Jan 2011 C2

1 i) $(1+2x)^7 = {}^7C_0 1^7 + {}^7C_1 1^6 (2x) + {}^7C_2 1^5 (2x)^2 + \dots$

$= 1 + 14x + 84x^2 + \dots$

ii) $(2-5x)(1+14x+84x^2+\dots)$

x^2// $-5x \times 14x = -70x^2$

$2 \times 84x^2 = 168x^2$

$98x^2$

2 $U_n = 3n+2$

i) $U_1 = 5$

$U_2 = 8$

$U_3 = 11$

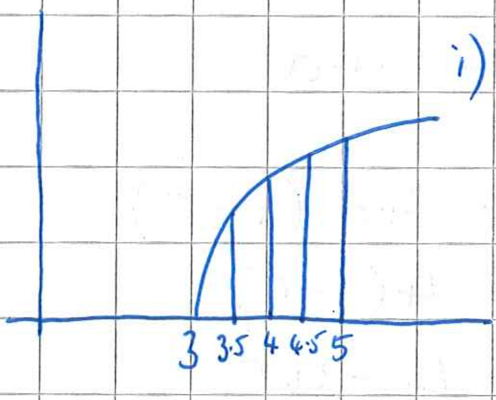
ii) Arithmetic Progression

iii) $\sum_{n=101}^{200} U_n = \frac{100}{2}(2a + (n-1)d)$

$= 50(305 \times 2 + 99 \times 3)$

$= 45350$

3

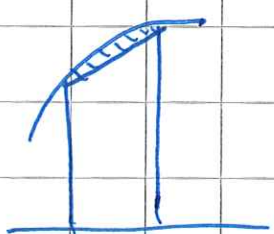


i) Area $\approx \frac{0.5}{2}\left(0 + \sqrt{2} + 2\left(\sqrt{\tfrac{1}{2}} + \sqrt{1} + \sqrt{1.5}\right)\right)$

$= 1.81947\dots$

$= 1.82$ (3sf).

ii) Underestimate as trapezium is under the curve:



4 a) $5^{x-1} = 120$

$$(x-1)\log 5 = \log 120$$

$$x-1 = \frac{\log 120}{\log 5}$$

$$x = \left(\frac{\log 120}{\log 5}\right) + 1$$

$$x = 3.97463\ldots$$

$$x = \underline{3.97} \text{ (3sf)}$$

b) $\log_2 x + 2\log_x 3 = \log_2 (x+5)$

$$\log_2 9x = \log_2 (x+5)$$

$$9x = x+5$$

$$x = 5/8$$

5 i) $S_\infty = \frac{a}{1-r}$ $\qquad \frac{a}{1-r} = 4a$

$$a = 4a(1-r)$$

$$1 = 4(1-r)$$

$$1 = 4 - 4r$$

$$3/4 = r$$

ii) $U_3 = ar^2$

$a \times \left(\frac{3}{4}\right)^2 = 9$

$a = 16$

iii) $\sum_{n=1}^{20} U_n = \frac{a(1-r^n)}{1-r}$

$= \frac{16\left(1 - \frac{3}{4}^{20}\right)}{1 - 3/4}$

$= 63.79704\ldots$

$= 63.8$ (3 s.f.)

⑥ a) $\frac{x^3 + 3x^{1/2}}{x} = \frac{x^3}{x} + \frac{3x^{1/2}}{x}$

$= x^2 + 3x^{-1/2}$

$\int x^2 + 3x^{-1/2}\,dx = \frac{1}{3}x^3 + 6x^{1/2} + c$

b) i) $\int_2^a 6x^{-4}\,dx = \left[-2x^{-3}\right]_2^a$

$= -2(a)^{-3} - -2(2)^{-3}$

$= -2a^{-3} + \frac{1}{4}$

ii) $\int_2^\infty 6x^{-4}\,dx = \frac{-2}{\infty^3} + \frac{1}{4}$

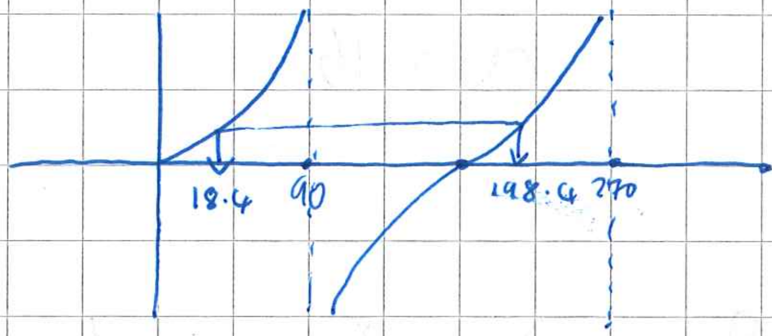
Hence $= 1/4$ as $\frac{-2}{\infty^3} \longrightarrow 0$

⑦ i) $3\tan 2x = 1$

$\tan 2x = 1/3$

$2x = 18.43, 198.4$

$x = 9.22°, 99.2°$



ii) $3\cos^2 x + 2\sin x - 3 = 0$

$3(1-\sin^2 x) + 2\sin x - 3 = 0$

$3 - 3\sin^2 x + 2\sin x - 3 = 0$

$3\sin^2 x - 2\sin x = 0$

$\sin x(3\sin x - 2) = 0$

$\sin x = 0$

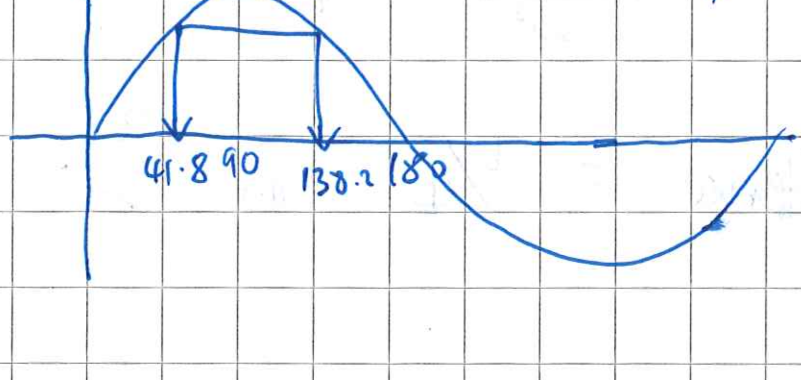
$x = 0°, 180°$

$3\sin x - 2 = 0$

$\sin x = 2/3$

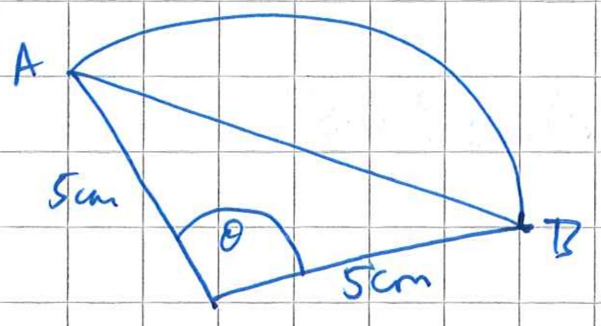
$x = 41.8103..$

$x = 41.8°, 138.2$



8

i)

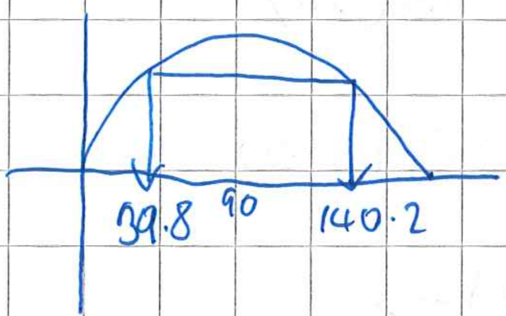


Area = $\frac{1}{2}ab\sin C$

$8 = \frac{1}{2} \times 5^2 \times \sin C$

$C = \sin^{-1}\left(\frac{16}{25}\right)$

$C = 39.79...$



C is obtuse $\therefore$ $C = 140.2^\circ$

$C = 140.2^\circ \equiv 2.45$ radians.

ii) Area of segment $= \frac{1}{2}r^2\theta - 8$

$= \frac{1}{2} \times 5^2 \times 2.45 - 8$

$= 30.625 - 8$

$= 22.63 \text{ cm}^2$

iii) length of Arc $= r\theta$

$= 5 \times 2.45$

~~Perimeter = 5×2.45 + 5 + 5~~

~~= 22.4~~ ~~22.25 cm~~

PTO

Length of Chord AB:

$$AB^2 = 5^2 + 5^2 - 2(5)(5)\cos 2.45^c$$
$$AB = \sqrt{88.5115\ldots}$$
$$= 9.40805\ldots$$

$\therefore$ Perimeter of segment $= 9.408 + 5 \times 2.45$
$= 21.65805839$
$= 21.7\text{cm} \ (3sf)$

9 i) $f(3) = 0$ if crossed x axis @ $(3, 0)$

$$f(3) = -4(3)^3 + 9(3)^2 + 10(3) - 3$$
$$= -108 + 81 + 30 - 3$$
$$= 0$$

Hence $(x-3)$ is a factor.

ii) $-4x^3 + 9x^2 + 10x - 3 = (x-3)(-4x^2 + bx + 1)$

$12x^2$
bx^2
$(12+b)$

$12 + b = 9$
$b = -3$

$$-4x^3 + 9x^2 + 10x - 3 = (x-3)(-4x^2 - 3x + 1)$$

$$(x-3)(-4x^2-3x+1)$$

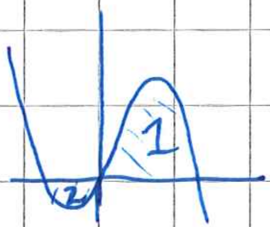
$$x = \frac{3 \pm \sqrt{9-4(-4)(1)}}{-8}$$

$$x = \frac{3 \pm \sqrt{25}}{8}$$

$$x = -1, \ 1/4$$

iv) $\int f(x)\,dx = -x^4 + 3x^3 + 5x^2 - 3x$

Area 1 : $\left[-x^4 + 3x^3 + 5x^2 - 3x\right]_{1/4}^{3}$



$$= 36 - \frac{-101}{256}$$

$$= \frac{9317}{256}$$

Area 2 : $\left[\int f(x)\right]_{-1}^{1/4}$

$$= \frac{-101}{256} - 4$$

$$= -\frac{1125}{256}$$

Total Area $= \frac{9317}{256} + \frac{1125}{256}$

$$= \frac{5221}{128}$$

$$= \underline{\underline{40.79}}$$