Mathematics Faculty

Applied Mathematics

Statistics & Mechanics

Booster

Contents

1. Formulae

Statistics

Standard deviation

Standard deviation = √(Variance)

Interquartile range = $\mathrm{IQR} = \mathrm{Q}_3 - \mathrm{Q}_1$

For a set of n values $x_1, x_2, \ldots x_i, \ldots x_n$

$$\text{Standard deviation} = \sqrt{\frac{\mathrm{S}_{xx}}{n}} \text{ or } \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

Mechanics

Kinematics

For motion in a straight line with constant acceleration:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u + v)t$

2. Data Collection

1.

(a) State two reasons why stratified sampling might be a more suitable sampling method than simple random sampling.

(2)

(b) State two reasons why stratified sampling might be a more suitable sampling method than quota sampling.

(2)

2.

A college manager wants to survey students' opinions of enrichment activities.
She decides to survey the students on the courses summarised in the table below.

Course	Number of students enrolled
Leisure and Sport	420
Information Technology	337
Health and Social Care	200
Media Studies	43

Each student takes only one course.

The manager has access to the college's information system that holds full details of each of the enrolled students including name, address, telephone number and their course of study. She wants to compare the opinions of students on each course and has a generous budget to pay for the cost of the survey.

(a) Give one advantage and one disadvantage of carrying out this survey using

(i) quota sampling,

(ii) stratified sampling.

(2)

The manager decides to take a stratified sample of 100 students.

(b) Calculate the number of students to be sampled from each course.

(3)

(c) Describe how to choose students for the stratified sample.

(2)

3.

(a) Explain what you understand by a random sample from a finite population.

(1)

(b) Give an example of a situation when it is not possible to take a random sample.

(1)

A college lecturer specialising in shoe design wants to change the way in which she organises practical work.

She decides to gather ideas from her 75 students.

She plans to give a questionnaire to a random sample of 8 of these students.

(c) (i) Describe the sampling frame that she should use.

(ii) Explain in detail how she should use a table of random numbers to obtain her sample.

(3)

3. Measures of Location and Spread

1.

On a randomly chosen day, each of the 32 students in a class recorded the time, t minutes to the nearest minute, they spent on their homework. The data for the class is summarised in the following table.

Time, t	Number of students
10 – 19	2
20 – 29	4
30 – 39	8
40 – 49	11
50 – 69	5
70 – 79	2

(a) Use interpolation to estimate the value of the median.

(2)

Given that

$$\sum t = 1414 \qquad \text{and} \qquad \sum t^2 = 69\,378$$

(b) find the mean and the standard deviation of the times spent by the students on their homework.

(3)

(c) Comment on the skewness of the distribution of the times spent by the students on their homework. Give a reason for your answer.

(2)

2.

The following table summarises the times, t minutes to the nearest minute, recorded for a group of students to complete an exam.

Time (minutes) t	11 – 20	21 – 25	26 – 30	31 – 35	36 – 45	46 – 60
Number of students f	62	88	16	13	11	10

[You may use $\sum ft^2 = 134281.25$]

(a) Estimate the mean and standard deviation of these data.

(5)

(b) Use linear interpolation to estimate the value of the median.

(2)

(c) Show that the estimated value of the lower quartile is 18.6 to 3 significant figures.

(1)

(d) Estimate the interquartile range of this distribution.

(2)

(e) Give a reason why the mean and standard deviation are not the most appropriate summary statistics to use with these data.

(1)

The person timing the exam made an error and each student actually took 5 minutes less than the times recorded above. The table below summarises the actual times.

Time (minutes) t	6 – 15	16 – 20	21 – 25	26 – 30	31 – 40	41 – 55
Number of students f	62	88	16	13	11	10

(f) Without further calculations, explain the effect this would have on each of the estimates found in parts (a), (b), (c) and (d).

(3)

4. Representing Data

1.

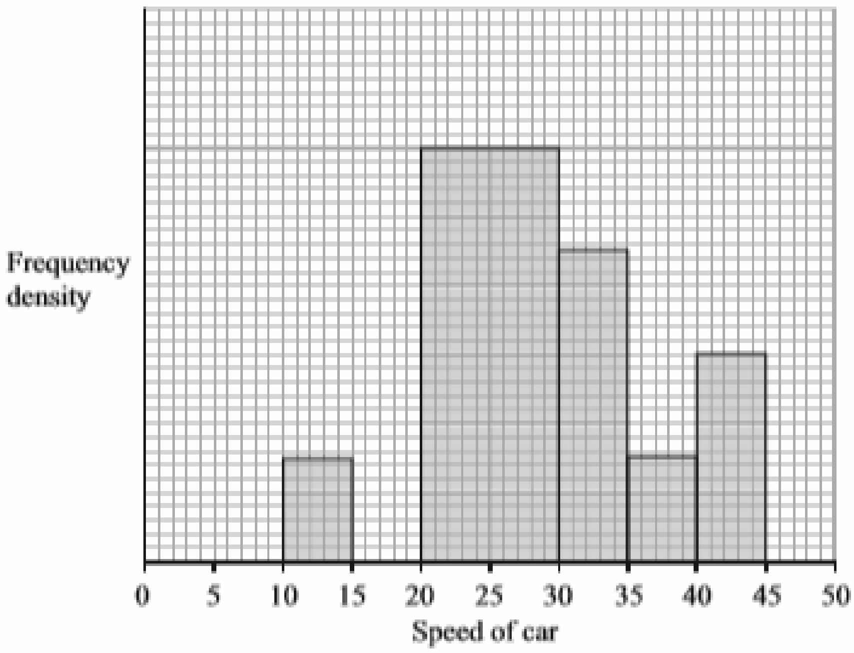

Figure 2

A policeman records the speed of the traffic on a busy road with a 30 mph speed limit. He records the speeds of a sample of 450 cars. The histogram in Figure 2 represents the results.

(a) Calculate the number of cars that were exceeding the speed limit by at least 5 mph in the sample.

(4)

(b) Estimate the value of the mean speed of the cars in the sample.

(3)

(c) Estimate, to 1 decimal place, the value of the median speed of the cars in the sample.

(2)

(d) Comment on the shape of the distribution. Give a reason for your answer.

(2)

(e) State, with a reason, whether the estimate of the mean or the median is a better representation of the average speed of the traffic on the road.

(2)

2.

A midwife records the weights, in kg, of a sample of 50 babies born at a hospital. Her results are given in the table below.

Weight (w kg)	Frequency (f)	Weight midpoint (x)
$0 \leqslant w < 2$	1	1
$2 \leqslant w < 3$	8	2.5
$3 \leqslant w < 3.5$	17	3.25
$3.5 \leqslant w < 4$	17	3.75
$4 \leqslant w < 5$	7	4.5

[You may use $\sum fx^2 = 611.375$]

A histogram has been drawn to represent these data.

The bar representing the weight $2 \leqslant w < 3$ has a width of 1 cm and a height of 4 cm.

(a) Calculate the width and height of the bar representing a weight of $3 \leqslant w < 3.5$
(3)

(b) Use linear interpolation to estimate the median weight of these babies.
(2)

(c) (i) Show that an estimate of the mean weight of these babies is 3.43 kg.

(ii) Find an estimate of the standard deviation of the weights of these babies.
(3)

3.

Over a long period of time a small company recorded the amount it received in sales per month. The results are summarised below.

	Amount received in sales (£1000s)
Two lowest values	3, 4
Lower quartile	7
Median	12
Upper quartile	14
Two highest values	20, 25

An outlier is an observation that falls
either $1.5 \times$ interquartile range above the upper quartile
or $1.5 \times$ interquartile range below the lower quartile.

(a) On the graph paper below, draw a box plot to represent these data, indicating clearly any outliers.

(5)

(b) State the skewness of the distribution of the amount of sales received. Justify your answer.

(2)

(c) The company claims that for 75% of the months, the amount received per month is greater than £10 000. Comment on this claim, giving a reason for your answer.

(2)

4.

The marks, x, of 45 students randomly selected from those students who sat a mathematics examination are shown in the stem and leaf diagram below.

Mark		Totals
3	6 9 9	(3)
4	0 1 2 2 3 4	(6)
4	5 6 6 6 8	(5)
5	0 2 3 3 4 4	(6)
5	5 5 6 7 7 9	(6)
6	0 0 0 0 1 3 4 4 4	(9)
6	5 5 6 7 8 9	(6)
7	1 2 3 3	(4)

Key	(3|6 means 36)

(a) Write down the modal mark of these students.

(1)

(b) Find the values of the lower quartile, the median and the upper quartile.

(3)

For these students $\sum x = 2497$ and $\sum x^2 = 143\,369$

(c) Find the mean and the standard deviation of the marks of these students.

(3)

(d) Describe the skewness of the marks of these students, giving a reason for your answer.

(2)

The mean and standard deviation of the marks of all the students who sat the examination were 55 and 10 respectively. The examiners decided that the total mark of each student should be scaled by subtracting 5 marks and then reducing the mark by a further 10 %.

(e) Find the mean and standard deviation of the scaled marks of all the students.

(4)

5. Constant Acceleration

1.

A ball is thrown vertically upwards with speed u m s^{-1} from a point P at height h metres above the ground. The ball hits the ground 0.75 s later. The speed of the ball immediately before it hits the ground is 6.45 m s^{-1}. The ball is modelled as a particle.

(a) Show that $u = 0.9$

(3)

(b) Find the height above P to which the ball rises before it starts to fall towards the ground again.

(2)

(c) Find the value of h.

(3)

2.

A car accelerates uniformly from rest for 20 seconds. It moves at constant speed v m s^{-1} for the next 40 seconds and then decelerates uniformly for 10 seconds until it comes to rest.

(a) For the motion of the car, sketch

(i) a speed-time graph,

(ii) an acceleration-time graph.

(6)

Given that the total distance moved by the car is 880 m,

(b) find the value of v.

(4)

3.

At time $t = 0$ a ball is projected vertically upwards from a point O and rises to a maximum height of 40 m above O. The ball is modelled as a particle moving freely under gravity.

(a) Show that the speed of projection is 28 m s^{-1}.

(3)

(b) Find the times, in seconds, when the ball is 33.6 m above O.

(5)

4.

A girl runs a 400 m race in a time of 84 s. In a model of this race, it is assumed that, starting from rest, she moves with constant acceleration for 4 s, reaching a speed of 5 m s^{-1}. She maintains this speed for 60 s and then moves with constant deceleration for 20 s, crossing the finishing line with a speed of V m s^{-1}.

(a) Sketch, in the space below, a speed-time graph for the motion of the girl during the whole race.

(2)

(b) Find the distance run by the girl in the first 64 s of the race.

(3)

(c) Find the value of V.

(5)

(d) Find the deceleration of the girl in the final 20 s of her race.

(2)

5.

A stone is projected vertically upwards from a point A with speed u m s^{-1}. After projection the stone moves freely under gravity until it returns to A. The time between the instant that the stone is projected and the instant that it returns to A is $3\frac{4}{7}$ seconds.

Modelling the stone as a particle,

(a) show that $u = 17\frac{1}{2}$,

(3)

(b) find the greatest height above A reached by the stone,

(2)

(c) find the length of time for which the stone is at least $6\frac{3}{5}$ m above A.

(6)

6.

A car is moving on a straight horizontal road. At time $t = 0$, the car is moving with speed 20 m s^{-1} and is at the point A. The car maintains the speed of 20 m s^{-1} for 25 s. The car then moves with constant deceleration 0.4 m s^{-2}, reducing its speed from 20 m s^{-1} to 8 m s^{-1}. The car then moves with constant speed 8 m s^{-1} for 60 s. The car then moves with constant acceleration until it is moving with speed 20 m s^{-1} at the point B.

(a) Sketch a speed-time graph to represent the motion of the car from A to B.

(3)

(b) Find the time for which the car is decelerating.

(2)

Given that the distance from A to B is 1960 m,

(c) find the time taken for the car to move from A to B.

(8)

6. Vectors

1.

A particle P of mass 2 kg is moving under the action of a constant force **F** newtons. The velocity of P is $(2\mathbf{i} - 5\mathbf{j})$ m s^{-1} at time $t = 0$, and $(7\mathbf{i} + 10\mathbf{j})$ m s^{-1} at time $t = 5$ s.

Find

(a) the speed of P at $t = 0$,

(2)

(b) the vector **F** in the form $a\mathbf{i} + b\mathbf{j}$,

(5)

(c) the value of t when P is moving parallel to **i**.

(4)

2.

[*In this question* **i** *and* **j** *are unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O.*]

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i} - 3\mathbf{j})$ km h^{-1} and ship Q moves with velocity $(3\mathbf{i} + 4\mathbf{j})$ km h^{-1}.

(a) Find, to the nearest degree, the bearing on which Q is moving.

(2)

At 2 pm, ship P is at the point with position vector $(\mathbf{i} + \mathbf{j})$ km and ship Q is at the point with position vector $(-2\mathbf{j})$ km.

At time t hours after 2 pm, the position vector of P is **p** km and the position vector of Q is **q** km.

(b) Write down expressions, in terms of t, for

(i) **p**,

(ii) **q**,

(iii) $\overrightarrow{PQ}$.

(5)

(c) Find the time when

(i) Q is due north of P,

(ii) Q is north-west of P.

(4)

3.

Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle P are given by

$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j})\ \text{N}$$
$$\mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j})\ \text{N}$$
$$\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})\ \text{N}$$

where p and q are constants.

Given that P is in equilibrium,

(a) find the value of p and the value of q.

(3)

The force $\mathbf{F}_3$ is now removed. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$.
Find

(b) the magnitude of $\mathbf{R}$,

(2)

(c) the angle, to the nearest degree, that the direction of $\mathbf{R}$ makes with $\mathbf{j}$.

(3)

4.

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving with constant velocity $(-12\mathbf{i} + 7.5\mathbf{j})$ km h^{-1}.

(a) Find the direction in which S is moving, giving your answer as a bearing.

(3)

At time t hours after noon, the position vector of S is $\mathbf{s}$ km. When $t = 0$, $\mathbf{s} = 40\mathbf{i} - 6\mathbf{j}$.

(b) Write down $\mathbf{s}$ in terms of t.

(2)

A fixed beacon B is at the point with position vector $(7\mathbf{i} + 12.5\mathbf{j})$ km.

(c) Find the distance of S from B when $t = 3$

(4)

(d) Find the distance of S from B when S is due north of B.

(4)