

Mathematics – Year 7

KNOWLEDGE ORGANISER

To support your revision for the End of Year Assessment

To effectively revise for a Maths assessment, you must finish, check, and correct many Maths questions.

- This document is to support your revision but remember the key with Maths revision is to **finish lots of questions**. Techniques like rewriting revision notes or copying from a revision guide, colour coding, and making posters can be enjoyable, but generally, they aren't the most effective use of revision time.
- Use your progress books and finish outstanding chapters or redo questions you struggled with.
- [Www.corbettmaths.com](http://www.corbettmaths.com) has lots of helpful videos and worksheets you can use as well.
- Use notes, and work through examples and questions in your book.
- Don't use your calculator unless the question specifically asks for it, you need to practise non-calculator skills as well. But checking answers with a calculator is very useful.
- If you struggle with anything, come to the support session during Tuesday lunchtime, in M51 or ask your teacher.
- The full-colour version can be found on www.smlmaths.com.



Year 7 Mathematics Knowledge Organiser – Unit 1: Representing data



- MEAN** means the fair share (**total of values ÷ number of values**).
- MEDIAN** is the **middle value** when the values are **put in order**.
- MODE** is the **most common value**.
- RANGE** is the **difference between the biggest and smallest values**.
- Frequency** is the number of times an event happens.
- Frequency table** is a table for a set of observations showing how frequently each event occurs.
- Grouped data** is data grouped into non-overlapping classes or intervals.
- Class** is an interval for grouping data.

AVERAGES

AVERAGES AND RANGE FROM LIST OF DATA

Example: Find all the averages and range from this list of data: 5, 6, 8, 9, 5, 8, 8, 7

Mean = total of values ÷ number of values

$$= (5 + 6 + 8 + 9 + 5 + 8 + 8 + 7) \div 8 = 56 \div 8 = 7$$

Mode (the most common value) = 8

Range = biggest – smallest value = 9 – 5 = 4

Median:

1. arrange data from the smallest to the biggest:

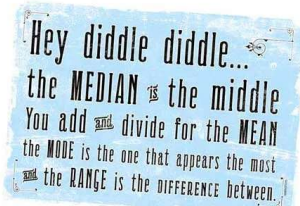
5, 5, 6, 7, 8, 8, 8, 9

2. find the middle value: 5, 5, 6, **7, 8**, 8, 8, 9

- the middle value is between 7 and 8

- find mean of 7 and 8 = $(7 + 8) \div 2 = 7.5$

- median = 7.5



Task1: Find mean, median, mode and range of these numbers: 9, 4, 9, 10, 13, 7, 11, 9

Task2: Choose 3 numbers with:



- a) mean 6
- b) mean 3 and range 4
- c) mean 5 and median 3

AVERAGES FROM FREQUENCY TABLE

Example: A team plays 20 games, the coach records the number of goals they score in each game in a frequency table. Find averages and range.

Mode: the most common number of goals is 1 (6 times in the table)

Mode = 1

Range: the highest value is 4 goals and the lowest 0 goals.

Range = 4 – 0 = 4

Mean:

1. Create the third column and multiply (**value × frequency**) to find the total number of values (goals)

2. Find total of frequencies and total of the 3rd column.

3. Divide $\frac{\text{total number of values (goals)}}{\text{total frequencies}} = \frac{31}{20} = 1.55$

Median:

1. Work out the **position of the median** = $\frac{\text{total frequency} + 1}{2} = \frac{20 + 1}{2} = 10.5^{\text{th}}$ position

2. There are 5 '0 goals' + 6 '1 goals', which makes 11 values. The median is 10.5th value, **median = 1**. (imagine values in a list: 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 2...)

Number of Goals	Frequency	Total number of goals
0	5	0 × 5 = 0
1	6	1 × 6 = 6
2	4	2 × 4 = 8
3	3	3 × 3 = 9
4	2	4 × 2 = 8
Total	20	31

10.5th value

AVERAGES FROM THE GROUPED DATA

Example: Find the estimate of mean, median and modal classes from the table below:

POCKET MONEY (£)	FREQUENCY (F)	MIDPOINT (X)	F × X = FX
0 < P ≤ 1	2	0.5	2 × 0.5 = 1
1 < P ≤ 2	5	1.5	5 × 1.5 = 7.5
2 < P ≤ 3	5	2.5	5 × 2.5 = 12.5
3 < P ≤ 4	9	3.5	9 × 3.5 = 31.5
4 < P ≤ 5	15	4.5	15 × 4.5 = 67.5
TOTAL	36	TOTAL	120

Modal class: 4 < P ≤ 5 (the most common class, 15 times)

Range: £5 – £0 = £5

Mean:

1. Create 3rd column (**midpoint** of the classes)
2. Create 4th column (**midpoint × frequency**)
3. Find total of frequencies and total of the 4th column.
4. Divide $\frac{\text{total number of values (£)}}{\text{total frequencies}} = \frac{120}{36} = \text{£}3.33$

Median class:

1. Position of the median = $\frac{\text{total frequency} + 1}{2} = \frac{36 + 1}{2} = 18.5^{\text{th}}$ position

POCKET MONEY (£)	FREQUENCY (F)	
0 < P ≤ 1	2	2
1 < P ≤ 2	5	2+5=7
2 < P ≤ 3	5	2+5+5=12
3 < P ≤ 4	9	2+5+5+9=21
4 < P ≤ 5	15	2+5+5+9+15=36

2. Median is 18.5th value, that is in **median class** 3 < P ≤ 4

TWO-WAY TABLES

Data collected in two-way tables is divided into more than one category.

Example: Some college students were asked to choose which of the three subjects, English, Maths or Science they enjoyed the most. Complete the two-table below.

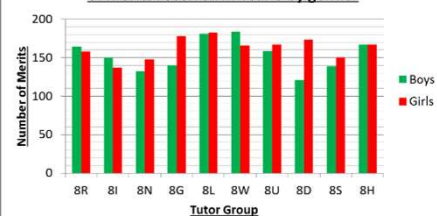
	E	M	S	Total
Girls	20	13	17	50
Boys	18	15	23	56
Total	38	28	40	106

- the black numbers in the table are filled in from the text
- the red numbers are added by calculation

BAR CHARTS

Bar charts represent statistical information. Bars, of **equal width**, represent frequencies by their height.

Comparative Bar Chart to compare the number of merits received in Year 8 by gender



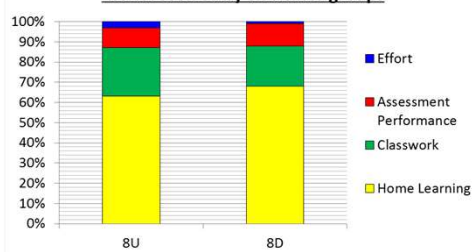
Comparative Bar Chart:

- Bars for each category side-by-side
- Gaps between each category

Composite Bar Chart:

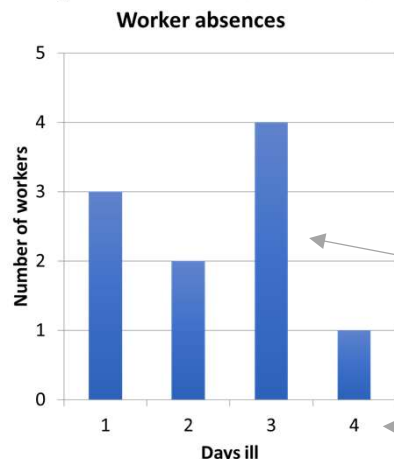
Bars show the size of individual categories split into their separate parts

Composite Bar Chart to compare the type of merit received by two tutor groups



AVERAGES FROM BAR CHART

Example: Find mean, median, mode and range from this bar chart.



- Mode is represented by the highest bar
Mode = 3 days
- Range = 4 - 1 = 3 days

3. To find mean and median use the table:

Days ill	Workers	Total days
1	3	1 × 3 = 3
2	2	2 × 2 = 4
3	4	3 × 4 = 12
4	1	4 × 1 = 4
Totals	10	23

$$\text{Mean} = \frac{\text{total number of values (day)}}{\text{total frequencies (workers)}} = \frac{23}{10} = 2.3$$

Median:

- the position of the median = $\frac{\text{total frequency} + 1}{2} = \frac{10 + 1}{2} = 5.5^{\text{th}}$ position

imagine values in a list: 1, 1, 1, 2, 2, 3, 3, 3, 3, 4
5.5th value lies between 2 and 3

Days ill	Workers	Running Total
1	3	3
2	2	3+2=5
3	4	3+2+4=9
4	1	3+2+4+1=10

- or, 5.5th position is located between 2nd and 3rd row of the table
- median = $(2 + 3) \div 2 = 2.5$

Task3: Find mean, median, mode and range from these frequency tables:

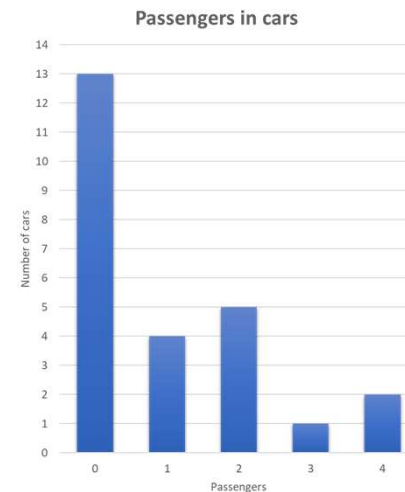
a)

Shoe size	Frequency
3	4
4	8
5	3
6	10

b)

Number of currants	Frequency
0 - 2	2
3 - 5	9
6 - 8	10
9 - 11	3

Task4: Find mean, median, mode and range from this bar chart:



Answers:

1. Mean = 9
Median = 9
Mode = 9
Range = 9

2. a) 5, 6, 7
Median = 6 - 8
Mode = 6 - 8
Range = 11

3a. Mean = 4.76
Median = 5
Mode = 6
Range = 3

3b. Mean = 5.75
Median = 6 - 8
Mode = 6 - 8
Range = 11

4. Mean = 1
Median = 0
Mode = 0
Range = 4

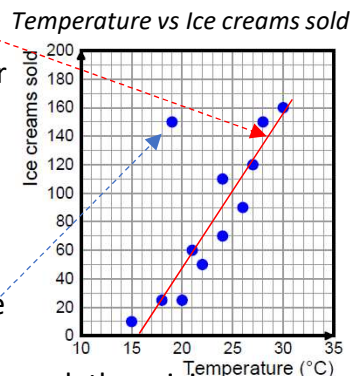
SCATTER GRAPHS

Scatter graph is a graph in which the values of two variables are plotted along two axes.

LINE OF THE BEST FIT

is a line drawn on a scatter graph to represent the best estimate of a linear relationship between the variables.

- It should not go too much further beyond the outer points.
- It does not have to run through the origin.
- Approximately the same amount of points should be above and below the line.



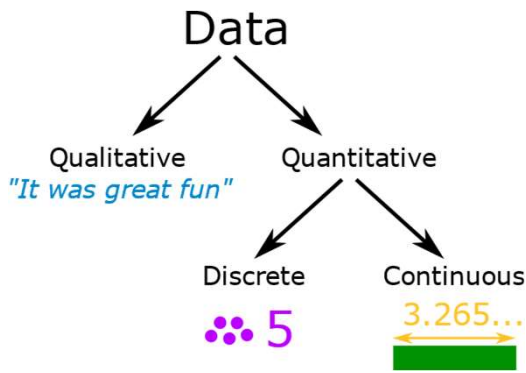
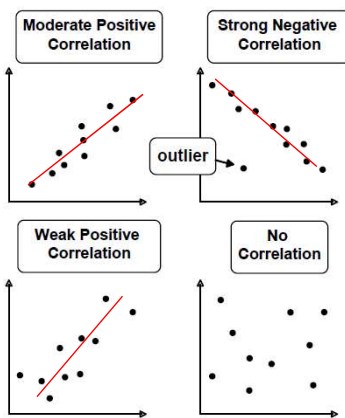
OUTLIER is an extreme value which is much higher or much lower than the rest of the values.

CORRELATION is a measure of the strength of the relationship between two variables. **Strong** correlation implies close relationship (the points make a relatively straight line).

Weak correlation: the points are further apart from each other.

Positive correlation: as one variable increases, so does the other. The line of the best fit has positive gradient (going uphill).

Negative correlation: as one quantity increases the other decreases. The line of the best fit has negative gradient (going downhill).



Discrete data is counted, it can only take certain values.

Example: the number of students in a class

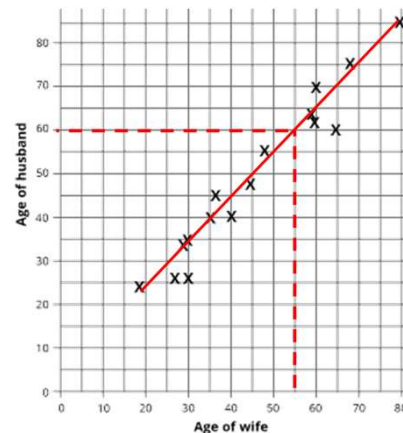
Continuous data is measured, it can take any value (within a range).

Example: a person's height.

Example: Based on the scatter graph below, predict the age of the husband of a 55 year old woman.

1. Draw a line of the best fit.
2. Draw a line from number 55 on the x-axis (age of wife axis) towards the line of the best fit.
3. Read the corresponding age of the husband from y-axis.

Answer: 60



PIE CHARTS

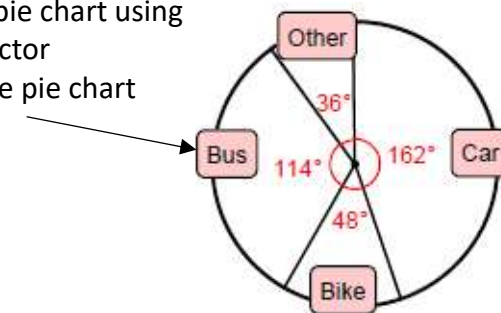
A type of graph in which a circle is divided into sectors that each represent a proportion of the whole.

The sum of the angles in pie chart is **360** degrees.

Transport	Frequency	Angle
Car	27	162°
Bike	8	48°
Bus	19	114°
Other	6	36°
Total	60	360°

Example: Create a pie chart from the table above.

- You will always need to calculate the total for the frequency: $27 + 8 + 19 + 6 = 60$
- Calculate the angle that represents one item in the list by sharing 360 into total of the frequency: $360 \div 60 = 6$ (means one car or bike or bus or other is represented by 6°)
- To calculate the angle of each category, multiply the frequency of the category by angle that represents one item only: for example angle for all cars = $27 \text{ cars} \times 6^\circ = 162^\circ$
- Always check if your angles are correct by seeing if they add up to 360: $162 + 48 + 114 + 36 = 360^\circ$
- Draw a pie chart using a protractor
- Label the pie chart





Integer

is any of the positive or negative whole numbers and zero.

Example: ...-2, -1, 0, +1, +2 ...

Even number

is an integer that is divisible by 2.

Example: 2, 4, 6, 8, 10, 12, 14, ...

Odd number

is an integer that has a remainder of 1 when divided by 2.

Example: 1, 3, 5, 7, 9, 11, 13, 15, ...

Product

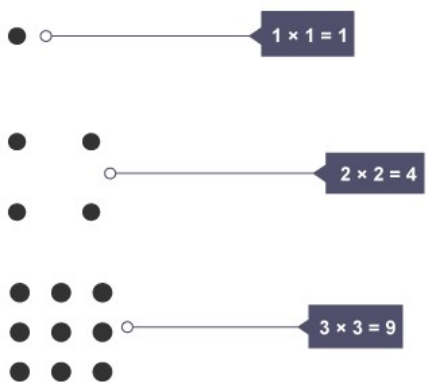
is the result of multiplying one number by another.

Index notation

is used as a shortcut for multiplication when a number is being multiplied by itself. Example: $5^4 = 5 \times 5 \times 5 \times 5$

SQUARE NUMBERS

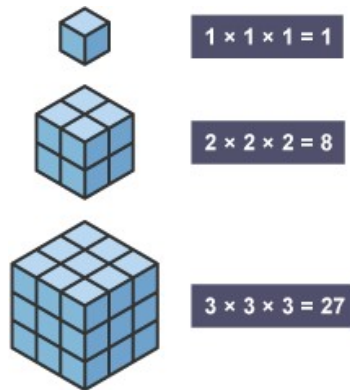
A **square number** is the result of multiplying integer by itself. It is called a square number because it gives the area of a square whose side length is an integer.



Notation of the square number: x^2
Example: $3^2 = 3 \times 3 = 9$

CUBE NUMBERS

A **cube number** is the result of multiplying integer by itself, and by itself again. It is called a cube number because it gives the volume of a cube whose side length is an integer.



Notation of the cube number: x^3
Example: $5^3 = 5 \times 5 \times 5 = 125$

ROOTS

A **square root** of a number is a value that, when multiplied by itself, gives the number. Example: $4 \times 4 = 16$, so a **square root** of 16 is 4. Notation for the square root is $\sqrt{\quad}$

The **cube root** of a number is a special value that, when used in a multiplication three times, gives that number. Example: $3 \times 3 \times 3 = 27$, so the **cube root** of 27 is 3. Notation for the square root is $\sqrt{\quad}$

Example: $\sqrt{81} = 9$ Extension: $-\sqrt{81} = -9$
 $\sqrt[3]{27} = 3$ $\sqrt[3]{-27} = -3$

Note: It is impossible to find square root of a negative number, for example $\sqrt{-81}$.

x	1	2	3	4	5	6	7	8	9	10	11	12
x^2	1	4	9	16	25	36	49	64	81	100	121	144
x^3	1	8	27	64	125	216	343	512	729	1000	1331	1728

* You should remember the first twelve square numbers and the first five cube numbers.

Examples:
 $4^2 = 4 \times 4 = 16$
 $(-4)^2 = (-4) \times (-4) = 16$
 $-4^2 = -(4 \times 4) = -16$

Examples:
 $4^3 = 4 \times 4 \times 4 = 64$
 $(-4)^3 = (-4) \times (-4) \times (-4) = -64$
 $-4^3 = -(4 \times 4 \times 4) = -64$

Task:
Evaluate
a) 7^2
b) 5^3
c) $\sqrt{36}$
d) $\sqrt[3]{8}$

Extension task:
Evaluate
e) $(-7)^2$
f) $(-5)^3$
g) $-\sqrt{36}$
h) $\sqrt[3]{-8}$

Answers:
a) 49
b) 125
c) 6
d) 2
e) 49
f) -125
g) 6
h) -2

PRIME NUMBER

Prime numbers (not shaded numbers on the hundred square below) are whole numbers greater than 1 that have **exactly** two factors, themselves and 1.

Composite numbers (shaded numbers on the 100 square below) are integers that are divisible without remainder by at least one positive integer other than themselves and one.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Example:

15 is divisible by 1, 15, 3, 5, therefore it is **not** a prime number.

2 is divisible by 1 and 2 only, therefore it is a **prime number** (the only even prime number)

Important note: Number **1** is not prime nor composite number.

Prime numbers that you should remember are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

FACTORS

Factors are numbers that divide another number without leaving a remainder.

Example:

Find factors of number 32

$$1 \times 32$$

$$2 \times 16$$

$$4 \times 8$$

Factors of number 32 are: 1, 2, 4, 8, 16 and 32

Task: Find factors of number 12

Answer: 1, 2, 3, 4, 6, 12

USING CALCULATOR

To work out 34^2 :

- Enter **34**
- Press the x^2 button
- Press **=**

You should get the answer 1156

To work out 26^3 :

- Enter **26**
- Press the x^3 button
- Press **=**

You should get the answer 17576



To work out $\sqrt{225}$:

- Press the $\sqrt{\square}$ button
- Enter **225**
- Press **=**

You should get the answer 15

To work out $\sqrt[3]{2197}$:

- Press **SHIFT**
- Press the $\sqrt[\square]{\square}$ button
- Enter **2197**
- Press **=**

You should get the answer 13

LCM or The Lowest / Least Common Multiple

LCM is smallest positive number that is a multiple of two or more numbers.

Example:

Find the lowest common multiple of 6 and 9.

Multiples of 6 are: 6, 12, 18, 24, 30, 36...

Multiples of 9 are: 9, 18, 27, 36, 45, 54, ...

Common multiples of 6 and 9 are: 18, 36 ...

The LCM is 18

HCF or the Highest Common Factor

HCF is the greatest number that is a factor of two (or more) other numbers.

Example:

Find the highest common factor of 18 and 24.

Factors of 18 are: 1, 2, 3, 6, 9, 18

Factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24

Common factors of 18 and 24 are: 1, 2, 3 and 6

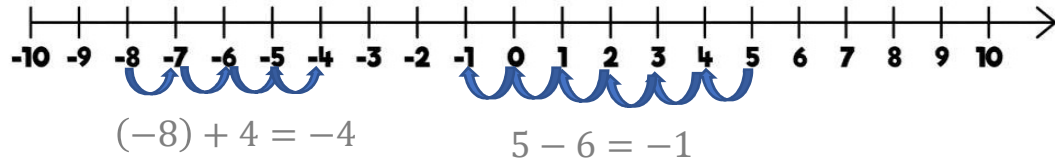
The HCF is 6

Task: Find the HCF and LCM of 12 and 15

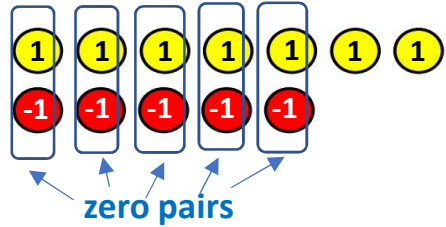
Answers:
 Multiples of 12 are: 12, 24, 36, 48, 60, 72...
 Multiples of 15 are: 15, 30, 45, 60, 75...
 The LCM is 60
 Factors of 12 are: 1, 2, 3, 4, 6, 12
 Factors of 15 are: 1, 3, 5, 15
 The HCF is 3

DIRECTED NUMBER

One of the methods which can help you to solve calculation with negative numbers, is a number line



Second method is using counters.



This represents calculation $7 + (-5)$

There are five **zero pairs**.

Removing the zero pairs leaves us with 2.

Therefore: $7 + (-5) = 2$

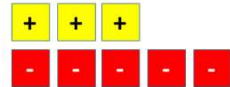
This leads us to realise that $7 + (-5)$ gives the same answer as $7 - 5$.

So $7 - 5 = 7 + (-5)$

Consequently also $7 - (-5) = 7 + (+5) = 7 + 5$

The subtraction symbol '-' means addition of the 'additive inverse'

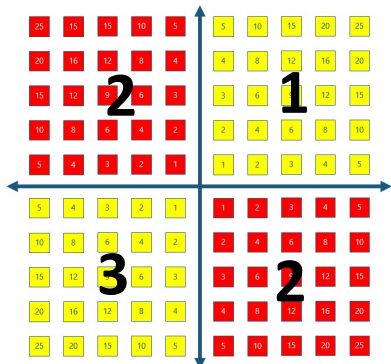
$$3 - 5 = 3 + (-5) = -2$$



$$2 - (-5) = 2 + 5 = 7$$



MULTIPLICATION AND DIVISION OF DIRECTED NUMBERS



1. \times or \div of positive numbers gives POSITIVE answer

Example: $2 \times 3 = 6$

2. \times or \div of positive AND negative numbers gives NEGATIVE answer

Example: $3 \times (-2) = -6$ or $(-6) \div 3 = -2$

3. \times or \div of negative numbers gives POSITIVE answer

Example: $(-6) \div (-2) = 3$

DIVISION

To divide two numbers, we use the bus stop method.

Short division

$$\begin{array}{r} 045 \\ 8 \overline{) 3640} \end{array}$$

Long division

$$432 \div 15 \text{ becomes } = 28.8$$

$$\begin{array}{r} 28.8 \\ 15 \overline{) 432.0} \\ \underline{30} \\ 132 \\ \underline{120} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

MULTIPLICATION

To multiply two numbers together, the grid method is useful to ensure that the calculation is completed correctly.

The second method to use is column method.

\times	100	80	7
9	900	720	63

$$900 + 720 + 63 = 1683$$

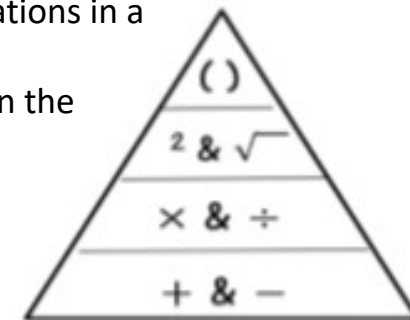
$$\begin{array}{r} 25 \\ \times 14 \\ \hline 100 \\ + 250 \\ \hline 350 \end{array}$$

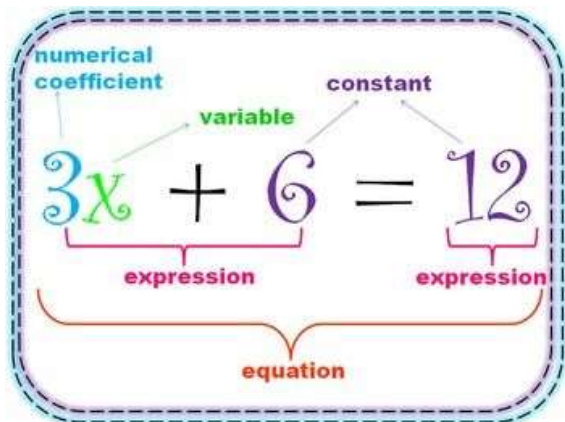
BIDMAS

The **order** in which we complete operations in a sum is important.

If operations of the same priority are in the same sum, we work from left to right

Example: $10 - 3 + 5 = 12$,
first $10 - 3 = 7$,
then $7 + 5 = 12$.





A **variable** is a letter or symbol that represents an unknown value.

When variables are used with other numbers, parentheses, or operations, they create an **algebraic expression**.

Equation is algebraic expression with equal sign, which can be solved (value of variable is found).

A **coefficient** is the number multiplied by the variable in an algebraic expression.

A **term** is the name given to a number, a variable, or a number and a variable combined by multiplication or division, including + or – symbol in front of it.

A **constant** is a number that cannot change its value.

Identity is an equation that is true no matter what values are chosen. (symbol \equiv)

A **formula** is an equation linking sets of physical variables.

SIMPLIFYING EXPRESSIONS

Multiplication of a number and variable is written without multiplication symbol, numbers first, letters in alphabetical order:

Example: $3 \times x = 3x$

$y \times 6 \times x = 6xy$

Division is written as a fraction:

Example: $6 \div x = \frac{6}{x}$

Multiplying and dividing variables

Example: $x \times x \times x = x^3$

$2 \times x \times y \times 3 =$

$2 \times 3 \times x \times y = 6xy$

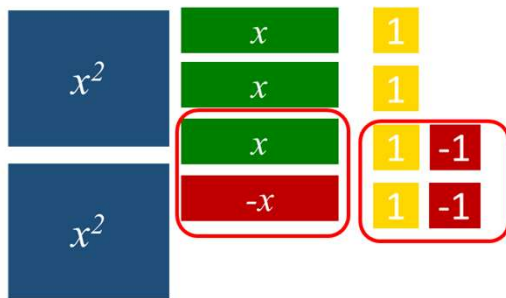
$6x \div 2 = 3x$

COLLECTING LIKE TERMS

‘Like terms’ are terms whose variables (and their powers) are the same, the **coefficients** can be different.

$3x + 2x^2 - x + 4 - 2 =$

$2x^2 + 2x + 2$



Example:

$x + x + 2x = 4x$

$x + 4 + 3x - 5 = 4x - 1$

$x + 3y + 2x - 2y + 3 = 3x + y + 3$

$9x^2 - 2x - 5x^2 - 5x = 4x^2 - 7x$

Task1: Simplify $2x^2 + xy + 3x^2 + xy =$

WRITING EXPRESSIONS AND EQUATIONS

The word phrases can be translated into algebraic expressions or equations with variables.

Examples:

- Eight less than the **quotient** of a number and two
 $\frac{x}{2} - 8$

- Nine times the **sum** of a number and fifteen
 $9(n + 15)$

- The sum of **twice** a number and seven
 $2x + 7$

- One plus the **product** of a number and five
 $1 + 5x$

- A number **less than** twenty-five
 $25 - x$

- I think of a number (x). When I multiply the number by two ($2x$) and add 3 ($2x + 3$), the answer is 11
 $2x + 3 = 11$

- I think of a number (x). When I add 3 ($x+3$) and multiply the result by 2 ($2(x + 3)$), the answers is 11
 $2(x+3) = 11$

Task2: Write an equation: I think of a number. When I add 7 and divide the result by 2, the answers is 5.



Expanding brackets means to remove the brackets.

Factorising means putting brackets back into expressions.

Factors of a number are the numbers that divide the original number without remainder.

Writing a number as a product of factors is called a **factorisation** of the number.

The **Highest Common Factor (HCF)** is the largest common factor (the factor that two or more numbers have in common).

WRITING FORMULAE

Example:

- Guy, Eric and Luke go Christmas shopping. Write an expression for how much money each man has left after shopping.

(a) Guy had £20 and spends £y on presents.

$$20 - y$$

(b) Eric had £m and spends £12 on presents.

$$m - 12$$

(c) Luke had £a and spends half £b on presents

$$a - \frac{1}{2}b$$

- Adult tickets to the cinema cost £7. Child tickets cost £4. Write an expression for the total cost of taking a adults and c children to the cinema. $7a + 4c$
- A phone company charges a monthly fee of £10.25 and £0.12 per minute. Write a formula for the monthly bill, b for m minutes. $b = 10.25 + 0.12m$

Task3: A delivery company charges according to the weight of each parcel.

They charge £2 per kilogram, plus 50p handling fee. Write an expression for the cost C of sending a parcel that weighs k kilograms.

EXPANDING SINGLE BRACKETS

- Multiply everything in the brackets by a number or variable in front of the bracket

Examples: Expand

$$4(a + 6) = 4a + 24$$

	a	+6
4	4a	+24

$$-2(b - 4) = -2b + 8$$

	b	-4
-2	-2b	+8

$$c(2c - 5) = 2c^2 - 5c$$

$$2d(3d - e) = 6d^2 - 2de$$

FACTORISING

- Find the HCF of the terms in the brackets (the highest numerical factor and the highest power of the variable).
- Put the HCF in front of the brackets. Terms divided by HCF stay in the brackets.
- Check your answers by expanding brackets.

Examples: Factorise

$$4x + 12 = 4(x + 3)$$

$$7x^2 + 3x = x(7x + 3)$$

$$8x^2 + 16x = 8x(x + 2)$$

SUBSTITUTION

If we are told what number a variable represents, we can **substitute** this into expressions to find their value.

Examples:

Find the value of expressions when $x = 5$, $y = 4$

$$7x = 7 \times 5 = 35$$

$$3(x + 1) = 3 \times (5 + 1) = 3 \times 6 = 18$$

$$\frac{2(x-1)}{4} = \frac{2(5-1)}{4} = \frac{2 \times 4}{4} = \frac{8}{4} = 2$$

$$x^2 = 5^2 = 25$$

$$2x^2 = 2 \times 5^2 = 2 \times 25 = 50$$

$$(2x)^2 = (2 \times 5)^2 = 10^2 = 100$$

$$xy = 5 \times 4 = 20$$

Task4: Expand $-2(2x - 3)$

Task5: Expand $3x(x + 1)$

Task6: Factorise $3x - 6$

Task7: Factorise $4x^2 + 6x$

Task8: Find the value of expression $3x - 1$ when $x = 2$

- Answers:
1. $5x^2 + 2xy$
 2. $\frac{x+7}{2} = 5$
 3. $C = 2k + 0.5$
 4. $-4x + 6$
 5. $3x^2 + 3x$
 6. $3(x - 2)$
 7. $2x(2x + 3)$
 8. $5 = 1 - 2 \times 2$



Fractions represent equal parts of a whole. *Example:*

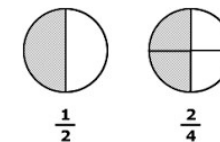
$\frac{3}{4}$ ← the **numerator** says how many parts we have.
 $\frac{3}{4}$ ← the **denominator** says how many equal parts the whole is divided into

Mixed number is a number consisting of an integer and a proper fraction. *Example:* $1\frac{1}{4}$

Improper fraction is a fraction in which the numerator is greater than the denominator. *Example:* $\frac{5}{4}$

Common denominator is a common multiple of the denominators of several fractions.

Equivalent fractions are fractions which have the same value. *Example:* $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.



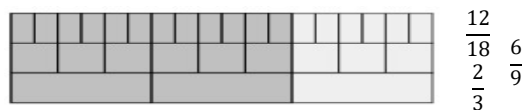
SIMPLIFYING FRACTIONS

Divide numerator and denominator by the same factor:

Example: Simplify/cancel down $\frac{12}{30}$ fully

$$\frac{12}{18} \xrightarrow{\div 2} \frac{6}{9} \xrightarrow{\div 3} \frac{2}{3}$$

$$\frac{12}{30} \xrightarrow{\div 3} \frac{4}{10} \xrightarrow{\div 2} \frac{2}{5}$$



All of these fractions are **equivalent**.

ORDERING FRACTIONS

Convert fractions into equivalent fractions with the lowest common denominator and compare.

Example: Put these fractions in ascending order

$$\frac{2}{3}, \frac{3}{5}, \frac{1}{2}$$

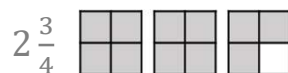
The lowest common multiple of 3, 5, and 2 is 30

$$\frac{2}{3} = \frac{20}{30}, \quad \frac{3}{5} = \frac{18}{30}, \quad \frac{1}{2} = \frac{15}{30}$$

The correct order is: $\frac{1}{2}, \frac{3}{5}, \frac{2}{3}$

CONVERTING MIXED NUMBERS TO IMPROPER FRACTIONS

Example: Convert $2\frac{3}{4}$ into improper fraction



- Imagine mixed number as an addition

$$2\frac{3}{4} = 2 + \frac{3}{4}$$

- Change integer part into fraction with the same denominator

$$2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4}$$

- Add the fractions

$$2\frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{8+3}{4} = \frac{11}{4}$$

Different method:

$$+ \begin{array}{l} \uparrow \\ \times \end{array} 2\frac{3}{4} = \frac{(4 \times 2) + 3}{4} = \frac{8 + 3}{4} = \frac{11}{4}$$

CONVERTING IMPROPER FRACTIONS TO MIXED NUMBERS

Example: Convert $\frac{11}{4}$ into improper fraction

- Divide numerator by denominator

- The answer is the integer part and the remainder is the numerator of fractional part

$$11 \div 4 = 2 \text{ r}3 \rightarrow 2\frac{3}{4}$$

- Denominator stays the same

ADDING AND SUBTRACTING FRACTIONS

Example: Add $\frac{2}{3}$ and $\frac{1}{2}$

1. Convert fractions into fractions with common denominator.

The lowest common denominator is 6

$$\frac{2}{3} = \frac{4}{6} \quad \frac{1}{2} = \frac{3}{6}$$

2. Add or subtract numerators and simplify, if needed. Convert improper fractions into mixed numbers.

$$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6} = 1\frac{1}{6}$$



To add or subtract mixed numbers, convert mixed numbers to improper fractions first, then add or subtract.

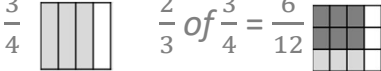

Example: Subtract $1\frac{3}{4}$ and $1\frac{1}{2}$

$$1\frac{3}{4} - 1\frac{1}{2} = \frac{7}{4} - \frac{3}{2} = \frac{7}{4} - \frac{6}{4} = \frac{1}{4}$$

↑ converting into improper fractions
 ↓ common denominator

MULTIPLYING FRACTIONS

Multiply numerators and denominators separately and simplify the answer if possible.

Example: $\frac{3}{4} \times \frac{2}{3}$  $\frac{2}{3}$ of $\frac{3}{4} = \frac{6}{12}$ 

$$\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12} = \frac{1}{2}$$

To multiply fraction by the whole number, convert whole number into the fraction and multiply.

Example: $\frac{2}{3} \times 4 = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3} = 2\frac{2}{3}$

$$\frac{2}{3} \times \frac{4}{1} = \frac{2 \times 4}{3 \times 1} = \frac{8}{3} = 2\frac{2}{3}$$


To multiply mixed numbers, convert mixed numbers to improper fractions and multiply. Convert the answer back to mixed number.

Example: $2\frac{1}{3} \times 3\frac{1}{2} = \frac{7}{3} \times \frac{7}{2} = \frac{7 \times 7}{3 \times 2} = \frac{49}{6} = 8\frac{1}{6}$

FRACTION OF A NUMBER

Divide a number by denominator and multiply by numerator.

Example: Find $\frac{2}{5}$ of £80

$(£80 \div 5) \times 2 = £16 \times 2 = £32$ 

ONE NUMBER AS A FRACTION OF ANOTHER

To write one number (x) as a fraction of another number (y), write number x as a numerator and number y as denominator of the fraction and simplify.

Example: Express 35 as fraction of 80 $\frac{35}{80} = \frac{7}{16}$

RECIPROCAL OR MULTIPLICATIVE INVERSE

The reciprocal of a number n is $1 \div n = \frac{1}{n}$

Example: the reciprocal of 5 is $\frac{1}{5}$

To find the reciprocal of a fraction, flip the fraction.

Example: the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$

Any number multiplied by its reciprocal is always equal to 1.

Example: $5 \times \frac{1}{5} = \frac{5}{5} = 1$ $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$

DIVIDING FRACTIONS

To divide two fraction, multiply the first fraction by the reciprocal of the second one.

Example: $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = \frac{2}{1} = 2$

Dividing fractions and whole numbers. Convert the whole number into a fraction and divide.

Example: $3 \div \frac{1}{4} = \frac{3}{1} \div \frac{1}{4} = \frac{3}{1} \times \frac{4}{1} = 12$

$\frac{2}{5} \div 2 = \frac{2}{5} \div \frac{2}{1} = \frac{2}{5} \times \frac{1}{2} = \frac{2}{10} = \frac{1}{5}$

Dividing mixed numbers: change the mixed numbers to an improper fractions and divide.

Example: $2\frac{2}{5} \div \frac{1}{2} = \frac{12}{5} \div \frac{1}{2} = \frac{12}{5} \times \frac{2}{1} = \frac{24}{5} = 4\frac{4}{5}$

CONVERTING BETWEEN FRACTIONS, TERMINATING DECIMALS AND PERCENTAGES

D → **F**

1. Write the decimal as a fraction 'over one'.
2. Convert the fraction into a fraction with whole number in the numerator by multiplying both numerator and denominator by the multiple of 10.
3. Simplify.

Example: Convert 0.84 into a fraction.

$0.84 = \frac{0.84}{1} = \frac{84}{100} = \frac{21}{25}$

F → **D**

1. Divide the numerator by denominator.
2. Sometimes, you can help yourself by converting the fraction into the fraction with multiple of 10 in the denominator.

Example1: Convert $\frac{2}{9}$ to decimal

$2 \div 9 = 0.22222.....$

Example2: Convert $\frac{2}{25}$ to decimal

$\frac{2}{25} = \frac{8}{100}$ $8 \div 100 = 0.08$

Multiply by 100
D → **P**
 ← **P**
 Divide by 100

Examples: $0.04 = (0.04 \times 100) \% = 4\%$

$1.2 = (1.2 \times 100)\% = 120\%$

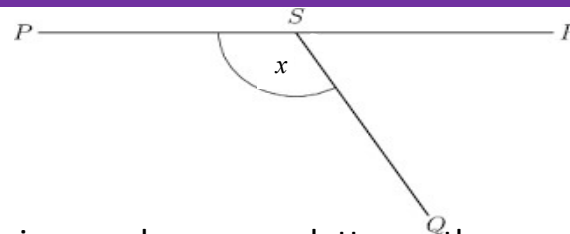
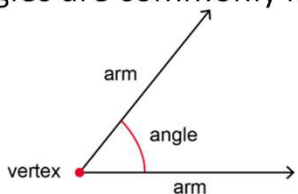
$23\% = \frac{23}{100} = 0.23$

$5\% = \frac{5}{100} = 0.05$

Year 7 Mathematics Knowledge Organiser – Unit 5: Angles



An **ANGLE** is the **amount of turn** between two **straight lines** joined (or **intersected**) at a point called a **VERTEX**. The two lines are called **arms**. Angles are commonly marked by an **arc** (part of a circle) between arms.

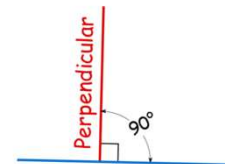


Angles are labelled using one lower case letter or three upper case letters, in which case the letter in the middle always represents the vertex.
 Example: Angle x is labelled as $\angle PSQ$ or $\angle QSP$

PARALLEL LINES are lines that **never cross** each other - they keep the same distance apart from each other. They are marked with arrows.



Lines that cross (intersect) each other at **right angles** (90°) are called **PERPENDICULAR LINES**.

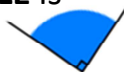


TYPES OF ANGLES

An **ACUTE ANGLE** is less than 90° .



A **RIGHT ANGLE** is exactly 90° .



An **OBTUSE ANGLE** is greater than 90° but less than 180° .



A **REFLEX ANGLE** is greater than 180° but less than 360° .



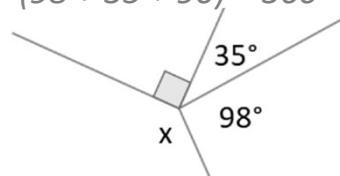
ANGLES AROUND THE POINT

Angles at a point add up to 360° .

Example: Find the missing angle x .

$$x = 360 - (98 + 35 + 90) = 360 - 223$$

$$x = 137^\circ$$



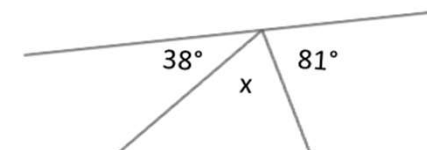
ANGLES ON THE STRAIGHT LINE

Angles on a straight line add up to 180° .

Example: Find the missing angle x .

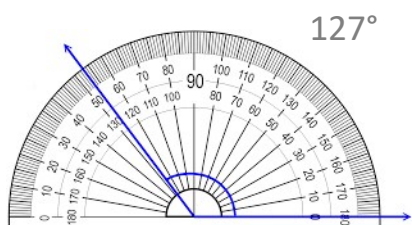
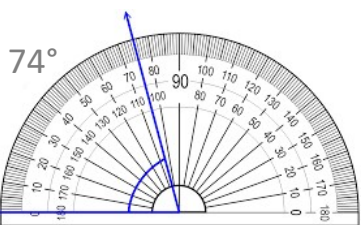
$$x = 180 - (38 + 81) = 180 - 119$$

$$x = 61^\circ$$



MEASURING ANGLES

- Place the midpoint of the protractor on the **VERTEX** of the angle.
- Line up one arm of the angle with the zero line of the protractor (where you see the number 0).
- Read the degrees where the other arm crosses the number scale.



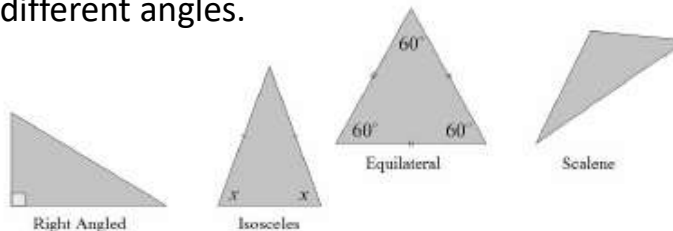
TRIANGLES

Right Angle Triangles have a 90° angle in.

Isosceles Triangles have 2 equal sides and 2 equal base angles.

Equilateral Triangles have 3 equal sides and 3 equal angles (60°).

Scalene Triangles have different sides and different angles.



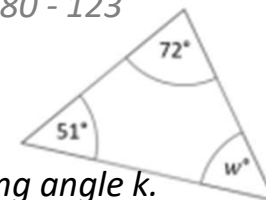
ANGLES IN THE TRIANGLE

Angles in the triangle add up to 180° .

Example: Find the missing angle w .

$$w = 180 - (72 + 51) = 180 - 123$$

$$w = 57^\circ$$



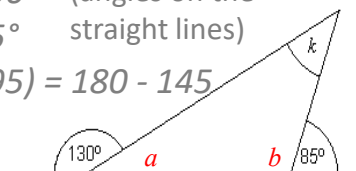
Example: Find the missing angle k .

$$a = 180 - 130 = 50^\circ$$

$$b = 180 - 85 = 95^\circ$$

$$w = 180 - (50 + 95) = 180 - 145$$

$$w = 35^\circ$$



QUADRILATERALS

<p>Square</p> <p>Four sides of equal length, four internal right angles.</p>	<p>Rectangle</p> <p>Four internal right angles, opposite sides of equal length.</p>	<p>Parallelogram</p> <p>Opposite sides are parallel and equal in length, opposite angles are equal.</p>	<p>Rhombus</p> <p>All four sides are the same length, like a square that has been squashed sideways.</p>
<p>Trapezium (or trapezoid)</p> <p>Two sides are parallel. Side lengths and angles are not equal.</p>	<p>Isosceles Trapezium (or trapezoid)</p> <p>Two sides are parallel and base angles are equal, non-parallel sides are equal length.</p>	<p>Kite</p> <p>Two pairs of adjacent sides are of equal length; the shape has an axis of symmetry.</p>	<p>Irregular Quadrilateral</p> <p>No sides are equal in length and no internal angles are the same.</p>

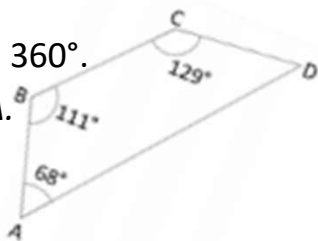
ANGLES IN THE QUADRILATERAL

Angles in the quadrilateral add up to 360°.

Example: Find the missing angle CDA.

$$\angle CDA = 360 - (111 + 129 + 68)$$

$$= 360 - 308 = 52^\circ$$



ANGLES IN PARALLEL LINES

When a transversal (a line that crosses at least two other lines) intersects **parallel lines**, angles with special properties are created.

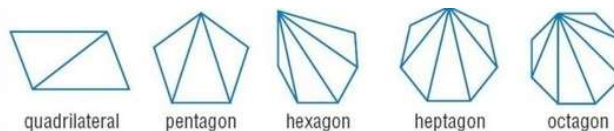
<p>Alternate angles are equal</p>	<p>Corresponding angles are equal</p>	<p>Co-interior angles sum to 180°</p>
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ANGLES IN THE POLYGONS (a polygon is a 2-D shape made of straight lines)

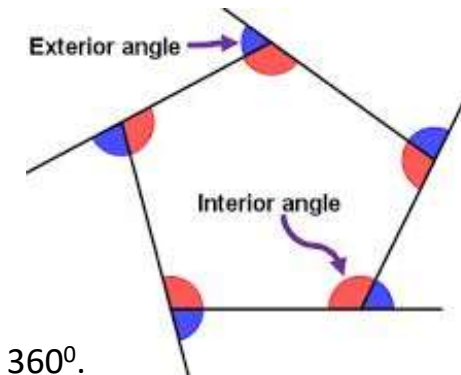
The sum of the interior angles of a polygon of n sides is found by the formula

$$(n - 2) \times 180$$

$(n-2)$ represents min. number of triangles, each n -sided polygon can be divided into



Convex Polygon	# of Sides	# of Triangles from 1 Vertex	Sum of Interior Angle Measures
Triangle	3	1	$1 \times 180 = 180$
Quadrilateral	4	2	$2 \times 180 = 360$
Pentagon	5	3	$3 \times 180 = 540$
Hexagon	6	4	$4 \times 180 = 720$
Heptagon	7	5	$5 \times 180 = 900$
Octagon	8	6	$6 \times 180 = 1080$
n-gon	n	$n - 2$	$(n - 2) \times 180$



The sum of the exterior angles of a polygon is always 360°.

Adjacent interior and exterior angles in polygons add up to 180°.

Regular polygon with n sides

In a regular polygon all sides and angles are equal.

In a regular polygon, the size of one interior angle is equal to $\frac{\text{sum}}{n} = \frac{(n-2) \times 180}{n}$

In a regular polygon, the size of one exterior angle is equal to $\frac{360}{n}$

Example: Find interior and exterior angles in a regular pentagon.

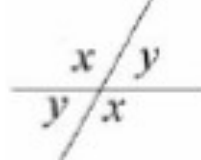
Pentagon can be divided into 3 triangles, so sum of interior angles = $(5 - 2) \times 180 = 3 \times 180 = 540^\circ$

In regular pentagon, each interior angle = $540 \div 5 = 108^\circ$

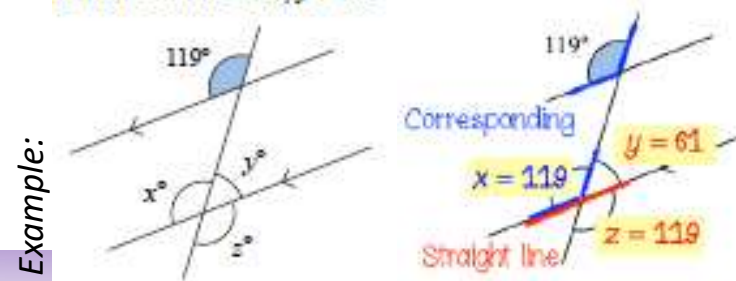
Sum of exterior angles = 360°, so each ext. angle in regular pentagon = $360 \div 5 = 72^\circ$

VERTICALLY OPPOSITE ANGLES

Vertically opposite angles are equal.



Find the values of x , y and z





Place Value
Estimate

is the value of each digit in the number based on its position.

means to find a value that is close enough to the right answer, usually with some thought or calculation involved.

Ascending
Descending

We use symbol \approx to mark estimation. *Example: $x \approx 10$ means x is approximately equal to 10.*

means increasing in value, ordered from the smallest to the largest value.

means decreasing in value, ordered from the largest to the smallest value.

Place Value Table

1000 thousands	100 hundreds	10 tens	1 units	.	0.1 tenths	0.01 hundredths	0.001 thousandths

ROUNDING TO DECIMAL PLACES

1. Find the digit with the value that you are rounding to.
2. If the decision digit (the next digit to the right) is having value
 - 5 or more – round up,
 - 4 or less – round down.
3. Do not write any more digits after the rounded number, not even 0s.

Example: Round 15.43 to the nearest tenths, or to 1 decimal place

$$15.43 \approx 15.4$$

the digit with the value that you are rounding to

the decision digit is having value 4 or less, that means rounding down

Note: Answer 15.43 rounded to 1dp is 15.40 is incorrect.

ADDING AND SUBTRACTING DECIMALS

1. Write one decimal beneath the other making sure that the digits with the same value are underneath each other.
2. Add 0s after the decimal point (place value holders) so that all the numbers have the same number of digits after the decimal point.
3. Add/subtract as you would with the whole numbers.
4. Copy the position of the decimal point into your answer.
5. Check your answer using estimation.

Example: Evaluate $65.3 - 42.45$

$$\begin{array}{r} 65.30 \\ - 42.45 \\ \hline 22.85 \end{array}$$

Check by estimation: $65.3 \approx 65$

$42.45 \approx 43$

$65 - 43 = 22$, which is very close to 22.85

ORDERING DECIMALS

1. Align numbers using the place value table or by writing one decimal beneath the other making sure that the digits with the same value are underneath each other.
2. Compare the digits starting with the digits that have the biggest value (from the left).

Example: Order numbers 0.21, 0.201 and 2.1 in descending order.

0.21

0.201 (the smallest number)

2.1 (the highest number - the biggest units digit)

Answer: 2.1, 0.21, 0.201



Factors are numbers which multiplied together get another number.
Product is the answer when two or more values are multiplied together.

Dividend a number to be divided.
Divisor a number by which another number is to be divided.
Quotient the answer after one number is divided by another.

Example: 2 and 3 are factors of 6, because $2 \times 3 = 6$
Example: 6 is a product of 2 and 3, because $2 \times 3 = 6$

$$\begin{array}{r}
 6 \leftarrow \text{quotient} \\
 4 \overline{) 24} \leftarrow \text{dividend} \\
 \uparrow \\
 \text{divisor}
 \end{array}$$

MULTIPLYING DECIMALS

1. Multiply decimals by powers of 10 (10, 100, 1000...) to change them into whole numbers.
2. Multiply the whole numbers.
3. 'Undo' the first step by using inverse operation - dividing the answer by the same powers of 10.

Example: Evaluate 12.3×6.11

$$\begin{array}{r}
 12.3 \times 10 = 123 \\
 6.11 \times 100 = 611 \\
 \hline
 \begin{array}{r}
 123 \\
 \times 611 \\
 \hline
 123 \\
 1230 \\
 + 73800 \\
 \hline
 75153
 \end{array}
 \end{array}$$

$75153 \div 10 \div 100 = 75.153$

*Check by estimation: $12.3 \approx 12$
 $6.11 \approx 6$
 $12 \times 6 = 72$, which is very close to 75.153*

ANOTHER METHOD

1. Multiply numbers as if there was no decimal point.
2. Count the number of digits after the decimal point in each factor.
3. Place the decimal point in the product by starting at the right and moving a number of places equal to the sum of the decimal places in both numbers multiplied.

*Numbers of digits after decimal point in both factors were $1 + 2 = 3$.
 Therefore the answer has to have 3 digits after decimal point.
 $12.3 \times 6.11 = 75.153$*

DIVIDING DECIMALS

1. Rewrite division as a fraction.
2. Find equivalent fraction with the whole number in denominator.
3. Divide numerator by denominator using bus method.

Example: Evaluate $0.12 \div 0.3$

$$\begin{array}{l}
 0.12 \div 0.3 = \frac{0.12}{0.3} = \frac{1.2}{3} = 1.2 \div 3 \dots\dots 3 \overline{) 1.2} \\
 0.12 \div 0.3 = 1.2 \div 3 = 0.4
 \end{array}$$

Using division of fractions to divide decimals

1. Convert decimals to fractions.
2. Divide fractions.
3. Convert the answer back to decimal.

Example: Evaluate $0.8 \div 0.01$

$$0.8 \div 0.01 = \frac{8}{10} \div \frac{1}{100} = \frac{8}{10} \times \frac{100}{1} = \frac{800}{10} = 80$$

RECURRING DECIMALS

Recurring decimal is a decimal number that has digits that repeat forever. The part that repeats is shown by placing a dot above the digit that repeats, or dots over the first and last digit of the repeating pattern.

Examples:

$$\begin{array}{ll}
 \frac{1}{3} = 0.333 \dots = 0.\dot{3} & 0.\dot{2}\dot{3} = 0.232323\dots \\
 \frac{1}{7} = 0.\underline{142857}\underline{142857} \dots = 0.\dot{1}4285\dot{7} & 0.5\dot{3} = 0.533333\dots \\
 & 0.\dot{1}2\dot{3} = 0.123123123\dots
 \end{array}$$

Key words **'Per cent'** means out of 100. *Example:* 3% means 3 out of 100, which can be written in a form of a fraction $\frac{3}{100}$ or as a decimal 0.03

Multiplier is a decimal that represents the percentage change.

VAT stands for Value Added Tax. This is 20% tax added on to the price of most of the things that you can buy.

Increase / reduce means to make something bigger / smaller (in size or quantity).

% OF AN AMOUNT

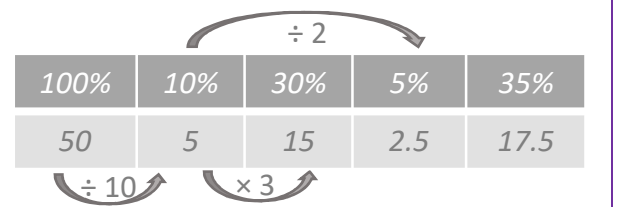
Finding 'easy' %s (without calculator)

- 50% by halving an amount
- 25% by dividing an amount by 4
- 10% by dividing an amount by 10
- 5% by halving 10%
- 1% by dividing an amount by 100
- ...and adding them together



Example: 35% of 50

10% of 50 = 5
5% of 50 = 2.5



Using multiplier (with calculator)

1. Change % into decimal (multiplier).
2. Multiply.

Example: 35% of 50 = 0.35 × 50 = 17.5

'of' means 'x'

% INCREASE AND DECREASE

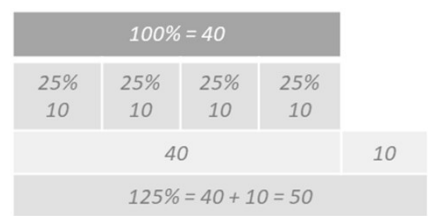
Finding % of an amount and adding (increase) or subtracting (decrease)

Example1:

Increase 40 by 25%.

25% of 40 = 10

40 + 10 = 50

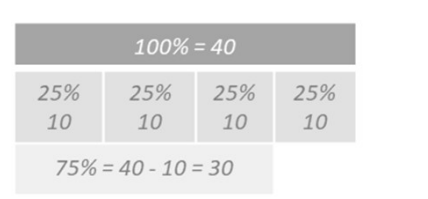


Example2:

Decrease 40 by 25%.

25% of 40 = 10

40 - 10 = 30



Using multiplier

Find the multiplier and **multiply**.

Example1: Increase 40 by 25%.

Multiplier = (100 + 25)% = 125% = 1.25

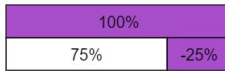
40 × 1.25 = 50



Example2: Decrease 40 by 25%.

Multiplier = (100 - 25)% = 75% = 0.75

40 × 0.75 = 30



EXPRESSING ONE NUMBER AS % OF ANOTHER

Example: What is 17 as a percentage of 25?

$\frac{17}{25} \times 100 = 68$ 17 is 68% of 25.

FINDING AN ORIGINAL AMOUNT

Using multiplication table

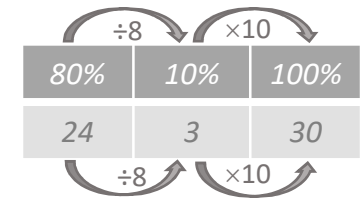
Example1: T-shirt costs £30 after VAT (20% increase). What is its original price?

20% increase:
(100 + 20)% = 120%
of the original price



Example2: T-shirt costs £24 after 20% discount. What is its original price?

20% discount:
(100 - 20)% = 80%
of the original price

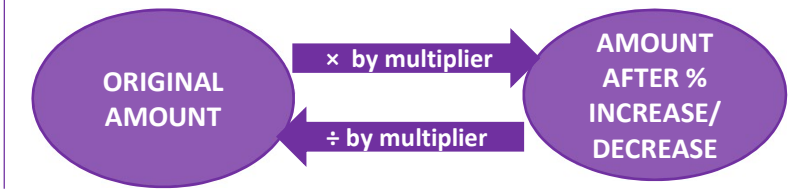


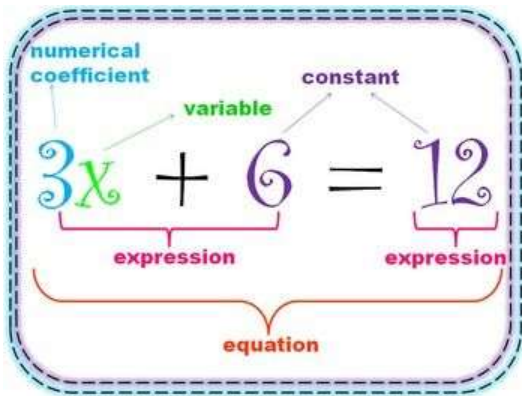
Using multiplier

Find the multiplier and **divide**.

Example1: Multiplier = (100 + 20)% = 120% = 1.2
£30 ÷ 1.2 = £25

Example2: Multiplier = (100 - 20)% = 80% = 0.8
£24 ÷ 0.8 = £30



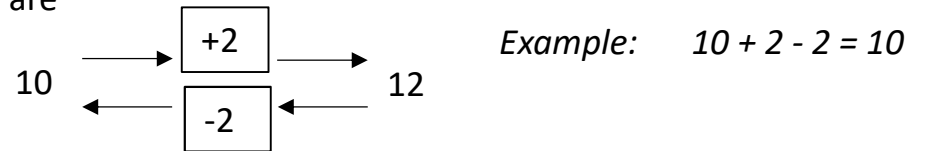


EQUATION is algebraic expression with equal sign, which can be solved (value of variable is found).

INVERSE OPERATIONS are opposite operations – one reverses the effect of the other.

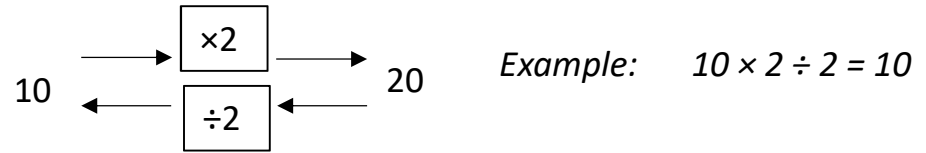
Two basic pairs of inverse operations are

ADDITION and **SUBTRACTION**



Example: $10 + 2 - 2 = 10$

MULTIPLICATION and **DIVISION**



Example: $10 \times 2 \div 2 = 10$

SOLVING ONE STEP EQUATIONS



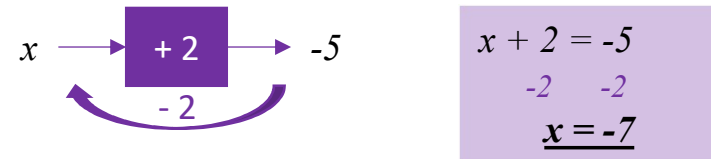
An equation is like a balance scale because it shows that two quantities are equal.

What you do to one side of the equation must also be done to the other side to keep it balanced.

Example: Solve $x + 2 = -5$

x	2
-5	

- What is the variable? x
- What operation is performed on the variable? $+ 2$
- What is the inverse operation? $- 2$



Example: Solve $x - 5 = 7$

x
5 7

- What is the variable? x
- What operation is performed on the variable? $- 5$
- What is the inverse operation? $+ 5$



Always indicate your steps (inverse operations).
Write your steps one under another, preferably aligning the equal symbols.

Example: Solve $4x = 14$

x	x	x	x
14			

- What is the variable? x
- What operation is performed on the variable? $\times 4$
- What is the inverse operation? $\div 4$



Example: Solve $\frac{x}{3} = 4$

x
4 4 4

- What is the variable? x
- What operation is performed on the variable? $\div 3$
- What is the inverse operation? $\times 3$



To solve one step equations, you need to ask three questions about the equation:

- What is the variable?
- What operation is performed on the variable?
- What is the inverse operation?

SOLVING 2-STEP EQUATIONS

Solving a two-step equation involves working backwards concerning the order of operations, using inverse operations.

You can always imagine equation as function machine or bar model to help you understand what is happening with variable.

Example: Solve $2x + 4 = 8$

x	x	4
8		



$$\begin{aligned}
 2x + 4 &= 8 \\
 -4 \quad -4 \\
 2x &= 4 \\
 \div 2 \quad \div 2 \\
 \underline{x = 2}
 \end{aligned}$$

Example: Solve $5x - 3 = 7$

x	x	x	x	x
3		7		



$$\begin{aligned}
 5x - 3 &= 7 \\
 +3 \quad +3 \\
 5x &= 10 \\
 \div 5 \quad \div 5 \\
 \underline{x = 2}
 \end{aligned}$$

Example: Solve $\frac{x}{5} - 9 = -2$



$$\begin{aligned}
 \frac{x}{5} - 9 &= -2 \\
 +9 \quad +9 \\
 \frac{x}{5} &= 7 \\
 \times 5 \quad \times 5 \\
 \underline{x = 35}
 \end{aligned}$$

Example: Solve $\frac{x-3}{5} = 4$



$$\begin{aligned}
 \frac{x-3}{5} &= 4 \\
 \times 5 \quad \times 5 \\
 x - 3 &= 20 \\
 +3 \quad +3 \\
 \underline{x = 23}
 \end{aligned}$$

SOLVING EQUATIONS WITH BRACKETS

Expand the brackets and solve as 2-step equation.

Example: Solve $3(2x - 1) = 13$

$$\begin{aligned}
 1. \text{ Expand brackets} &\longrightarrow 3(2x - 1) = 13 \\
 2. \text{ Solve as 2-step equation} &\longrightarrow \begin{aligned}
 6x - 3 &= 13 \\
 +3 \quad +3 \\
 6x &= 16 \\
 \div 6 \quad \div 6 \\
 x &= \frac{16}{6} = 2\frac{2}{3}
 \end{aligned}
 \end{aligned}$$

EXPANDING SINGLE BRACKETS

- Multiply everything in the brackets by a number or variable in front of the bracket

Examples: Expand

$$4(a + 6) = 4a + 24$$

a	+6
4	4a + 24

If needed, leave your answers as fractions rather than decimals (never leave your answer as recurring decimal).
Check your answer by substitution.

SOLVING EQUATIONS WITH UNKNOWN ON BOTH SIDES

Rearrange all the variables onto one side of the equation (the one with more variables) and all numbers onto the other side.

Example: Solve $5x + 3 = 15 + 2x$

x	x	x	x	x	3
x		15			

$$\begin{aligned}
 5x + 3 &= 15 + 2x \\
 -2x \quad -2x \\
 3x + 3 &= 15 \\
 -3 \quad -3 \\
 3x &= 12 \\
 \div 3 \quad \div 3 \\
 \underline{x = 4}
 \end{aligned}$$

Example: Solve $22 - 3x = 2 + 2x$

22		
3x	2x	2

$$\begin{aligned}
 22 - 3x &= 2 + 2x \\
 +3x \quad +3x \\
 22 &= 2 + 5x \\
 -2 \quad -2 \\
 20 &= 5x \\
 \div 5 \quad \div 5 \\
 4 &= x \text{ or } \underline{x = 4}
 \end{aligned}$$

FORM AND SOLVE EQUATIONS

Example: I think of a number. When I add 3 and multiply the result by 2, the answer is 11, form the equation. $2(x + 3) = 11$

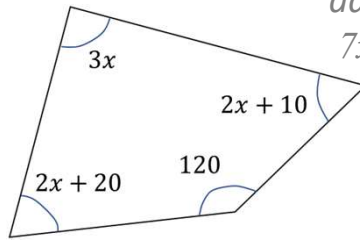
Example: After tennis training, Andy collects twice as many balls as Roger and five more than Maria. They collect 35 balls in total. How many balls does Andy collect?

Let x be the number of balls Roger collects. Then Andy collects $2x$ balls and Maria collects $2x - 5$.

$$\begin{aligned} \text{Total balls collected: } x + 2x + 2x - 5 &= 35 \\ 5x - 5 &= 35 \\ x &= 8 \end{aligned}$$

So Andy collected $2 \times x = 2 \times 8 = 16$ balls.

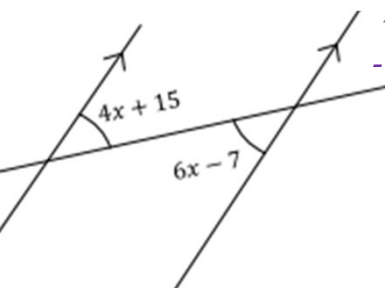
Example: Find x



Angles in the quadrilateral add up to 360° .

$$\begin{aligned} 7x + 150 &= 360 \\ -150 & \quad -150 \\ 7x &= 210 \\ \div 7 & \quad \div 7 \\ x &= 30^\circ \end{aligned}$$

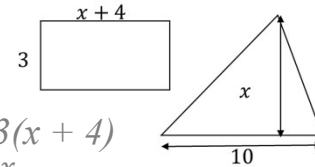
Example: Find x



Alternate angles are equal.

$$\begin{aligned} 4x + 15 &= 6x - 7 \\ -4x & \quad -4x \\ 15 &= 2x - 7 \\ +7 & \quad +7 \\ 22 &= 2x \\ \div 2 & \quad \div 2 \\ x &= 11 \end{aligned}$$

Example: The rectangle and the triangle have the same area. Determine the width of the rectangle.



$$\begin{aligned} \text{Area of rectangle} &= 3(x + 4) \\ \text{Area of triangle} &= \frac{10x}{2} \end{aligned}$$

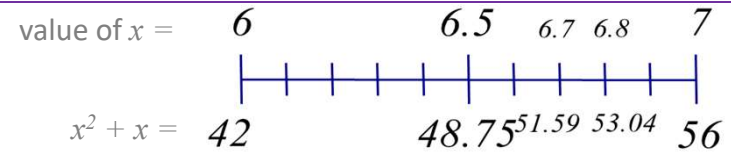
$$\begin{aligned} 3(x + 4) &= \frac{10x}{2} \\ 3x + 12 &= 5x \\ -3x & \quad -3x \\ 12 &= 2x \\ \div 2 & \quad \div 2 \\ x &= 6 \end{aligned}$$

Width of the rectangle = $x + 4 = 6 + 4 = 10$

TRIAL AND IMPROVEMENT

Example: Solve $x^2 + x = 53$ correct to 1 decimal place

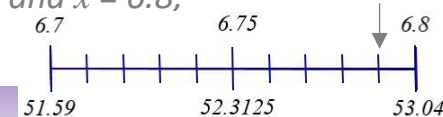
- First find the **two whole number values** of x that will give the closest answers to 53: one will be a bit too small, the other will be a bit too big.
- Then ZOOM in between these two numbers to find the **two numbers with one decimal place** that give the closest answers to 53: again one will be a bit too small, the other will be a bit too big.
- To work out which one of those two numbers is the BEST answer you need to ZOOM in again to **HALFWAY** between them, then you will be able to see which one was closest to 53. (This last step is the most commonly missed out part because sometimes it seems unnecessary, but it is important.)



when $x = 6$ $x^2 + x = 6^2 + 6 = 42$ too small
 when $x = 7$ $x^2 + x = 7^2 + 7 = 56$ too big
 - answer is between 6 and 7

when $x = 6.5$ $x^2 + x = 6.5^2 + 6.5 = 48.75$ too small
 when $x = 6.8$ $x^2 + x = 6.8^2 + 6.8 = 53.04$ too big
 when $x = 6.7$ $x^2 + x = 6.7^2 + 6.7 = 51.59$ too small
 - answer is between 6.7 and 6.8

when $x = 6.75$ $x^2 + x = 6.75^2 + 6.75 = 52.3125$
 - the answer 53 is to the right side from 52.3125 on the number line, between $x = 6.75$ and $x = 6.8$, so the answer is $x = 6.8$





- Length** is the distance of something measured.
- Capacity** how much something can hold (generally liquid), commonly known as VOLUME
- Mass** is a measure of how much matter is in an object, commonly known as WEIGHT.
- The Imperial system** is a system of weights and measures originally developed in England. It consists of many different units of measurement, which are named differently and have different conversion factors.
- The Metric system** is a system of measurement that uses the **meter**, **litre**, and **gram** as base units of length (distance), capacity (volume), and weight (mass) respectively.

CONVERTING BETWEEN IMPERIAL AND METRIC UNITS

LENGTH

- 1 inch \approx 2.5 cm
- 1 foot \approx 30 cm
- 1 yard \approx 0.9 metre
- 1 mile \approx 1.6 km

Example: Estimate a length of half a metre in inches?

Half a metre is 50 cm.

1 inch \approx 2.5 cm.

$50 \div 2.5 \approx 20$ inches

inch	1	2	20
cm	2.5	5	50

CAPACITY / VOLUME

- 1 pint \approx 0.6 litres (l)
- 1 litre (l) \approx 1.75 pints (pt)
- 1 gallon \approx 4.5 litres.

Example: How many litres are in 8 gallons of petrol?

1 gallon \approx 4.5 litres

8 gallons = $8 \times 4.5 = 36$ l

gallon	1	2	8
litre	4.5	9	36

MASS / WEIGHT

- 1 kilogram (kg) \approx 2.2 pounds (lb).
- 1 pound (lb) \approx half a kilogram.
- 1 stone (st) \approx 6.5 kilograms (kg).

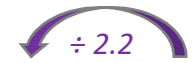
Example: Estimate a mass of 7 pounds (lb) in kilograms (kg).

1 pound (lb) \approx half a kilogram.

7 lb = $0.5 \times 7 = 3.5$ kg

pound	1	2	6	7
kg	0.5	1	3	3.5

To convert from pounds to kg: **divide** by 2.2.



1kg \approx 2.2 pounds



To convert from kg to pounds: **multiply** by 2.2.

CONVERTING METRIC UNITS

LENGTH - basic unit is metre (m)

- 1 cm = 10 mm**
- 1 m = 100 cm**
- 1 km = 1000 m**

Example: Convert 3 m to millimetres.

3 m = 300 cm = 3000 mm

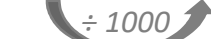


CAPACITY - basic unit is litre (l)

1 l = 1000 ml

Example: Convert 5700 ml to litres.

5700 ml = 5.7 l



metre	gram	litre
1m = 1000 mm	1g = 1000mg	1l = 1000 ml
1 km = 1000 m	1 kg = 1000 g	

MASS - basic unit is gram (g)

- 1 g = 1000 mg**
- 1 kg = 1000 g**
- 1 t = 1000 kg**

Example: Convert 8700 mg to kilograms.

8700 mg = 8.7 g = 0.0087 kg



Prefix	Meaning	Length	Capacity	Mass
kilo-	thousand (1,000)	kilometre		kilogram
hecto-	hundred (100)			
deka-	ten (10)			dekagram
BASE UNIT	ones (1)	metre	litre	gram
deci-	tenths (0.1)		decilitre	
centi-	hundredths (0.01)	centimetre		
milli-	thousandths (0.001)	millimetre	millilitre	milligram

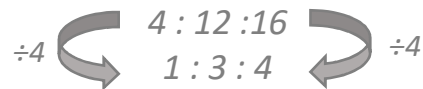


A **ratio** shows the relative sizes of two or more values.

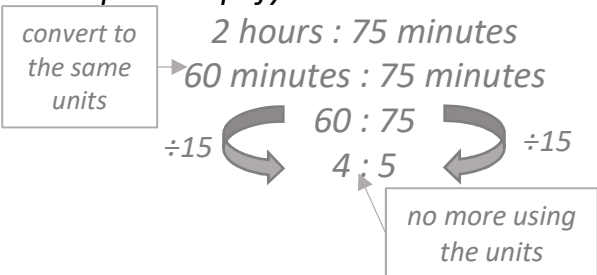
SIMPLIFYING RATIO

To simplify ratio, divide all parts of the ratio by the highest common factor.

Example: Simplify ratio 4 : 12 : 16.



Example: Simplify ratio 2 hours : 75 minutes.



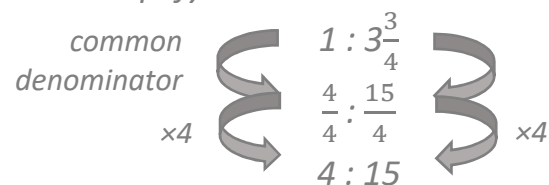
Example: Simplify ratio 1.2 : 3.

1. Convert to whole numbers.
2. Simplify.



Example: Simplify ratio 1 : 3 $\frac{3}{4}$.

1. Convert to fractions with common denominator.
2. Simplify.



SHARING IN THE RATIO

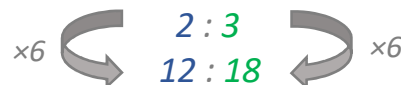
A ratio can also be used to share a quantity into parts.

Example: Share 30 sweets into a ratio of 2 : 3.

Total amount of parts = 2 + 3 = 5.



If 30 sweets are shared equally in 5 parts, each part contains $30 \div 5 = 6$ sweets.



Example: Alison and Henry shared some money in a ratio of 3 : 5. Henry got £10 as a result. How much money did they share?

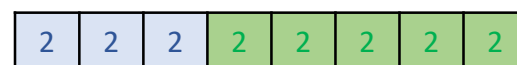
Total amount of parts = 3 + 5 = 8.



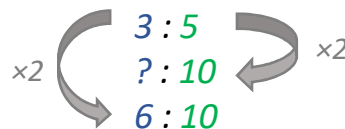
If Henry received £10, this amount is shared in 5 parts. One part is worth $10 \div 5 = £2$.



Therefore Alison received $3 \times £2 = £6$.



They shared $£10 + £6 = £2 \times 8 = £16$.



RELATIONSHIP BETWEEN RATIO AND PROPORTION

RATIO is a **part to part comparison**.

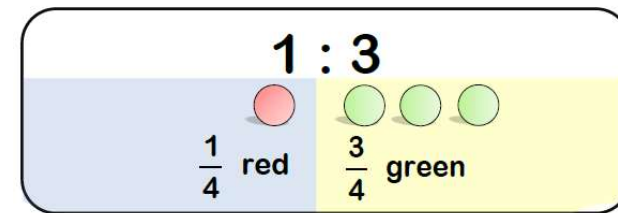
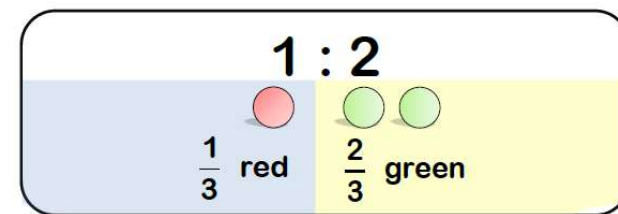
PROPORTION is a **part to whole comparison**.

Proportions can be expressed as fractions or percentages.

Example: To make the colour I need for my painting I mix the blue and red paint in the ratio 3 : 7. What fraction is blue paint? What percentage is red paint?

Blue paint is represented by 3 parts out of 10, therefore the proportion of blue paint is $\frac{3}{10}$.

The rest of the paint is red = $\frac{7}{10}$, so the red paint makes 70% of the whole paint.



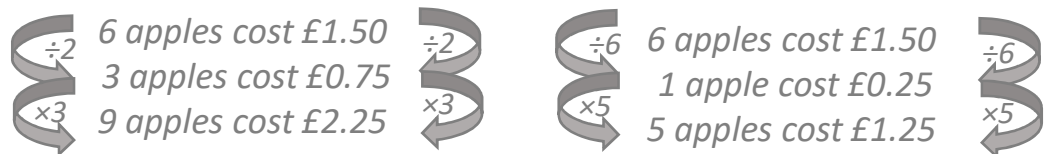
SOLVING SIMPLE DIRECT PROPORTION WORD PROBLEMS

Two quantities are said to be in DIRECT PROPORTION if they increase or decrease at the same rate. That is, if the ratio between the two quantities is always the same.

Example: The cost of 6 apples is £1.50. What is the cost for 9 apples? What would be the cost of 5 apples?

apples	6	3	9	1	5
cost	£1.50	£0.75	£2.25	£0.25	£1.25

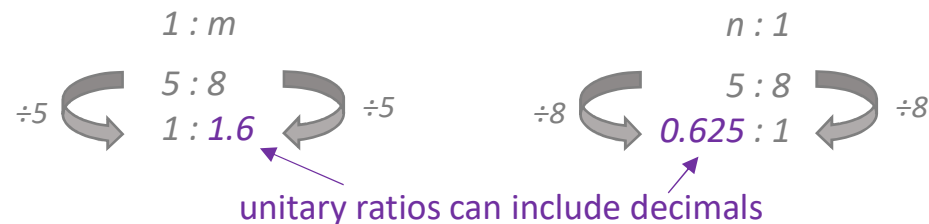
Diagram showing multiplicative relationships: 6 to 3 is $\div 2$, 6 to 9 is $\times 3$, 6 to 1 is $\div 6$, 6 to 5 is $\times 5$.



USING UNITARY METHOD

Using unitary method means find the value of 1 part.

Example: Write the ratio 5 : 8 in the form 1 : m and n : 1.



BEST BUYS

We can compare the value of products using the UNITARY METHOD, either through

- the cost per unit amount, or
- the amount per unit cost.

Example: Kayden buys an ice cream from Mr Whipcream. He gets 200ml of ice cream for £1.50. Lilly buys an ice cream from Iced Delights. She gets 300ml of ice cream for £2.40.

Who gets better value for money?

Cost per unit amount method:

Mr Whipcream: $1.50 \div 200 = \text{£}0.0075$ per ml → $\text{£}1.50 : 200 \text{ ml}$
 $\text{£}0.0075 : 1 \text{ ml}$

Iced Delights: $2.40 \div 300 = \text{£}0.008$ per ml → $\text{£}2.40 : 300 \text{ ml}$
 $\text{£}0.008 : 1 \text{ ml}$

Mr Whipcream is cheaper per ml, therefore better value for money.

Amount per unit cost method:

Mr Whipcream: $200 \div 1.50 = 133.33$ ml per £ → $\text{£}1.50 : 200 \text{ ml}$
 $\text{£}1 : 133.33 \text{ ml}$

Iced Delights: $300 \div 2.40 = 125$ ml per £ → $\text{£}2.40 : 300 \text{ ml}$
 $\text{£}1 : 125 \text{ ml}$

Mr Whipcream gives more ice cream per £, therefore better value for money.

SOLVING SIMPLE INVERSE PROPORTION WORD PROBLEMS

Two quantities are said to be INVERSELY PROPORTIONAL if, as one quantity increases, the other quantity decreases at the same rate.

Example: I want to build a wall in my garden. 2 builders would do it in 108 hours. How long would it take 9 builders to build the wall?

builders	2	1	9
hours	108	216	24

Diagram showing multiplicative relationships: 2 to 1 is $\div 2$, 2 to 9 is $\times 9$, 108 to 216 is $\times 2$, 108 to 24 is $\div 9$.

