# Core 1 – Jan 2010 – Miss Watson – Worked Solutions.

**1**

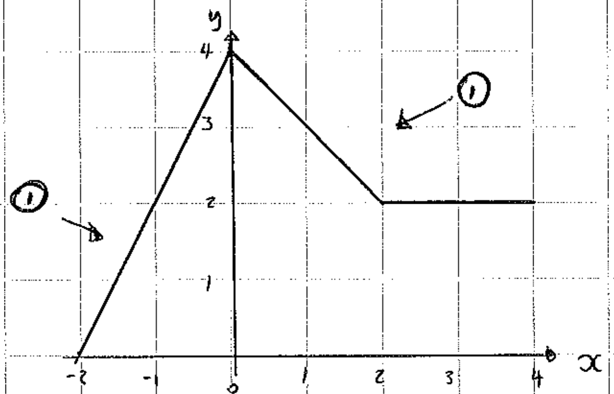
$x^2 - 12x + 1$

(1) $(x-6)^2 - 36 + 1$ (1)

$(x-6)^2 - 35$ (1)

$\boxed{3}$

**2** (i)




$\boxed{2}$

(ii) Translation (1)

One unit to the right (1)

$\boxed{2}$

**3**

$y = x^3 - 4x^2 + 7$

(1) $\dfrac{dy}{dx} = 3x^2 - 8x$ (1)

at $x = 2$ (1) $\dfrac{dy}{dx} = 3 \times 2^2 - 8 \times 2$

$= -4$ (1) tangent gradient.

normal gradient $= \frac{1}{4}$ (1)

$y = mx + c$

$y = \frac{1}{4}x + c$ sub in $(2, -1)$ (1)

$-1 = \frac{1}{4} \times 2 + c$

$-1 = \frac{1}{2} + c$

$c = -3/2$

$y = \frac{1}{4}x - \frac{3}{2}$ $(\times 4)$

$4y = x - 6$

$0 = x - 4y - 6$ (1)

$\boxed{7}$

(4) (i) $3^m = 81$

$3^4 = 81$

① $\underline{\underline{m = 4}}$ $\boxed{1}$

(ii) $(36p^4)^{1/2} = 24$

① $6p^2 = 24$

$p^2 = 4$

$p = \pm\sqrt{4}$

$\underline{\underline{p = \pm 2}}$ ①① $\boxed{3}$

(iii) $5^n \times 5^{n+4} = 25$

① $5^{2n+4} = 25$

$5^{2n+4} = 5^2$

① $2n + 4 = 2$

$2n = -2$

① $\underline{\underline{n = -1}}$ $\boxed{3}$

(5) $x - 8\sqrt{x} + 13 = 0$ sub in $t = \sqrt{x}$

$t^2 = x$

① $t^2 - 8t + 13 = 0$

quadratic formula, $a = 1$, $b = -8$, $c = 13$

①① $t = \dfrac{8 \pm \sqrt{(-8)^2 - 4 \times 1 \times 13}}{2 \times 1} = \dfrac{8 \pm \sqrt{12}}{2} = \dfrac{8 \pm 2\sqrt{3}}{2} = 4 \pm \sqrt{3}$ ①

$x = t^2$

① $x = (4 + \sqrt{3})^2 = (4 + \sqrt{3})(4 + \sqrt{3})$

$x = 16 + 4\sqrt{3} + 4\sqrt{3} + 3$

$x = 19 + 8\sqrt{3}$

$x = t^2$

① $x = (4 - \sqrt{3})^2 = (4 - \sqrt{3})(4 - \sqrt{3})$

$x = 16 - 4\sqrt{3} - 4\sqrt{3} + 3$

$x = 19 - 8\sqrt{3}$

$\underline{\underline{x = 19 \pm 8\sqrt{3}}}$ ① $\boxed{7}$

(6) (i) $y = x^2 + 5$

$\frac{dy}{dx} = 2x$ ①

at $x = 1$ $\frac{dy}{dx} = 2 \times 1 = \underline{2}$ ① $\boxed{2}$

(ii) $\frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{a^2 + 5 - 6}{a - 1} = 2.3$ ①

$a^2 + 5 - 6 = 2.3a - 2.3$

$a^2 - 2.3a + 1.3 = 0$ ①

$(a - 1)(a - 1.3) = 0$ ①

$\cancel{a = 1}$ $\quad \underline{\underline{a = 1.3}}$ ①

would produce gradient of $\frac{1^2 + 5 - 6}{1 - 1} = \frac{0}{0}$

and question states a is <u>greater</u> than 1.

$\boxed{4}$

(iii) At A gradient is 2, the line joining A and B has a gradient of 2.3.

Possible gradient at C is $\underline{2.1}$. ①

$2 <$ value $< 2.3$

$\boxed{1}$

(7) (i) (a) fig 3 ① $\boxed{1}$

(b) fig 1 ① $\boxed{1}$

(c) fig 4 ① $\boxed{1}$

(ii) $\underline{y = -(x - 3)^2}$ ① ① $\boxed{2}$

(8) (i) $x^2+y^2+6x-4y-4=0$

$x^2+6x+y^2-4y=4$

$(x+3)^2-9+(y-2)^2-4=4$ ①

$(x+3)^2+(y-2)^2=17$

Centre $=(-3,2)$ ① radius $=\sqrt{17}$ ① $\boxed{3}$

(ii) sub in $y=3x+4$

$(x+3)^2+(3x+4-2)^2=17$ ①

$(x+3)^2+(3x+2)^2=17$

$x^2+6x+9+9x^2+12x+4=17$ ①

$10x^2+18x-4=0$ $(\div 2)$ ①

$5x^2+9x-2=0$

$(5x-1)(x+2)=0$ ①

$x=\frac{1}{5}$ $x=-2$ ①

$y=3x+4$ $y=3x+4$

$y=\frac{3}{5}+4$ $y=3\times -2+4$

$y=\frac{23}{5}$ $y=-2$ ① $\boxed{6}$

(9) (i) $f(x)=x^{-1}-x^{1/2}+3$

$f'(x)=-x^{-2}-\frac{1}{2}x^{-1/2}$ $\boxed{3}$

① attempt ① ①

(ii) ① $f''(x)=2x^{-3}+\frac{1}{4}x^{-3/2}$ ① ①

$f''(x)=\frac{2}{x^3}+\frac{1}{4(\sqrt{x})^3}$

① substitution

$f''(4)=\frac{2}{4^3}+\frac{1}{4\times(\sqrt{4})^3}=\frac{2}{64}+\frac{1}{32}=\frac{2}{32}=\frac{1}{16}$ ①

$\boxed{5}$

(10) $Kx^2 - 30x + 25K = 0$

$a = K \quad b = -30 \quad c = 25K$

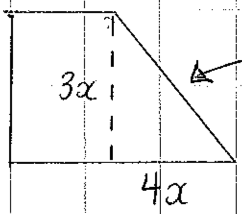
Discriminant $b^2 - 4ac = (-30)^2 - 4 \times K \times 25K = 0$ ①

$= 900 - 100K^2 = 0$ ①

$\underline{\underline{K = \pm 3}}$ ① ①

$\boxed{4}$

(11) (i)




$(3x)^2 + (4x)^2 = 25x^2$

$\sqrt{25x^2} = 5x$ ①

$P = 2 + x + 3x + 2 + 5x + 5x$

$\underline{\underline{P = 14x + 4}}$ ①

$\boxed{2}$

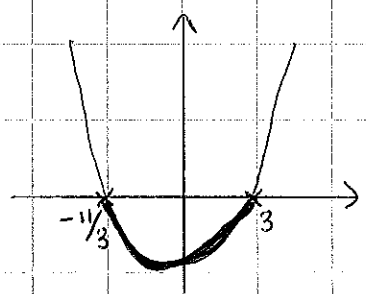
(ii) Area $= \dfrac{(a+b) \times h}{2}$ ① $= \dfrac{(2 + x + 2 + 5x) \times 3x}{2}$ ①

$\Rightarrow \dfrac{(6x+4) \times 3x}{2} = \dfrac{18x^2 + 12x}{2} = \underline{\underline{9x^2 + 6x}}$

$\boxed{2}$

(iii) $14x + 4 \geq 39$ ①

$x \geq 5/2$ ①




$9x^2 + 6x < 99$ ①

$9x^2 + 6x - 99 < 0$

$3x^2 + 2x - 33 < 0$ ① $\quad -11/3 < x < 3$ ①

$(3x+11)(x-3) = 0$

$x = -11/3 \quad x = 3$ ①

combining both answers $\quad \underline{\underline{5/2 \leq x < 3}}$ ①

$\boxed{7}$