Core 2: January 2008

(1) Area of the sector $= 121\pi \times \dfrac{0.7}{2\pi}$ (1)

area of the full circle $= \pi r^2$

fraction of the circle we want.

$= \dfrac{84.7\cancel{\pi}}{2\cancel{\pi}}$ ← π's cancel out

$= 42.35$ (1)

Area of the triangle $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 11 \times 11 \times \sin 0.7$ (1)

$= 38.975\ldots$

$= 38.98$

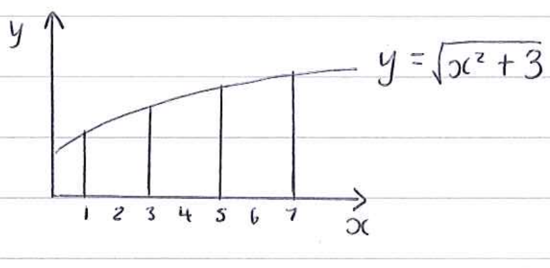
Make sure calculator is in radians mode!

Area of the shaded segment $= 42.35 - 38.98$

$= \underline{\underline{3.37}}$ (1)

$\boxed{4}$

(2) $\int_1^7 \sqrt{x^2+3}\, dx$



height $= \dfrac{7-1}{3} = 2$

(1)
$x = 1 \quad y = 2$
$x = 3 \quad y = \sqrt{12}$
$x = 5 \quad y = \sqrt{28}$
$x = 7 \quad y = \sqrt{52}$

$$\underbrace{\int_1^7 \sqrt{x^2+3}\, dx \approx \frac{1}{2} \times 2 \times \left[(2+\sqrt{52}) + 2(\sqrt{12}+\sqrt{28})\right]}_{(2)}$$

$\approx 26.722\ldots$

$\approx \underline{\underline{26.7}}$ (1)

$\boxed{4}$

(3)

(i) $\log_a 2 + \log_a 3 = \log_a (2 \times 3) = \underline{\underline{\log_a 6}}$ (1)

$\boxed{1}$

(ii) $2\log_{10} x - 3\log_{10} y$

$\log_{10} x^2 - \log_{10} y^3$ (1)

$\underline{\underline{\log_{10}\left(\frac{x^2}{y^3}\right)}}$ (2)

$\boxed{3}$

(4)

(i) Using sine rule.

$\frac{c}{\sin C} = \frac{d}{\sin D}$

$\frac{x}{\sin 62^\circ} = \frac{16}{\sin 50^\circ}$ (1)

$x = \frac{16}{\sin 50^\circ} \times \sin 62^\circ \leftarrow$ make sure your calculator is in degrees mode!

$x = 18.441\ldots$

$\underline{\underline{x = 18.4\text{ cm}}}$ (1)

$\boxed{2}$

(ii) Using cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

$18.4^2 = 20^2 + 10^2 - 2 \times 20 \times 10 \times \cos A$ (1)

$\frac{18.4^2 - 20^2 - 10^2}{-2 \times 20 \times 10} = \cos A$

$\cos A = 0.4036$ (1)

$A = \cos^{-1}(0.4036) \leftarrow$ make sure your calculator is in degrees mode.

$A = 66.196\ldots$

$\underline{\underline{A = 66.2^\circ}}$ (1)

$\boxed{3}$

(5) $\frac{dy}{dx} = 12\sqrt{x} = 12x^{1/2}$ (1)

(1) $y = \frac{12}{3/2}x^{3/2} + C$ ← Integrate to find y= .

(1) $y = 8x^{3/2} + C$

(1) $50 = 8(4)^{3/2} + C$ ← through point (4,50)

$50 = 8 \times 8 + C$

$50 = 64 + C$

$50 - 64 = C$

(1) $-14 = C$

(1) $\underline{\underline{y = 8x^{3/2} - 14}}$ $\boxed{6}$

(6) $u_n = 2n + 5$

(i) $u_1 = 2 \times 1 + 5 = 7$ (1)

$\left.\begin{array}{l} u_2 = 2 \times 2 + 5 = 9 \\ u_3 = 2 \times 3 + 5 = 11 \end{array}\right\}$ (1) $\boxed{2}$

(ii) $\underbrace{\text{Arithmetic}}_{1}$ Progression $\boxed{1}$

(iii) $\sum_{n=1}^{N} u_n = 2200$

$\boxed{\begin{array}{l} l = a + (n-1)d \\ S = \dfrac{(a+l)n}{2} \end{array}}$

$a = 7$

$d = 2$

$n = ?$

$l = ?$

$S = 2200$

$l = 7 + (n-1)2$

$l = 7 + 2n - 2$

$\underline{l = 5 + 2n}$

now sub this into here.

(iii) (continued)

$$S = \frac{(a + (5+2n))n}{2}$$

2 $\quad 2200 = \frac{(7 + 5 + 2n)n}{2} \qquad (\times 2)$

$$4400 = (12 + 2n)n$$

$$4400 = 12n + 2n^2$$

$$0 = 2n^2 + 12n - 4400 \qquad (\div 2)$$

1 $\quad 0 = n^2 + 6n - 2200$

1 $\quad 0 = (n-44)(n+50)$

$$n = 44 \text{ OR } n = -50$$

1 Hence $n = 44$ $\boxed{5}$

7

(i) Some of the area is below the x-axis which is 'negative area'. 1 $\boxed{1}$

(ii) $\displaystyle\int_3^5 (x^2 - 3x)\,dx = \overbrace{\left[\tfrac{1}{3}x^3 - \tfrac{3}{2}x^2\right]}^{2}{}_3^5$

$$= \left(\frac{125}{3} - \frac{75}{2}\right) - \left(9 - \frac{27}{2}\right) \quad 1$$

$$= 8\tfrac{2}{3} \quad 1$$

$$\int_0^3 (x^2 - 3x)\,dx = \left[\tfrac{1}{3}x^3 - \tfrac{3}{2}x^2\right]_0^3$$

$$= \left(9 - \frac{27}{2}\right) - (0 - 0) \quad 1$$

$$= -4\tfrac{1}{2} \quad 1$$

Hence the total area $= 8\tfrac{2}{3} + 4\tfrac{1}{2}$

$$= 13\tfrac{1}{6} \quad 1$$

$\boxed{7}$

(8)

(i) $\boxed{u_n = a \times r^{n-1}}$ ← green booklet

$a = 10$

$r = 0.8$

$u_4 = 10 \times 0.8^{4-1}$ (1)

$= 10 \times 0.8^3$

$= 5.12$ (1) $\boxed{2}$

(ii) $\boxed{S_n = \frac{a(1-r^n)}{(1-r)}}$ ← green booklet

$a = 10$

$r = 0.8$

$n = 20$

$S = \frac{10(1-0.8^{20})}{(1-0.8)}$ (1)

$S = 49.423\ldots$

$S = 49.4$ (1)

(iii) $\boxed{S_\infty = \frac{a}{1-r}}$ for $|r| < 1$ ← green booklet

$$S_\infty - S_N = \frac{a}{1-r} - \frac{a(1-r^N)}{1-r} < 0.01$$

(2) $$= \frac{10}{1-0.8} - \frac{10(1-0.8^N)}{1-0.8} < 0.01$$

$$= \frac{10}{0.2} - \frac{10(1-0.8^N)}{0.2} < 0.01$$

(1) $$= 50 - 50(1-0.8^N) < 0.01$$

$$= 50 - 50 + 50 \times 0.8^N < 0.01$$

$$= 50(0.8)^N < 0.01$$

$50(0.8^N) < 0.01 \qquad (\div 50)$

1 $\quad 0.8^N < 0.0002$

Now solve inequality using logs.

1 $\quad \log 0.8^N < \log 0.0002$

1 $\quad N \log 0.8 < \log 0.0002$

$$N < \frac{\log 0.0002}{\log 0.8}$$

$$N < 38.169\ldots$$

1 Hence $\underline{N = 39}$ is the smallest possible value of N.

$\boxed{7}$

9

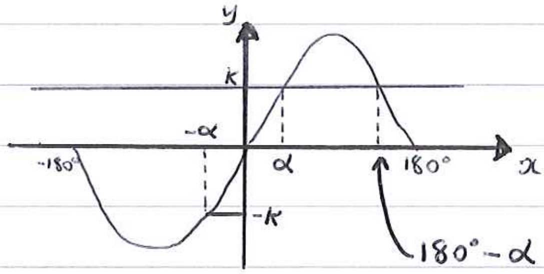
(i) Minimum $= (-90°, 2)$ 1

Maximum $= (90°, 2)$ 1 $\quad \boxed{2}$

(ii)

(a) $180° - \alpha$ 1

(b) $-\alpha$ 1



$\boxed{2}$

(iii) $y = 2\sin x$ intersects $y = 2 - 3\cos^2 x$

Put equal to each other

$2\sin x = 2 - 3\cos^2 x$

remember $\sin^2 x + \cos^2 x = 1$

1 $\quad 2\sin x = 2 - 3(1 - \sin^2 x)$

$2\sin x = -1 + 3\sin^2 x$

Make sure calculator in degrees mode.

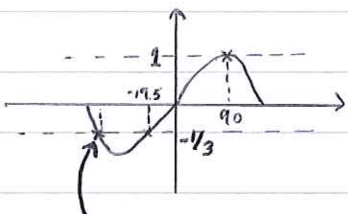
1 $\quad 0 = 3\sin^2 x - 2\sin x - 1$

1 $\quad 0 = (3\sin x + 1)(\sin x - 1)$

1 $\quad \sin x = -\frac{1}{3}$ $\quad$ 1 $\quad \sin x = 1$

$x = \sin^{-1}(-\frac{1}{3})$ $\quad x = \sin^{-1}(1)$

$\underline{x = -19.5°}$ $\quad \underline{x = 90°}$



$-180 + 19.5 = -160.5$

$\underline{x = -160.5°}$ 1 $\quad \boxed{6}$

(10)

(i) $(2x+5)^4$

Row 4 of pascal = 1, 4, 6, 4, 1. (1)

$= 1(2x)^4 5^0 + 4(2x)^3 5^1 + 6(2x)^2 5^2 + 4(2x)^1 5^3 + 1(2x)^0 5^4$ (2)

$= \underline{16x + 160x^3 + 600x^2 + 1000x + 625}$ (1)

$\boxed{4}$

(ii) Expansion for $(2x-5)^4$ will be the same apart from all odd powers of 5 which will become negative terms (1)

$$(-)\ (2x+5)^4 = \cancel{16x} + 160x^3 + \cancel{600x^2} + 1000x + \cancel{625}$$
$$(2x-5)^4 = \cancel{16x} - 160x^3 + \cancel{600x^2} - 1000x + \cancel{625}$$

subtract

$$(2x+5)^4 - (2x-5)^4 = 160x^3 - -160x^2 + 1000x - -1000x$$
$$= \underline{320x^3 + 2000x}$$ (1)

$\boxed{2}$

(iii) $(2x+5)^4 - (2x-5)^4 = 3680x - 800$

at $x = 2$

$$9^4 - (-1)^4 = 7360 - 800$$
$$6560 = 6560 \checkmark$$ (1)

therefore $(x-2)$ is a factor.

(iii) continued

$$(2x+5)^4-(2x-5)^4=3680x-800$$

$$320x^3+2000x=3680x-800$$

$(\div 80)$ $$320x^3-1680x+800=0 \quad 1$$

$$4x^3-21x+10=0 \quad 1$$

$$(x-2)(Ax^2+Bx+C)=0$$

$\times$	Ax^2	$+Bx$	$+C$
x	Ax^3	$+Bx^2$	$+Cx$
-2	$-2Ax^2$	$-2Bx$	$-2C$

By equating coefficients.

$Ax^3=4x^3$ therefore $A=4$

$-2Ax^2+Bx^2=0x^2$

$-8x^2+Bx^2=0x^2$ therefore $B=8$

$-2C=10$ therefore $C=-5$

$$(x-2)(4x^2+8x-5) \quad 1$$

$$(x-2)(2x-1)(2x+5) \quad 1$$

$$\underbrace{x=2,\ x=1/2,\ x=-5/2}_{1}$$

$\boxed{6}$

Total marks $=\boxed{72}$