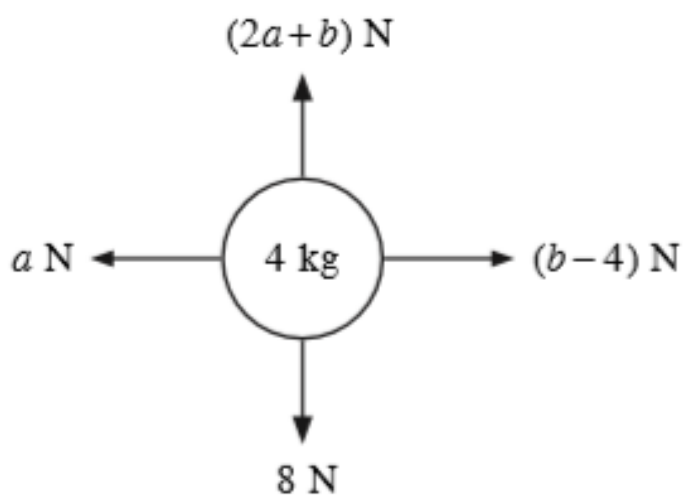
(ii) Explain the restriction of k.

Given that $k = 0.4$,

(b) find the magnitude of the tension in the string,

(c) calculate the magnitude of the resultant force acting on the pulley.

The diagram below shows the forces that act on the particle P which has mass 4 kg.



The particle P accelerates to the left at 8 m s^{-2}.

Find the values of a and b.

A particle P moves in a straight line with a constant acceleration of a m s^{-2}. Given that at time $t = 0$ s, the velocity of the particle is u m s^{-1} and the displacement of the particle is 0 m,

(a) prove that the velocity, v m s^{-1}, of P at time t can be given by $v = u + at$,

(b) prove that the displacement, s m, of P at time t can be given by $s = ut + 0.5at^2$.

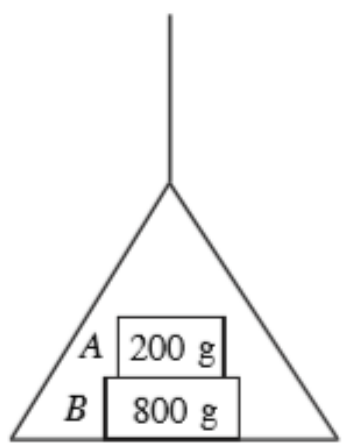
(c) Using the results in parts (a) and (b), deduce that $v^2 - u^2 = 2as$.

At time $t = 0$, a train passes through the point A with a speed of 48 km h^{-1}. The train then accelerates at a constant rate until it reaches a velocity of 69 km h^{-1} at time $t = T$ minutes and then moves at a constant velocity of 69 km h^{-1}. After 10 minutes, the train passes through the point B.

(a) Draw a velocity-time graph for the motion of the train.

Given that the train travels a distance of 11.325 km between A and B,

(b) find the value of T.



A light lift is attached to a vertical light inextensible string. The lift carries the masses A and B. The mass of A is 200 g and the mass of B is 800 g. Mass A rests on top of mass B as shown in the diagram above. The lift is raised vertically at 3 m s^{-2}.

(a) Find the tension in the string.

(b) Calculate the magnitude of the force exerted on A by B.

(c) Write down the magnitude of the force exerted on B by A.

(d) Explain how you have used the fact that lift is light in your calculations.

The particle P is moving to the right in a straight line and on a smooth horizontal table.

At time $t = 0$, the particle passes through the point A and is moving with a velocity of 8 m s^{-1}.

The particle P decelerates at 1 m s^{-2} until it passes through the point B, which is 2 m from A.

After the particle passes the point B, it moves with constant speed for 5 seconds until it passes through the point C.

(a) Find the total time taken for the particle to move from A to C.

(b) Find the total distance travelled by the particle between A and C.

Two particles P and Q hang freely under the influence of gravity. The particles are connected by a light inextensible string that passes over a fixed small smooth pulley. The particle P has mass 2 kg and the particle Q has mass 5 kg. The particles P and Q are 3 m above the ground.

The system is released from rest.

(a) Find the acceleration of the masses.

(b) Find the time taken for the particle Q to hit the ground.

(c) Calculate the maximum height reached by the particle P above the ground.

Statistics Practice Questions

The random variable Y has the probability function

$$P(Y=y)=\begin{cases} k^{-1}y & y=1,\ 2,\ 3 \\ 0 & \text{otherwise} \end{cases}$$

(i) Find the value of the constant k. **[2]**

(ii) Calculate $P(-2 \leq Y \leq 2.5)$. **[1]**

Jasmine records the speed of cars, in miles per hour (mph), on a stretch of a UK motorway. Her results are given in the table below.

Speed (s mph)	Frequency (f)	Speed midpoint (x)
$40 \leq s < 55$	67	47.5
$55 \leq s < 65$	102	60
$65 \leq s < 70$	255	67.5
$70 \leq s < 75$	483	72.5
$75 \leq s < 85$	192	80

(You may use $\sum x^2 f = 5\ 447\ 781.25$.)

A histogram has been drawn to represent these data.

The bar representing the speed $55 \leq s < 65$ has width 2 cm and height 4 cm.

(i) Calculate the width and height of the bar representing the speed $65 \leq s < 70$. **[2]**

(ii) **(a)** Show that the mean speed of the cars on the motorway is 70 mph. **[1]**

(b) Find an estimate for the standard deviation of the speeds of the cars on the motorway. **[2]**

Jasmine thinks she has miscounted and left out a number of cars that were travelling at 70 mph from the table. She wants to include these missing data points into the calculation of the mean and standard deviation.

(iii) Without further calculations, state, giving a reason, what effect including these data points will have on your estimate of the standard deviation. **[1]**

(i) State **two** conditions for a random variable X to have a binomial distribution. **[1]**

In a regular deck of cards, the probability of taking a clubs card is a quarter.

Jeremy takes a deck of cards. Jeremy takes, at random, 32 cards from the deck. He writes down whether the card is a clubs card or not. After each individual card selection, the card is replaced and the deck is shuffled before the next card is picked. Jeremy suspects that the deck is not a regular deck of cards. Jeremy found that 14 of the 32 cards randomly selected from the deck were clubs cards.

(ii) Test Jeremy's suspicion at the 5% level of significance. You should state your hypotheses clearly and the p-value of the test. **[7]**

It turns out that after Jeremy forgot to shuffle the deck after several card selections from the deck.

(iii) Comment on the validity of the model used to obtain the answer to part **(ii)**, giving a reason for your answer. **[1]**

Lauren wants to find the average daily mean windspeed in Hurn in 1987.

She only has access to the large data set. She uses it to obtain a random sample of the daily mean windspeeds, t knots, on n days in Hurn in 1987.

The data collected by Lauren are summarised as follows

$$\sum(t-5)=55 \qquad \bar{t}=10$$

(a) Find n. **(2)**

Lauren uses the same sampling method to estimate that the average daily mean windspeed in Hurn in 2015 was 11 mph.

(b) Convert 11 mph into knots. **(1)**

(c) Hence, compare the average daily mean windspeed in Hurn in 1987 and 2015. **(1)**

(d) With reference to the large data set, state **one** limitation of your conclusion in part (c). **(1)**

(e) Explain how Lauren can

(i) improve her data collection method **(1)**

(ii) improve her data processing **(1)**

to allow for a more reliable comparison in part (c).

Chris investigates the price of petrol, p pence, per litre at fuel stations r miles from his house. He collects data from his local petrol stations and summarises his data in the scatter graph in **Figure 1**.

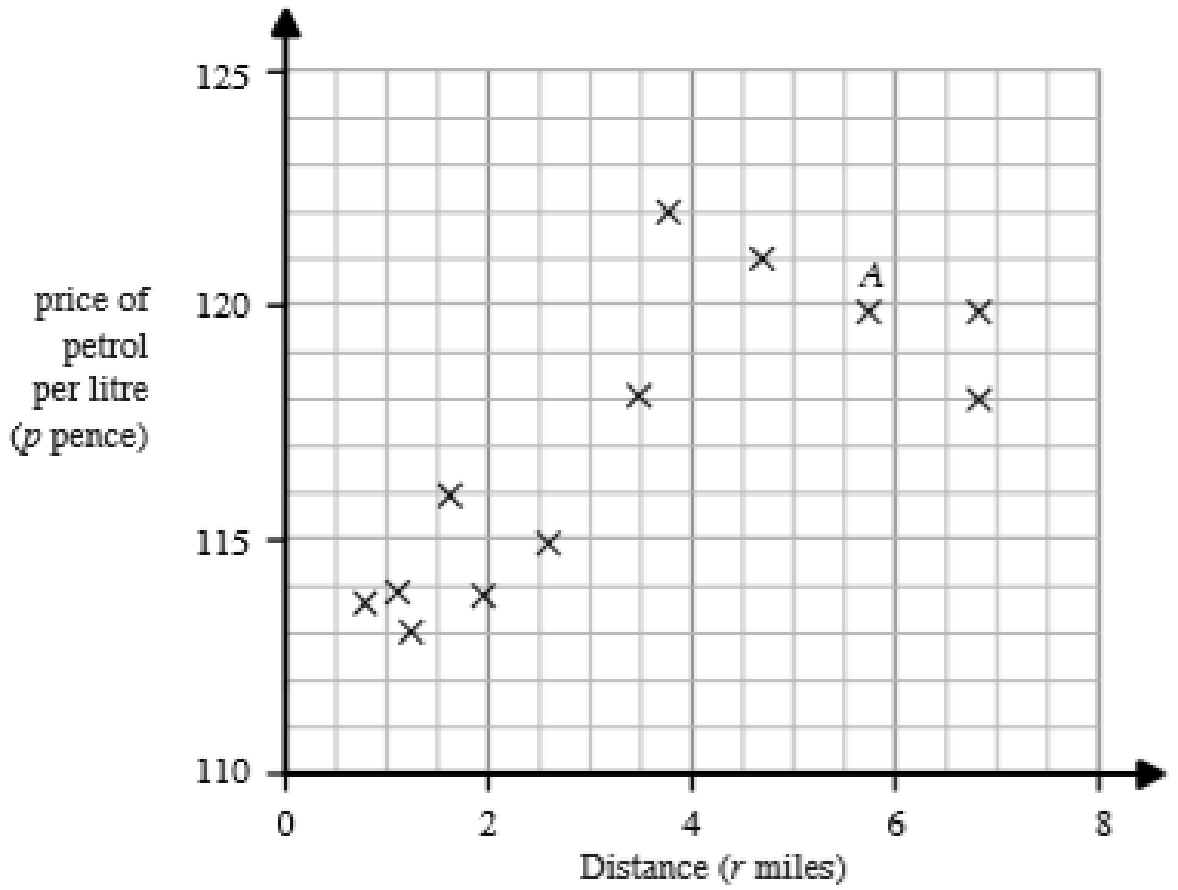


Figure 1

The equation of the regression line of p on r is $p = 113 + 1.13r$.

(a) Give an interpretation of the gradient of this regression line. **(1)**

The petrol station A, labelled on the graph, is 5.70 miles from Chris' home. A new petrol station B has opened 5.80 miles from Chris' home.

(b) Show that the station B is likely to charge less for petrol than station A. **(2)**

Chris is going to visit a friend 100 miles away and wants to estimate the price of petrol near his friend.

(c) Explain, with the aid of a suitable calculation, why Chris should not use his current model to estimate the price of petrol near his friend. **(2)**

A fair four-sided die has faces numbered 1, 2, 3 and 4. A coin is biased so that the probability of tossing heads is $\frac{1}{5}$. The die is thrown once and the number n that it lands on is recorded. The biased coin is then thrown $(n + 2)$ times. So, for example, if the die lands on 3, the coin is thrown 5 times.

(a) Find the probability that the die lands on 4 and the coin shows heads 4 times. **(3)**

(b) Find the probability that the number the die lands on is the same as the number of times the coin shows heads. **(3)**

Emma collects information on the number of hours it took individuals to pass their driving test. To collect her data, Emma uses an opportunity sample. She samples 53 individuals and obtained 50 data points. All of Emma's data is summarised by the histogram in **Figure 2**.

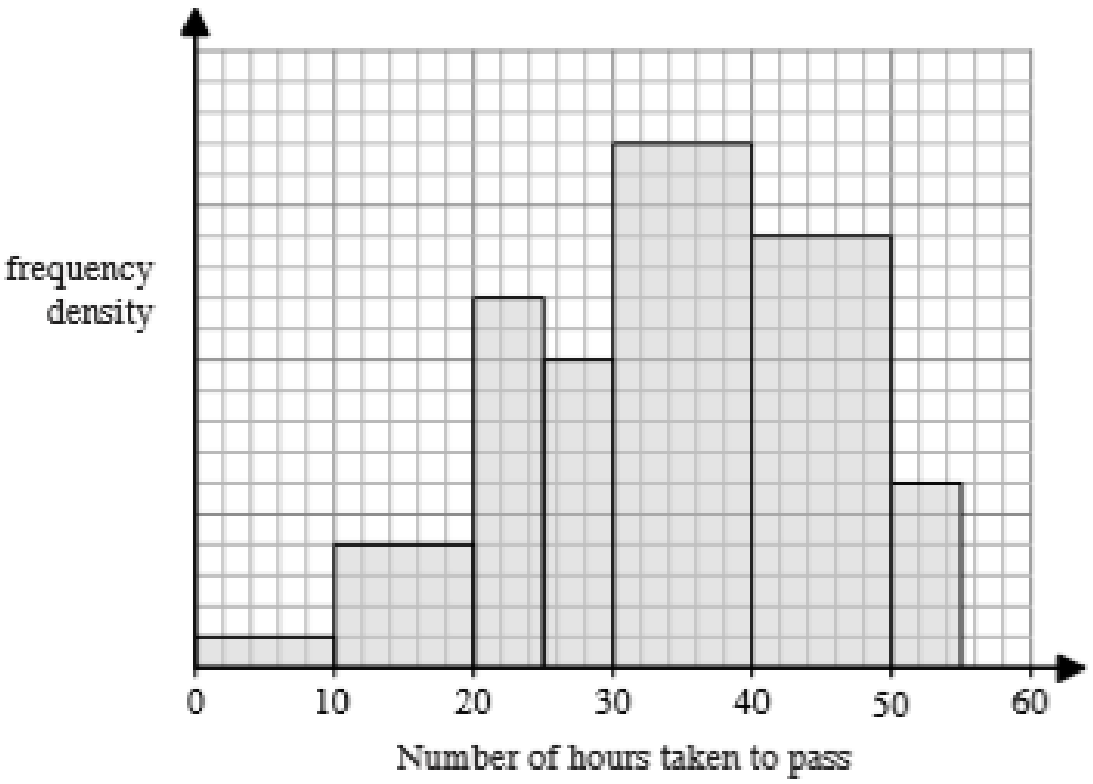


Figure 2

(a) Suggest why Emma did not obtain 53 data points. **(1)**

(b) Find the number of individuals in Emma's sample that took between 25–38 hours to pass their driving test. **(4)**

(c) Calculate an estimate for the median of these data. **(2)**

(a) Explain briefly what you understand by the critical region of a test statistic. **(1)**

(b) A commercial airline reports that two in every thirty of its passengers do not turn up to their flight, and therefore the airline routinely overbooks their flights. An investigative journalist disputes the airline's claims, believing that the proportion is much lower. The journalist picks 50 scheduled passengers at random and finds that one passenger failed to turn up their flight. Investigate the journalist's claims at the 5% level of significance. **(4)**