AS Pure Practice Paper A.

1. $x^2 + 6x + 18 > 2 - \frac{1}{2}x$.

$x^2 + 6x + 16 > -\frac{1}{2}x$.

$2x^2 + 12x + 32 > -x$.

$2x^2 + 13x + 32 > 0$.

$2\left[x^2 + \frac{13}{2}x\right] + 32 > 0$

$2\left[\left(x + \frac{13}{4}\right)^2 - \left(\frac{13}{4}\right)^2\right] + 32 > 0$

$2\left[\left(x + \frac{13}{4}\right)^2 - \frac{169}{16}\right] + 32 > 0$

$2\left(x + \frac{13}{4}\right)^2 - \frac{169}{8} + 32 > 0$

$+\frac{87}{8}$

↑ will always be positive as squared.

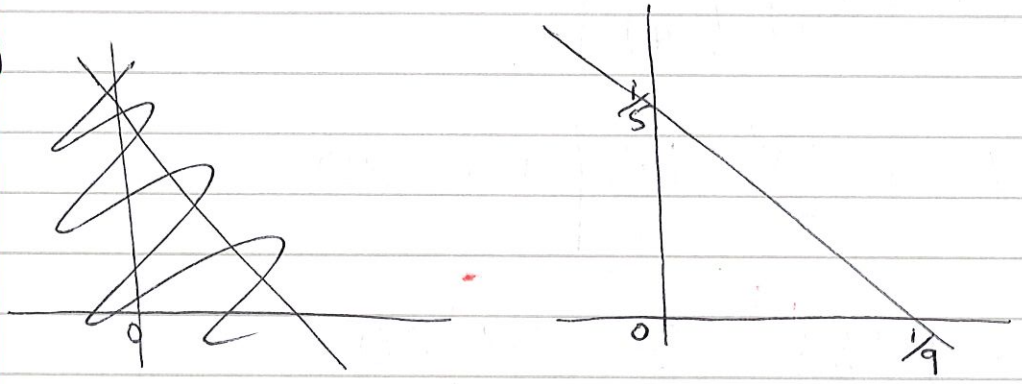
Positive + positive > 0.

2a) (4, -7) and (-6, 11)

$$m = \frac{11 - -7}{-6 - 4} = \frac{18}{-10} = \frac{-9}{5}.$$

$$y - y_1 = m(x - x_1).$$
$$y - 11 = \frac{-9}{5}(x + 6)$$
$$5y - 55 = -9x - 54.$$
$$5y + 9x + 1 = 0.$$

b)



$x = 0$, $5y - 1 = 0$, $5y = 1$, $y = \frac{1}{5}$.

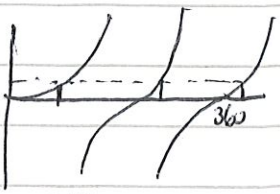
$y = 0$, $9x - 1 = 0$, $9x = 1$, $x = \frac{1}{9}$.

$$\text{Area} = \frac{1}{2} \times \frac{1}{5} \times \frac{1}{9} = \frac{1}{90}.$$

3a) $4\sqrt{3}\sin(3\theta + 20) = 4\cos(3\theta + 20)$

$$4\sqrt{3}\frac{\sin(3\theta + 20)}{\cos(3\theta + 20)} = 4.$$
$$4\sqrt{3}\tan(3\theta + 20) = 4.$$
$$\tan(3\theta + 20) = \frac{1}{\sqrt{3}}.$$

$3\theta + 20 = x$ $\quad \tan x = \frac{1}{\sqrt{3}}$.



$x = 30, 210, 390$

3(a) $30\theta + 20 = 30, 210, 390.$

$\theta = \frac{10}{3}, \frac{190}{3}, \frac{370}{3}.$

$\theta = 3.3, 63.3, 123.3$

4. $\log_{11}(2x-1) = 1 - \log_{11}(x+4).$

$\log_{11}(2x-1) + \log_{11}(x+4) = 1$

$\log_{11}(2x-1)(x+4) = 1$

$3^2 = 9$

$\log_3 9 = 2.$

$11^1 = (2x-1)(x+4)$

$11 = 2x^2 + 8x - x - 4$

$11 = 2x^2 + 7x - 4$

$0 = 2x^2 + 7x - 15$

$0 = (2x-3)(x+5)$

$2x - 3 = 0$ or $x + 5 = 0$

$\underline{x = \frac{3}{2}}$

~~$x = -5$~~

Can't take a log of a negative

5(a) $a = 2pi - 5j$

$b = 6i - 3pj$

$c = 4i - 5j$

Resultant of a and b $= (2p+6)i + (-5-3p)j$

$x(4i - 5j) = (2p+6)i + (-5-3p)j$

5a) $2p + 6 = 4x$.
$-5 - 3p = -5x$.

$\rightarrow$ $2p - 4x = -6 \rightarrow \times 3$
$3p - 5x = -5 \rightarrow \times 2$.

$6p - 12x = -18$
$6p - 10x = -10$
$-2x = -8$
$x = 4$.

$2p + 6 = 4x$
$2p + 6 = 16$.
$2p = 10$
$p = 5$.

b) $(2p + 6)i + (-5 - 3p)j$.
$(10 + 6)i + (-5 - 15)j$
$16i - 20j$

6. a) $P = 100e^{0.4t}$

$P = 100e^{0.4 \times 7} = 1644.46 = \underline{1644}$.

b) Initial bacteria population.

c) $100e^{0.4t} > 1{,}000{,}000$
$e^{0.4t} > 10{,}000$.
$\log_e e^{0.4t} > \log_e 10{,}000$
$0.4t > \ln 10{,}000$
$t > \frac{\ln 10{,}000}{0.4}$.
$t > 23.02$.

$\therefore$ 24 hours

7. $mx - y - 2 = 0$. $\quad x^2 + 6x + y^2 - 8y = 4$.

$y = mx - 2$.

$x^2 + 6x + (mx-2)^2 - 8(mx-2) = 4$.

$x^2 + 6x + (mx-2)(mx-2) - 8mx + 16 = 4$.

$x^2 + 6x + m^2x^2 - 4mx + 4 - 8mx + 12 = 0$.

$m^2x^2 + x^2 + 6x - 12mx + 16 = 0$.

$(m^2+1)x^2 + (6-12m)x + 16 = 0$.

$b^2 - 4ac = 0$.

$(6-12m)^2 - 4 \times (m^2+1) \times 16 = 0$.

$(6-12m)(6-12m) - 64m^2 - 64 = 0$.

$36 - 144m + 144m^2 - 64m^2 - 64 = 0$.

$80m^2 - 144m - 28 = 0$.

$20m^2 - 36m - 7 = 0$.

$a = 20$, $b = -36$, $c = -7$

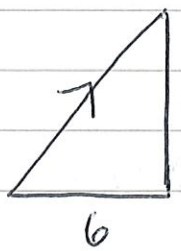
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{36 \pm \sqrt{36^2 - (4 \times 20 \times -7}}{2 \times 20}.$$

$$m = \frac{9 + 2\sqrt{29}}{10} \qquad m = \frac{9 - 2\sqrt{29}}{10}.$$

8. A $4i + 7j$ $\quad$ B $10i + qj$

a) $\overrightarrow{AB} = 6i + (q-7)j$

b) $|\overrightarrow{AB}| = 2\sqrt{13}$.



$6^2 + (q-7)^2 = (2\sqrt{13})^2$.

$36 + (q-7)(q-7) = 4 \times 13$.

$36 + q^2 - 14q + 49 = 52$.

$q^2 - 14q + 33 = 0$.

$(q-3)(q-11) = 0$

$q = 3$ or $q = 11$

9.a) $(2+px)^9$

$= 2^9 + {}^9C_1 2^8 (px)^1 + {}^9C_2 2^7 (px)^2 + {}^9C_3 2^6 (px)^3$

$= 512 + 2304px + 4608p^2x^2 + 5376p^3x^3$

b)i) $5376p^3x^3 = -84x^3$

$5376p^3 = -84$

$p^3 = \frac{-1}{64}$.

$p = \sqrt[3]{\frac{-1}{64}}$.

$p = \frac{-1}{4}$.

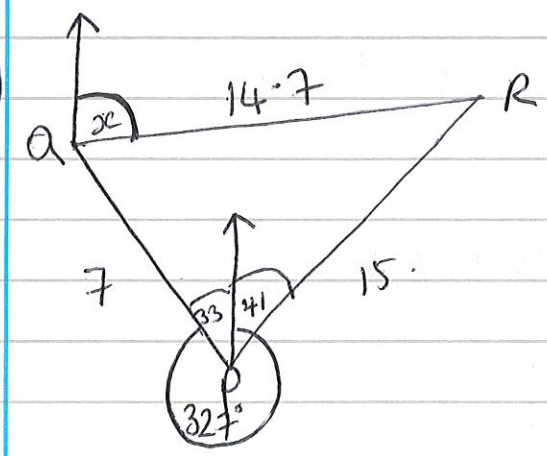
ii) $2304px$

$2304 \times -\frac{1}{4} = -576$.

$4608p^2x^2$

$4608 \times \left(-\frac{1}{4}\right)^2 = 288$.

10.a)



$a^2 = b^2 + c^2 - 2bc\cos A$.

$a^2 = 7^2 + 15^2 - (2 \times 7 \times 15 \times \cos 74)$

$a^2 = 216.11\ldots$

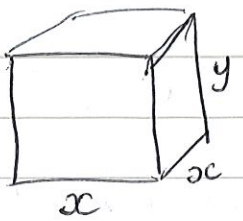
$a = 14.7\text{km}$

$\underline{QR = 14.7\text{km}}$.

b) $\frac{\sin Q}{15} = \frac{\sin 74}{14.7}$ $\Rightarrow$ $Q = 78.77°$.

$x + 78.77 + 33 = 180$. $x = 68.23° \Rightarrow \underline{068°}$

11. a) $SA = 1600\,cm^2$



$SA = (x \times x) + 2(x \times y) + 2(xy)$.

$1600 = x^2 + 4xy$.

$\frac{1600 - x^2}{4x} = y$

$V = x^2 y$

$V = x^2 \times \frac{1600 - x^2}{4x}$.

$= \frac{1600x^2 - x^4}{4x}$

$= \frac{1600x}{4} - \frac{x^3}{4}$

$= 400x - \frac{x^3}{4}$.

b) $V = 400x - \frac{x^3}{4}$.

$\frac{dV}{dx} = 400 - \frac{3x^2}{4}$.

$\frac{dV}{dx} = 0, \quad 400 - \frac{3x^2}{4} = 0$.

$\frac{3x^2}{4} = 400$

$3x^2 = 1600$

$x^2 = \frac{1600}{3}$.

$x = \frac{\sqrt{1600}}{\sqrt{3}} = \frac{40}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{40\sqrt{3}}{3}$.

c) $\frac{d^2V}{dx^2} = -\frac{6x}{4} = -\frac{3x}{2}$, $x = \frac{40\sqrt{3}}{3}$ $\frac{d^2V}{dx^2} = -\frac{3}{2} \times \frac{40\sqrt{3}}{3}$

$< 0 \therefore$ max

12. $y = -x^3 + 2x^2 + 8x$.

a) $y = -x(-x^2 - 2x - 8)$

$y = -x(x+2)(x-4)$

(0, 0) A(−2, 0) B(4, 0).

b) $\int_{-2}^{0} -x^3 + 2x^2 + 8x$

$= \left[\frac{-x^4}{4} + \frac{2x^3}{3} + \frac{8x^2}{2}\right]_{-2}^{0}$

$= (0) - \left(4 - \frac{16}{3} + 16\right) = \frac{-20}{3}$ Area $= \frac{20}{3}$.

$\int_{0}^{4} -x^3 + 2x^2 + 8x$

$= \left[\frac{-x^4}{4} + \frac{2x^3}{3} + \frac{8x^2}{2}\right]_{0}^{4}$

$= \left(-64 + \frac{128}{3} + 64\right) - (0) = \frac{128}{3}$.

Total area $= \frac{20}{3} + \frac{128}{3} = \frac{148}{3}$

13a) $p(x) = 3 - \frac{1}{2}x$ $\quad q(x) = x^2 - 10x - 20$.

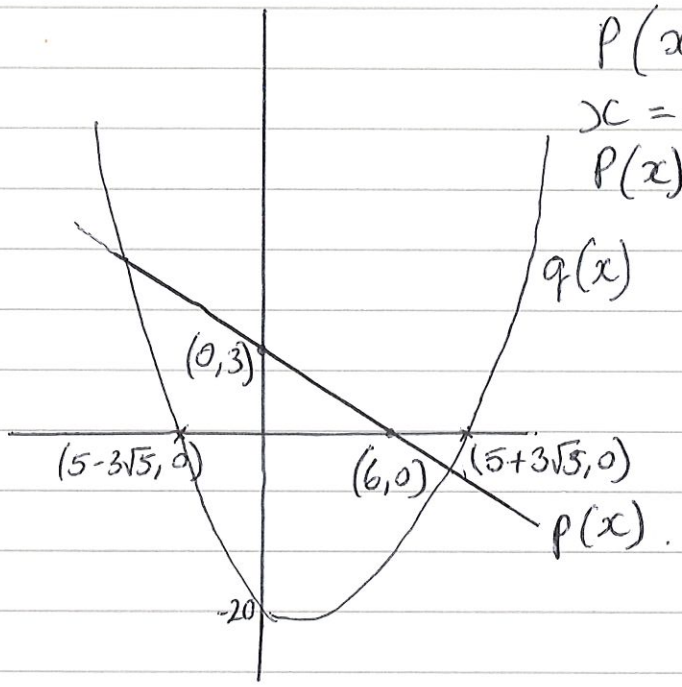
$x^2 - 10x - 20 = 0$

$a = 1$
$b = -10$
$c = -20$

$x = \frac{+10 \pm \sqrt{10^2 - (4 \times 1 \times -20}}{2}$.

$x = 5 \pm 3\sqrt{5}$.

b)



$p(x) = 3 - \frac{1}{2}x$.
$x = 0$, $p(x) = 3$.
$p(x) = 0$, $3 - \frac{1}{2}x = 0$.
$3 = \frac{1}{2}x$
$6 = x$.

c) $x^2 - 10x - 20 = 3 - \frac{1}{2}x$.
$2x^2 - 20x - 40 = 6 - x$.
$2x^2 - 19x - 46 = 0$.
$(2x - 23)(x + 2) = 0$
$x = \frac{23}{2}$ $\quad x = -2$.

$\left(\frac{23}{2}, \frac{-11}{4}\right)$ and $(-2, 4)$

d) $x < -2$ or $x > \frac{23}{2}$.

$\{x : x \in \mathbb{R}, x < -2\} \cup \{x : x \in \mathbb{R}, x > 11.5\}$