

Please write clearly in block capitals.

Centre number

Candidate number

Surname _____

Forename(s) Worked Solutions

Candidate signature _____

GCSE MATHEMATICS

H

Higher Tier Paper 3 Calculator

Monday 11 November 2019 Afternoon Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use	
Pages	Mark
2-3	
4-5	
6-7	
8-9	
10-11	
12-13	
14-15	
16-17	
18-19	
20-21	
22-23	
24-25	
26	
TOTAL	

Advice

In all calculations, show clearly how you work out your answer.



Answer **all** questions in the spaces provided

Do not write
outside the
box

- 1 Circle the relative frequency that represents 13 successes out of 50 trials. [1 mark]

0.13

26

13 : 50

0.26

- 2 The equation of a straight line is $2y = 3x + 5$
Circle the gradient of the line. $y = \frac{3}{2}x + \frac{5}{2}$ [1 mark]

 $\frac{2}{3}$ $\frac{3}{2}$

3

5

- 3 $(2x - 4)(3x + 5)$ is expanded and simplified. $6x^2 + 10x - 12x - 20$
Circle the term which is part of the answer. $6x^2 - 2x - 20$ [1 mark]

2x

-2x

22x

-22x

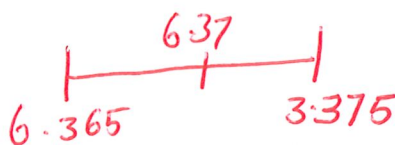


4 When rounded to 3 significant figures, $x = 6.37$

Circle the correct error interval.

$$6.365 \leq x < 6.375$$

$$6.369 \leq x < 6.379$$



$$6.36 \leq x < 6.38$$

$$6.365 \leq x < 6.3749$$

[1 mark]

5 Solve the simultaneous equations

$$\begin{array}{r} 7x + 2y = 36 \\ - (3x + 2y = 16) \\ \hline 4x = 20 \end{array}$$

$$x = 5$$

$$7 \times 5 + 2y = 36$$

$$2y = 1$$

$$y = \frac{1}{2}$$

$$x = \underline{\hspace{2cm}} \quad y = \underline{\hspace{2cm}}$$

[3 marks]



6 (a) Tom is tiling a wall.

He needs to buy at least 100 tiles.

The tiles are sold in large packs and small packs.

Large pack 40 tiles £18

Small pack 28 tiles £14

Special offer

25% reduction when you buy 3 or more **large** packs

Work out the cheapest cost for Tom to buy the packs of tiles he needs.

[3 marks]

$$£18 \times 3 \times 0.75 = £40.50$$

or

$$£14 \times 1 + 18 \times 2 = £50$$

3 large packs

Answer £ 40.50



6 (b) Tom is also tiling a floor.

The floor is a rectangle with length 600 cm and width 240 cm

Each tile is a square with side 40 cm

Tom uses this method to work out the number of tiles he needs.

$$\begin{aligned} \text{Number of tiles that will fit along the length} &= 600 \div 40 \\ &= 15 \end{aligned}$$

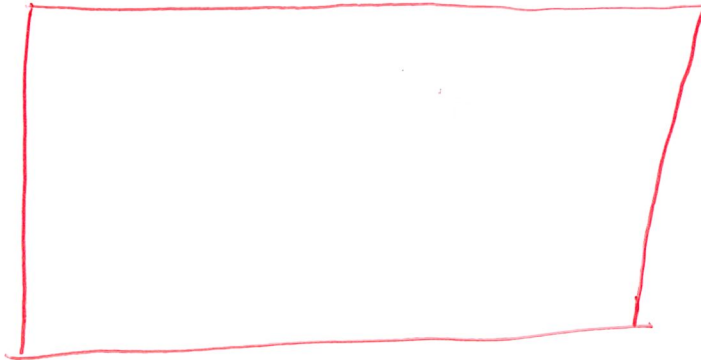
$$\begin{aligned} \text{Number of tiles that will fit along the width} &= 240 \div 40 \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{Total number of tiles needed} &= 15 + 6 \\ &= 21 \end{aligned}$$

Give a reason why Tom's method is wrong.

[1 mark]

*Tom has done 1 length and 1 width,
rather than the whole area.*



Turn over for the next question



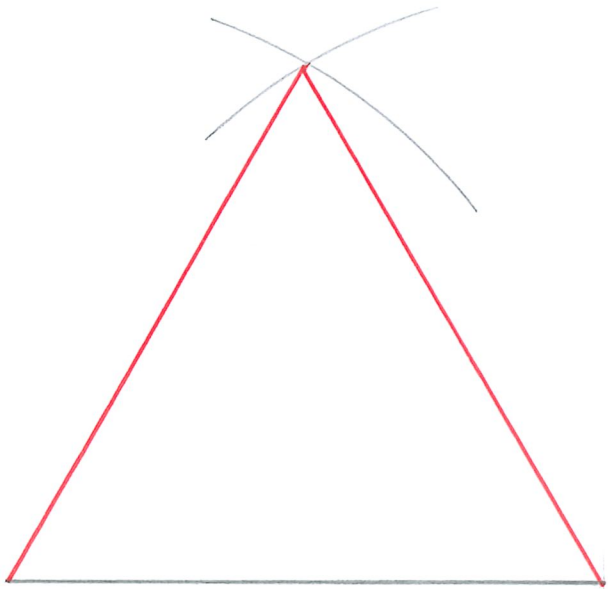
7 An equilateral triangle has side length 16 metres.

Using ruler and compasses only, construct a scale drawing of the triangle.

Use the scale 1 centimetre represents 2 metres.

[3 marks]

Scale: 1 cm represents 2 m



8 In a choir there are 35 men and 48 women.

The probability that a man chosen at random wears glasses is $\frac{2}{5}$

The probability that a woman chosen at random wears glasses is $\frac{3}{8}$

8 (a) Work out the number of people in the choir who wear glasses.

[3 marks]

$$35 \times \frac{2}{5} + 48 \times \frac{3}{8} = 32$$

Answer _____

8 (b) A person is chosen at random from the choir.

Work out the probability that the person does **not** wear glasses.

[2 marks]

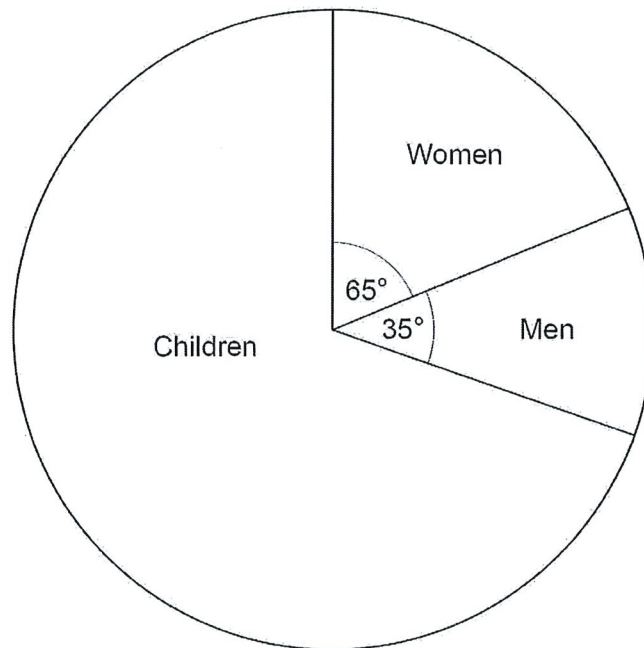
$$\frac{48 + 35 - 32}{48 + 35} = \frac{51}{83}$$

Answer _____



9

The pie chart shows information about people at a theme park.

There were 450 **more** women than men.

Work out the number of children.

[3 marks]

$$65 - 35 = 30$$

$$30^\circ \text{ is } 450$$

$$360 - 65 - 35 = 260$$

$$450 \div 30 = 15$$

$$15 \times 260 = \underline{\underline{3900}}$$

Answer _____



10

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\frac{\div 2}{4} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

The mass is divided by 2 and the volume is multiplied by 4

What happens to the density?

Circle your answer.

[1 mark]

× 2

÷ 2

× 8

÷ 8

11

Work out

cube root of 512 : reciprocal of 0.4

Give your answer in the form $n : 1$

[3 marks]

$$8 : 2.5$$

$$3.2 : 1$$

Answer

3.2

:

1

Turn over for the next question

Turn over ►



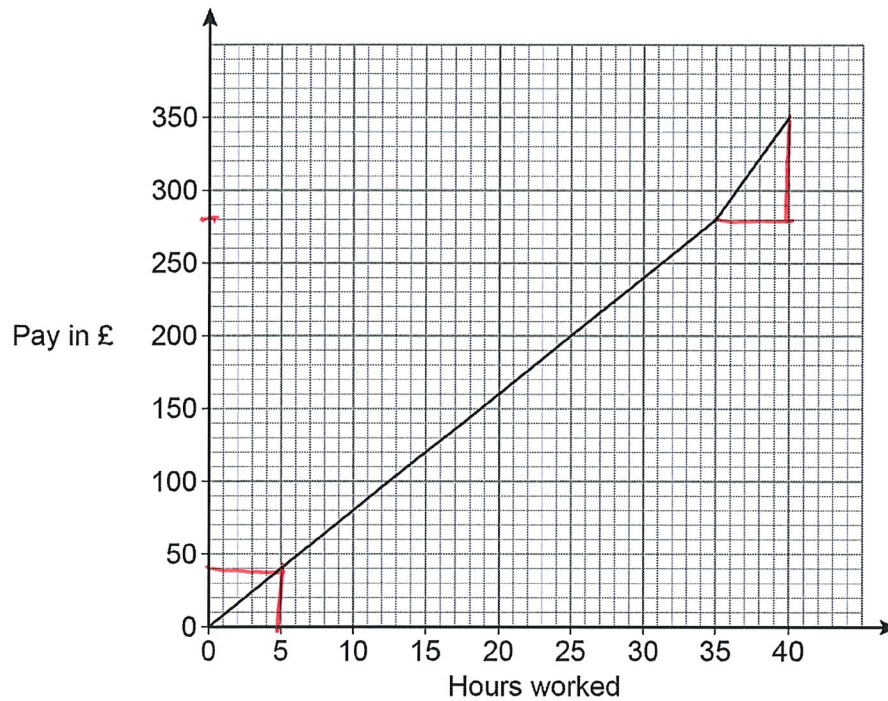
12

The graph shows how much Molly is paid for working for up to 40 hours.

She receives

a basic rate of pay for the first 35 hours worked

a higher rate of pay for the next 5 hours worked.



Work out the difference between the higher rate of pay and the basic rate of pay.

Give your answer in £ per hour.

[3 marks]

$$\begin{array}{r}
 5 \text{ hrs} \quad \pounds 40 \quad \quad 40/5 = \pounds 8 \text{ per hour} \\
 \hline
 5 \text{ hrs} \quad \pounds 70 \quad \quad 70/5 = \pounds 14 \\
 \hline
 \text{difference} \quad 14 - 8 = 6
 \end{array}$$

Answer £ 6 per hour



13 Naga states a hypothesis.

“Most people read more than 100 books a year.”

She asks a sample of five people in a book club how many books they read last month.
The table shows the results.

	Lynn	Ali	Paul	Chen	Ruth
Number of books	10	11	8	10	13

13 (a) Show how Naga could use the data to support her hypothesis.

[2 marks]

Multiply by 12 120, 132, 96, 120, 156

4 out of 5 more than 100

13 (b) Give two reasons why this sample should **not** be used to support her hypothesis.

[2 marks]

Reason 1

Too few people asked

Reason 2

She has asked people in a book club,
so more likely to read a lot of books.



14

A graph has equation $y = x^3 + a$ where a is an integer.

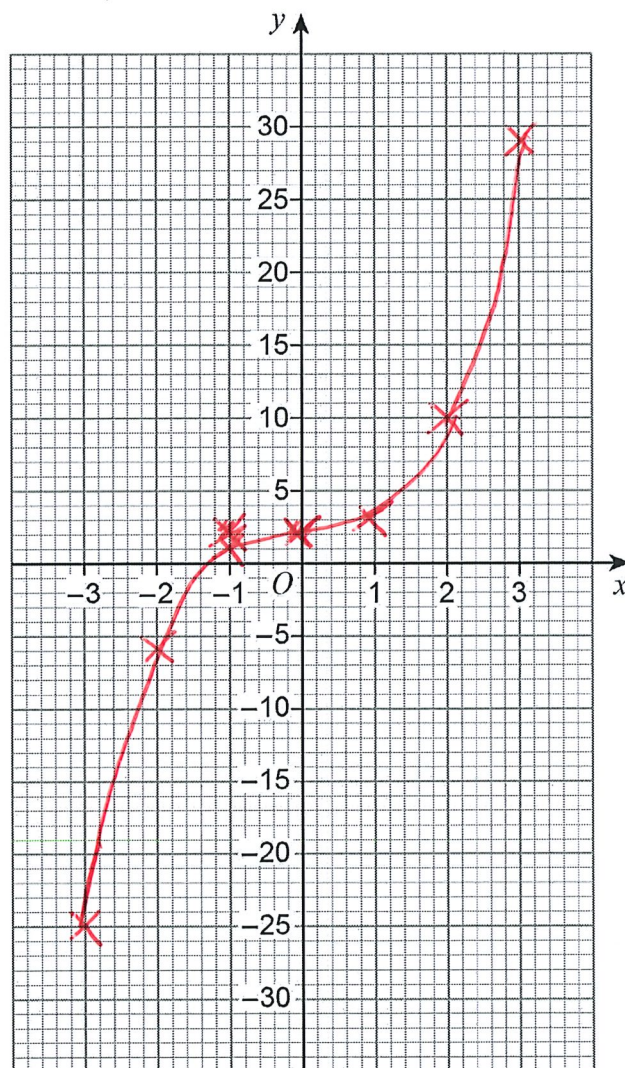
The graph passes through the point (3, 29)

Draw the graph for values of x from -3 to 3

$$y = x^3 \quad \text{if } x = 3 \quad y = 27$$

$$\text{so } a = 2 \quad y = x^3 + 2$$

[3 marks]



15

When you earn money you pay income tax.

The amount you pay depends on how much you earn that year.

You pay

0% on the first £12 500 you earn

20% on the next £37 500 you earn

40% on the next £112 500 you earn.

One year, Kim paid £9260 income tax.

Work out how much she earned that year.

[4 marks]

$$37500 \times 0.2 = 7500$$

$$9260 - 7500 = 1760$$

$$x \times 0.4 = 1760 \Rightarrow x = \underline{\underline{4400}}$$

$$12,500 + 37,500 + 4,400 = \underline{\underline{54,400}}$$

Answer £ _____



- 16** A building company employs
2 labourers
14 joiners
9 electricians
8 plumbers.

For a job, the company needs one of each type of worker.

- 16 (a)** In how many ways can the company choose the four workers?

[2 marks]

$$2 \times 14 \times 9 \times 8 = \underline{2016}$$

Answer _____

- 16 (b)** One labourer and two plumbers are on holiday.

In how many ways can the company now choose the four workers?

[2 marks]

$$1 \times 14 \times 9 \times 6 = 756$$

Answer _____



17

$$f(x) = 3x^2 - 4x + 8 \quad \text{for all values of } x$$

Jenny says,

" $f(10)$ must equal $2 \times f(5)$, because 10 is 2×5 "

Is Jenny correct?

Show working to support your answer.

[2 marks]

$$f(10) = 3 \times 10^2 - 4 \times 10 + 8 = 268$$

$$f(5) = 3 \times 5^2 - 4 \times 5 + 8 = 63$$

$$268 \neq 63 \times 2$$

so not correct.

18

Work out the **two** roots of $(7x + 1)(2x - 3) = 0$

Circle **both** roots.

[1 mark]

$$\frac{1}{7}$$

$$\frac{1}{7}$$

$$-\frac{3}{2}$$

$$\frac{3}{2}$$

$$7x + 1 = 0$$

$$x = -\frac{1}{7}$$

$$2x - 3 = 0$$

$$x = \frac{3}{2}$$

Turn over ►

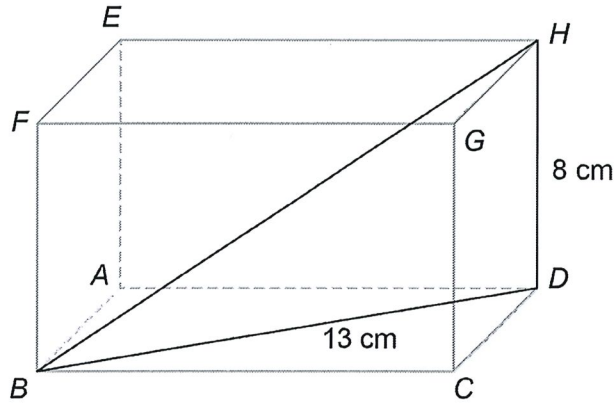


19

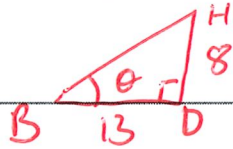
Here is a cuboid.

$$DH = 8 \text{ cm}$$

$$DB = 13 \text{ cm}$$

19 (a) Work out the size of angle DBH .

[2 marks]

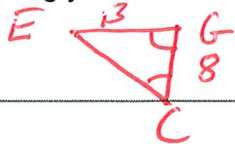


$$\tan \theta = \frac{8}{13}$$

$$\theta = \tan^{-1}\left(\frac{8}{13}\right) = 31.6950225^\circ$$

Answer 31.6 degrees19 (b) Using your answer to part (a), work out the size of angle ECG .

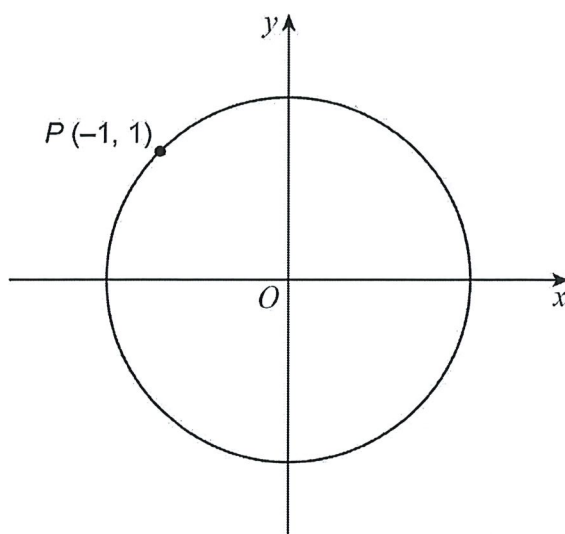
[1 mark]



$$180 - 31.6 - 90 =$$

Answer 58.4 degrees

20

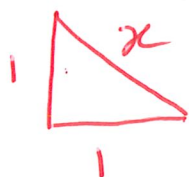
 $P(-1, 1)$ is a point on the circle, centre O , radius r .Not drawn
accuratelyWork out the value of r .

Circle your answer.

[1 mark]

1

2

 $\sqrt{2}$ $2\sqrt{2}$ 

$$1^2 + 1^2 = x^2$$

$$2 = x^2$$

$$x = \sqrt{2}$$

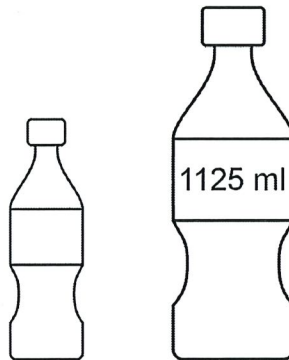
Turn over ►



21

Juice is sold in small bottles and large bottles.

The volume of the large bottle is 1125 ml.



volume of small bottle : volume of large bottle = 2 : 5

A café has small glasses and large glasses.

volume of small glass : volume of large glass = 4 : 7

A small bottle fills 6 small glasses with no juice left over.

How many large glasses can be filled by a large bottle?

You **must** show your working.

[4 marks]

SG SM LB LG

2 : 5

$\frac{2}{6}$

$\frac{2}{6} \times \frac{7}{4} = \frac{7}{12}$

$5 \div \frac{7}{12} = \frac{60}{7}$ or 8.571428

8 with 0.571428 over

Answer _____



22 The only solution to $x^2 + bx + c = 0$ is $x = 5$

Work out the values of b and c .

[2 marks]

$$(x-5)^2 = 0$$

$$x^2 - 10x + 25 = 0$$

$$b = \underline{-10} \quad c = \underline{25}$$

23

$$x : y = \frac{1}{4} : \frac{2}{3}$$

What is x as a fraction of y ?

Circle your answer.

[1 mark]

$$\frac{8}{3}$$

$$\frac{1}{6}$$

$$\frac{3}{7}$$

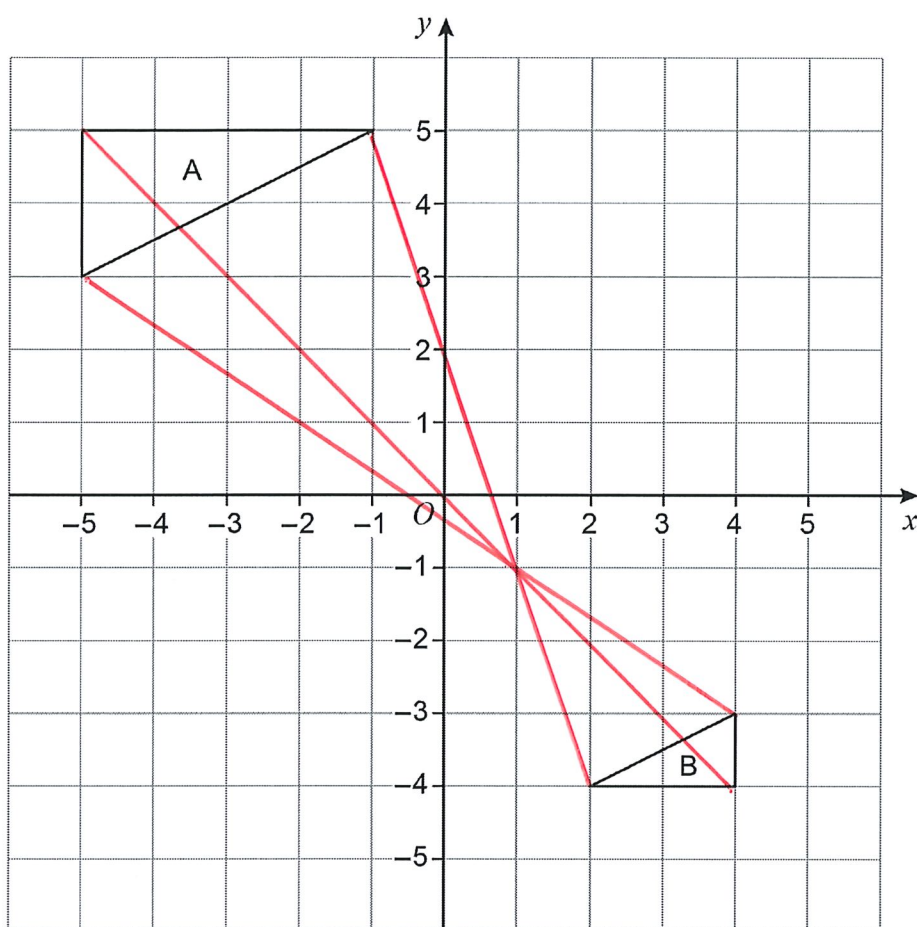
$$\frac{3}{8}$$

$$\frac{1}{4} \div \frac{2}{3} = \frac{3}{8}$$



24

Shape A and shape B are shown on the grid.

Describe the **single** transformation that maps shape A to shape B.

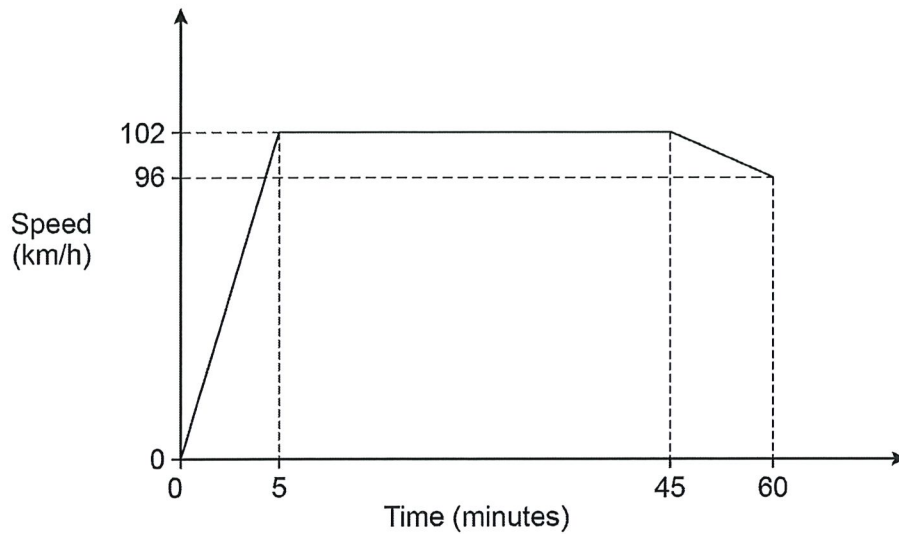
[3 marks]

Enlargement scale factor -2 , centre of
enlargement $(1, -1)$



25

Here is a sketch of a speed-time graph for the first part of a journey.



The total distance for the journey is 130 kilometres.

How far is left to travel?

[4 marks]

$$\frac{5 \times 102}{2} + 40 \times 102 + 15 \times 96 + \frac{6 \times 15}{2}$$

But time in hours not minutes so all times need dividing by 60.

$$\frac{5820}{60} = 97 \quad 97 \text{ km}$$

$$130 - 97 = 33 \text{ km}$$

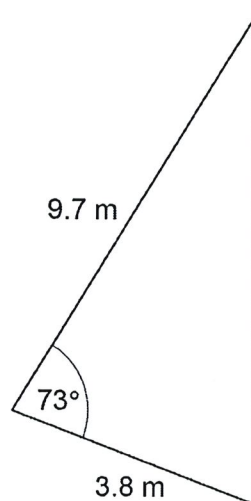
Answer 33 km

Turn over ►



26

Here is a triangular sail.

Not drawn
accurately

26 (a) Vicky needs to buy waterproofing liquid for the sail.

She will put **3 coats** of liquid on **each** side of the sail.

A litre of liquid covers 8.5 square metres of sail.

How many 1-litre bottles of liquid does Vicky need?

[3 marks]

$$\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2} \times 9.7 \times 3.8 \times \sin 73$$

$$= 17.62469665 \text{ m}^2$$

$$\text{Area} \times 2 \times 3 = 105.7481799$$

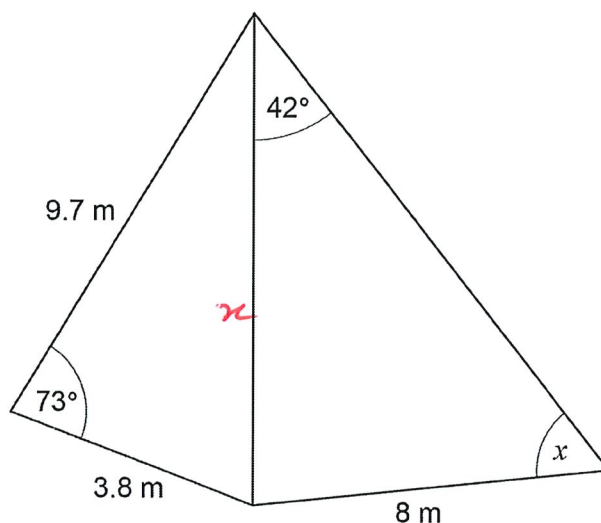
$$105.7481799 \div 8.5 = 12.44096234$$

Answer

13



- 26 (b) Another sail is joined to the first sail as shown.



Not drawn
accurately

x is an acute angle.

Work out the size of angle x .

[5 marks]

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$x^2 = 9.7^2 + 3.8^2 - 2 \times 9.7 \times 3.8 \times \cos 73$$

$$x = 9.32611619$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{8}{\sin 42} = \frac{x}{\sin x} \quad \text{or} \quad \frac{\sin 42}{8} = \frac{\sin x}{9.32611619}$$

$$x = \sin^{-1} \left(\frac{9.32611619 \sin 42}{8} \right) = 51.2650016$$

Answer 51.3 degrees

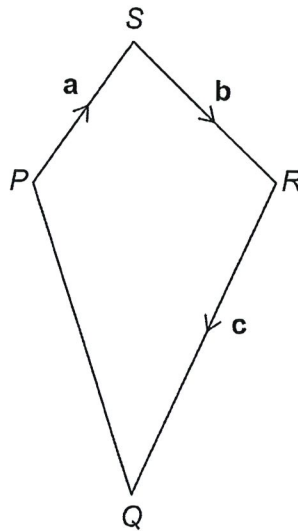
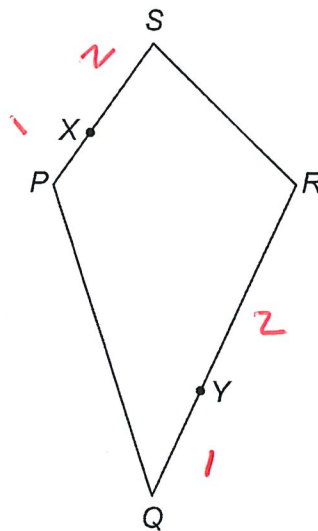
Turn over ►



27

Here is quadrilateral $PQRS$.

$$\overrightarrow{PS} = \mathbf{a} \quad \overrightarrow{SR} = \mathbf{b} \quad \overrightarrow{RQ} = \mathbf{c}$$

Not drawn
accuratelyX is a point on PS where $PX : XS = 1 : 2$ Y is a point on RQ where $RY : YQ = 2 : 1$ Not drawn
accurately

Is XY parallel to PQ?

Show working to support your answer.

[3 marks]

$$\vec{XY} = \vec{XS} + \vec{SR} + \vec{RY}$$

$$\frac{2}{3}\underline{a} + \underline{b} + \frac{2}{3}\underline{c}$$

$$\vec{PQ} = \underline{a} + \underline{b} + \underline{c}$$

as \vec{PQ} is not a multiple of \vec{XY} they are
not parallel.

Turn over for the next question

Turn over ►



28

$$f(x) = 2x - 3 \text{ and } g(x) = x^2$$

Show that $f^{-1}(55) = fg(4)$

[4 marks]

$$\text{let } y = 2x - 3 \Rightarrow y + 3 = 2x$$

$$\text{and } \frac{y+3}{2} = x \quad \text{so } f^{-1}(x) = \frac{x+3}{2}$$

$$f^{-1}(55) = \frac{55+3}{2} = \underline{\underline{29}}$$

$$g(4) = 16 \quad fg(4) = f(16) = 2 \times 16 - 3 = \underline{\underline{29}}$$

so both the same, therefore $f^{-1}(55) = fg(4)$

END OF QUESTIONS

