Miss Watson Solutions

Core 1: January 2008

1 $\dfrac{4}{3-\sqrt{7}} \times \dfrac{3+\sqrt{7}}{3+\sqrt{7}} = \dfrac{4(3+\sqrt{7})}{\underbrace{(3-\sqrt{7})(3+\sqrt{7})}}$ 1

$\times$	3	$-\sqrt{7}$
3	9	$-3\sqrt{7}$
$+\sqrt{7}$	$+3\sqrt{7}$	-7

$\Rightarrow \dfrac{12+4\sqrt{7}}{9-7}$ 1 $= \dfrac{12+4\sqrt{7}}{2}$

$= 6 + 2\sqrt{7}$ 1

$\boxed{3}$

2 centre $(0,0)$ radius 7

General form of a circle $(x-a)^2+(y-b)^2=r^2$

(i) $(x-0)^2+(y-0)^2 = 7^2$

$x^2+y^2=49$ 1

$\boxed{1}$

(ii) Centre $(3,5)$

$x^2+y^2-6x-10y-30=0$

Complete the square for x's and y's

$(x-3)^2-9+(y-5)^2-25-30=0$ 1

$(x-3)^2+(y-5)^2-64=0$

$(x-3)^2+(y-5)^2=64$

$(x-3)^2+(y-5)^2=8^2$

radius $= 8$ 1

$\boxed{2}$

③ $3x^2 + bx + 10 = a(x+3)^2 + c$

can see $a = 3$ from completed square form. ①

Multiply out the brackets

$3(x+3)^2 + c$

$3(x^2 + 6x + 9) + c$ ①

$3x^2 + 18x + 27 + c$

$3x^2 + 18x + 27 + c = 3x^2 + bx + 10$ ①

$a = 3$

$b = 18$

$c = 10 - 27 = -17$ ①

$\boxed{4}$

④

(i) $10^p = 0.1$

$10^{-1} = \frac{1}{10^1} = 0.1$

$p = -1$ ①

$\boxed{1}$

(ii) $(25k^2)^{1/2} = 15$

$\sqrt{25k^2} = 15$

$25k^2 = 15^2$ ①

$25k^2 = 225$

$k^2 = 9$ ①

$k = \pm 3$ ①

$\boxed{3}$

(iii) $t^{-1/3} = \frac{1}{2}$

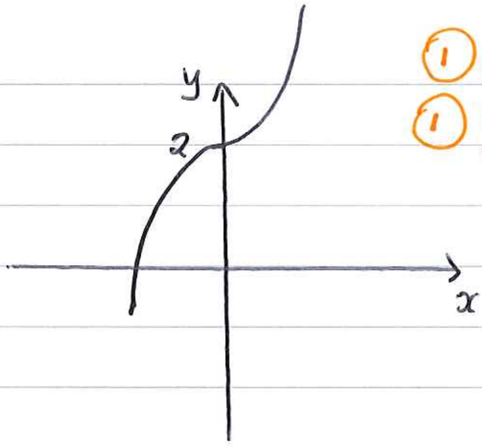
$\frac{1}{\sqrt[3]{t}} = \frac{1}{2}$ ①

$t = 8$ ①

$\boxed{2}$

(5)

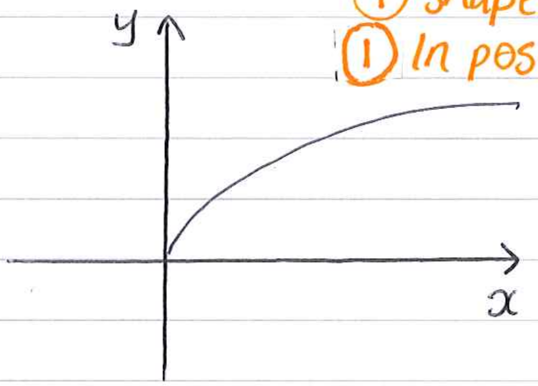
(i) $y = x^3 + 2$



(1) cubic shape
(1) y-intercept

$\boxed{2}$

(ii) $y = 2\sqrt{x}$



(1) shape of curve
(1) In positive quadrant

$\boxed{2}$

(iii) $y = 2\sqrt{x} \longrightarrow y = 3\sqrt{x}$

stretch (1) of scale factor 1.5 (1) parallel to y-axis. (1)

$\boxed{3}$

(6)

(i) $x^2 + 8x + 10 = 0$

$a = 1$ (1)

$b = 8$

$c = 10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

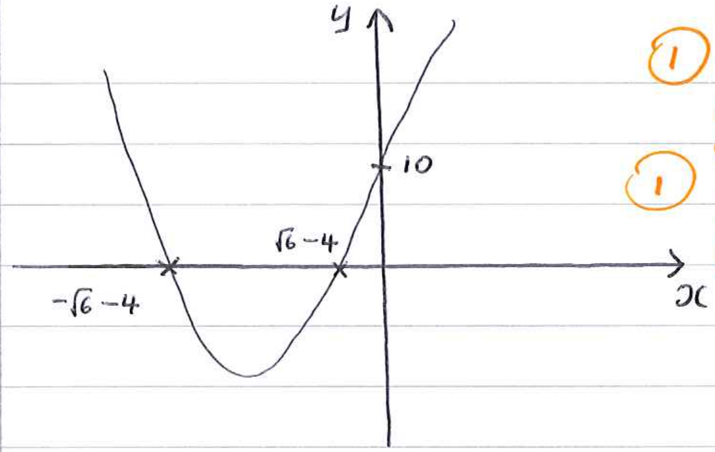
$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times 10}}{2 \times 1}$$

$$x = \frac{-8 \pm \sqrt{24}}{2} \quad (1)$$

$$x = \frac{-8 \pm 2\sqrt{6}}{2} = \underline{\underline{-4 \pm \sqrt{6}}} \quad (1)$$

$\boxed{3}$

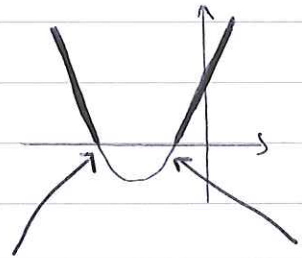
(ii)



① Positive quadratic graph = parabola
① y intercept
① the two roots.

$\boxed{3}$

(iii) $x^2+8x+10 \geq 0$



$x \leq -\sqrt{6}-4$ ①

$x \geq \sqrt{6}-4$ ①

$\boxed{2}$

⑦

(i) $x+2y=4$

$2y=-x+4$

$y=-\frac{1}{2}x+2$

Gradient $= -\frac{1}{2}$ ①

$\boxed{1}$

(ii) $y=-\frac{1}{2}x+c$ parallel, Point $(6,5)$

$5=-\frac{1}{2}(6)+c$ ①

$5=-3+c$

$c=8$

(x2) $y=-\frac{1}{2}x+8$ ① In form $ax+by+c=0$

$2y=-x+16$

$x+2y-16=0$ ①

$\boxed{3}$

(iii) $y = x^2 + x + 1$ and $x + 2y = 4$

$x + 2y = 4$
$x = 4 - 2y$

$y = x^2 + x + 1$ (sub in $x = 4 - 2y$)
$y = (4-2y)^2 + (4-2y) + 1$ 1
$y = 16 - 16y + 4y^2 + 4 - 2y + 1$
$y = 21 - 18y + 4y^2$
$0 = 4y^2 - 19y + 21$

$0 = (4y - 7)(y - 3)$ 1

$y = 7/4$, $y = 3$

when $y = 7/4$, $x = 4 - 2y$
$x = 4 - 2 \times 7/4$
$x = 1/2$

$\underline{\underline{(1/2, 7/4)}}$ 1

when $y = 3$, $x = 4 - 2y$
$x = 4 - 2 \times 3$
$x = -2$

$\underline{\underline{(-2, 3)}}$ 1

$\boxed{4}$

(8)

(i) $y = x^3 + x^2 - x + 3$

$\frac{dy}{dx} = 3x^2 + 2x - 1$ (2)

$0 = 3x^2 + 2x - 1$ (1)

$0 = (3x - 1)(x + 1)$ (1)

$x = 1/3 \quad , \quad x = -1$

when $x = 1/3$, $y = (1/3)^3 + (1/3)^2 - 1/3 + 3$

$= 1/27 + 1/9 - 1/3 + 3$

$= 1/27 + 3/27 - 9/27 + 3$

$= -5/27 + 3$

$= 76/27$

$\underline{\underline{(1/3 , 76/27)}}$ (1)

when $x = -1$ $y = (-1)^3 + (-1)^2 - (-1) + 3$

$= -1 + 1 + 1 + 3$

$= 4$

$\underline{\underline{(-1, 4)}}$ (1)

$\boxed{6}$

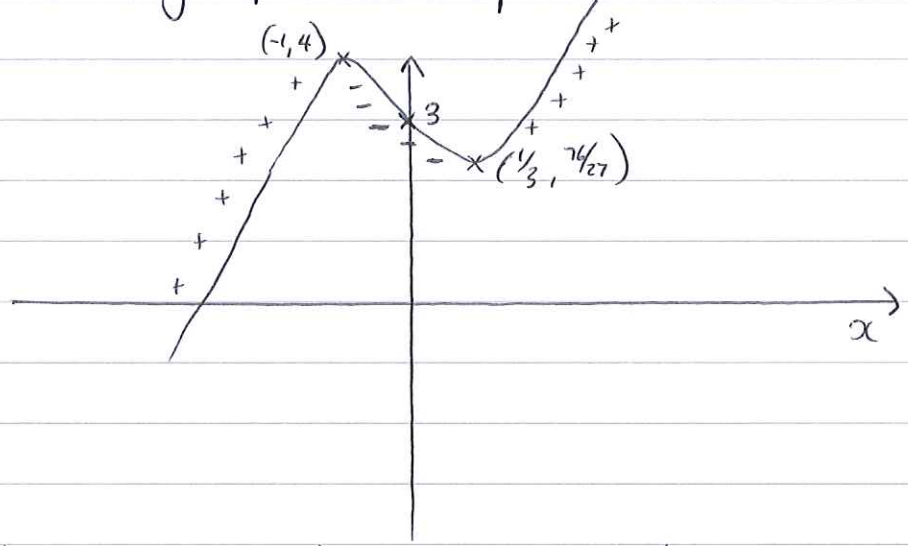
(ii) $\frac{d^2y}{dx^2} = 6x + 2$ (1)

at $x = 1/3$, $\frac{d^2y}{dx^2} = 6 \times 1/3 + 2 = 4 > 0$ Minimum (1)

at $x = -1$, $\frac{d^2y}{dx^2} = 6 \times -1 + 2 = -4 < 0$ Maximum (1)

$\boxed{3}$

(iii) Sketch graph to help $x^3 + x^2 - x + 3$

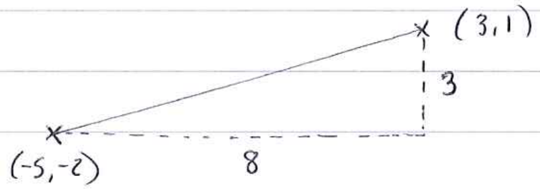


The graph is decreasing between

$$\underline{\underline{-1 < x < 1/3}} \quad 2$$

2

9
(i) $A(-5, -2) \quad B(3, 1)$



$$\text{Gradient} = \frac{1-(-2)}{3-(-5)} = \frac{3}{8} \quad 1$$

$y = mx + c$
$y = 3/8\, x + c$ (sub in (3,1))
$1 = 3/8(3) + c$ 1
$1 = 9/8 + c$ (×8)
$8 = 9 + c$
$c = -1$

$y = 3/8\, x - 1$ (×8)
$8y = 3x - 1$
$\underline{\underline{0 = 3x - 8y - 1}}$ 1

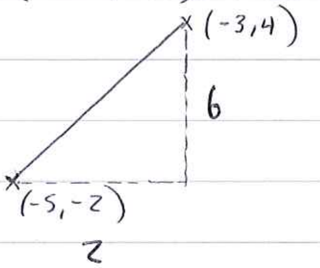
3

(ii) Midpoint $(-5,-2)(3,1)$

$$\left(\frac{-5+3}{2}, \frac{-2+1}{2}\right) = \underline{\underline{\left(-1, -\tfrac{1}{2}\right)}}$$ 1 1

$\boxed{2}$

(iii) $A(-5,-2)$ $C(-3,4)$



$$AC = \sqrt{(-5-(-3))^2 + (-2-4)^2}$$ 1

$$= \sqrt{2^2 + 6^2}$$

$$= \sqrt{40}$$ 1

$$= \underline{\underline{2\sqrt{10}}}$$ 1

$\boxed{3}$

(iv) Is AC perpendicular to BC?

Gradient $AC = \dfrac{-2-4}{-5-(-3)} = 3$ 1

Gradient $BC = \dfrac{4-1}{-3-3} = -\tfrac{1}{2}$ 1

2 $3 \times -\tfrac{1}{2} \neq -1$ so lines are not perpendicular.

OR

3 is not the negative reciprocal of $-\tfrac{1}{2}$ and therefore AC is not perpendicular to BC.

$\boxed{4}$

10

(i) $f(x) = 8x^3 + \frac{1}{x^3}$

$f(x) = 8x^3 + x^{-3}$ 1

$f'(x) = 24x^2 - 3x^{-4}$ 2

$f''(x) = 48x + 12x^{-5}$ 2

$\boxed{5}$

(ii) $-9 = 8x^3 + \frac{1}{x^3} \quad (\times x^3)$

$-9x^3 = 8x^6 + 1$

$0 = 8x^6 + 9x^3 + 1$ 1

Now, use substitution $y = x^3$

$0 = 8y^2 + 9y + 1$ 1

$0 = (8y + 1)(y + 1)$

$y = -1/8 \quad , \quad y = -1$ 1

Now, find the values of x.

If $y = x^3$ then $x = \sqrt[3]{y}$ 1

when $y = -1/8 \quad x = \sqrt[3]{-1/8} \quad x = -1/2$

when $y = -1 \quad x = \sqrt[3]{-1} \quad x = -1$

1

$\boxed{5}$

Total Marks = $\boxed{72}$