A2 Pure Paper 1

October 2021

(1) $(x-1)$ $f(x) = ax^3 + 10x^2 - 3ax - 4$

$f(1) = a + 10 - 3a - 4 = 0$

$6 = 2a$

$\underline{\underline{a = 3}}$

(2) (a) $f(x) = x^2 - 4x + 5$ $x \in \mathbb{R}$

$f(x) = (x-2)^2 + 1$

(b) (i) at P $y = 5$ $(0, 5)$

(ii) $x = 2$ $y = 1$ $(2, 1)$

(3) $u_{n+1} = k - \frac{24}{u_n}$ $u_1 = 2$

$u_2 = k - 12$ $u_3 = k - \frac{24}{k-12}$

$2 + 2(k-12) + k - \frac{24}{k-12} = 0$

$2 + 2k - 24 + k - \frac{24}{k-12} = 0$

$-22 + 3k - \frac{24}{k-12} = 0$

$-22(k-12) + 3k(k-12) - 24 = 0$

$-22k + 264 + 3k^2 - 36k - 24 = 0$

$3k^2 - 58k + 240 = 0$

(4) $f(x) = x^2 + \ln(2x^2 - 4x + 5)$

$f'(x) = 2x + \dfrac{4x - 4}{2x^2 - 4x + 5}$

$f'(x) = 0$ when $x = \alpha$

$0 = 2\alpha + \dfrac{4\alpha - 4}{2\alpha^2 - 4\alpha + 5}$

$0 = 2\alpha(2\alpha^2 - 4\alpha + 5) + 4\alpha - 4$

$0 = 4\alpha^3 - 8\alpha^2 + 10\alpha + 4\alpha - 4$

$0 = 4\alpha^3 - 8\alpha^2 + 14\alpha - 4$

$0 = 2\alpha^3 - 4\alpha^2 + 7\alpha - 2$

So α is a solution to $0 = 2x^3 - 4x^2 + 7x - 2$

$x_1 = 0.3$ $x_2 = 0.3294$ $x_3 = 0.3375$

$x_4 = 0.3398$

(c) Try ~~$x = 0.3405$ 1.4691~~
~~$x = 0.3415$~~

function $2x^3 - 4x^2 + 7x - 2 = 0$

Try $x = 0.3405$ $-ve$
$x = 0.3415$ $+ve$
$\Big\}$ As sign change the real root must be between these values

All numbers between these round to $\underline{\underline{0.341}}$ to 3 dp

⑤ $20000, \; 20{,}000 \times 1.08, \; 20{,}000 \times 1.08^2$

(a) $20000 \times 1.08^2 = £23{,}328$

(b) $20{,}000 \times 1.08^{n-1} > 65000$

$$1.08^{n-1} > 13/4$$

$$n-1 > \frac{\ln(13/4)}{\ln 1.08}$$

~~n > 2.29~~

~~n = 28~~

$$n-1 > 15.3149$$

$$n > 16.3$$

$$\underline{\underline{n = 17}}$$

(c) $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{20{,}000(1-1.08^{20})}{1-1.08}$

$= 915239.286$

£915000 to the nearest £1000.

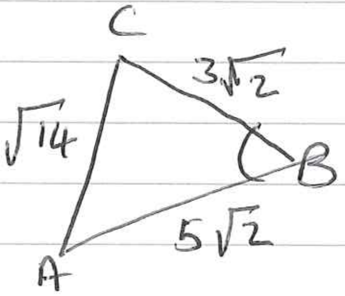
(6) $\vec{AB} + \vec{BC} = \vec{AC}$

$$\begin{pmatrix}-3\\-4\\-5\end{pmatrix} + \begin{pmatrix}1\\1\\4\end{pmatrix} = \begin{pmatrix}-2\\-3\\-1\end{pmatrix} \qquad -2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}$$

$|AB| = 5\sqrt{2}$

$|BC| = 3\sqrt{2}$

$|AC| = \sqrt{14}$



$c^2 = a^2 + b^2 - 2ab\cos C$

$b^2 = a^2 + c^2 - 2ac\cos B$

$\cancel{\cos B = 14 - 18 - 50 \quad 2\times 3\sqrt{2}\times}$

$$\cos B = \frac{b^2 - a^2 - c^2}{-2ac} = \frac{14 - 18 - 50}{-2\times 3\sqrt{2}\times 5\sqrt{2}}$$

$$\cos B = \frac{-54}{-60} = \frac{9}{10}$$

7(i) $x^2+y^2-10x+4y+11=0$

$x^2-10x+y^2+4y+11=0$

$(x-5)^2-25+(y+2)^2-4+11=0$

$(x-5)^2+(y+2)^2-18=0$

$(5,-2)$

(ii) $\sqrt{18}=3\sqrt{2}$

(b) $l \rightarrow y=3x+k$

As at a tangent, only one solution to the line and curve.

$(x-5)^2+(3x+k+2)^2-18=0$

$(x-5)(x-5)+(3x+k+2)(3x+k+2)-18=0$

$x^2-10x+25+9x^2+3xk+6x+3xk+k^2+2k$
$+6x$ $+2k+4$
$-18=0$

$10x^2+x(2+6k)+(11+4k+k^2)=0$

Discriminant should be zero $\quad b^2-4ac=0$

$(2+6k)^2-4\times10\times(11+4k+k^2)=0$

$(2+6k)(2+6k)$

$4+12k+36k^2-440-160k-40k^2=0$

$4k^2+136k+436=0$

$k^2+34k+109=0 \qquad \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\dfrac{-34\pm\sqrt{34^2-4\times1\times109}}{2\times34} = \dfrac{-17\pm6\sqrt{5}}{34}$

8. $N = Ae^{kt} \qquad t \geq 0$

(a)

$t = 0 \qquad N = 1000$

$t = 5 \qquad N = 2000$

So $A = 1000$

$2000 = 1000\, e^{5k}$

$\ln 2 = 5k \Rightarrow K = \frac{\ln 2}{5} = 0.1386294361$

So $N = 1000\, e^{\left(\frac{\ln 2}{5}\right)t}$

(b) $\frac{dN}{dt} = 1000\left(\frac{\ln 2}{5}\right) e^{\left(\frac{\ln 2}{5}\right)t}$

$t = 8 \quad \frac{dN}{dt} = 1000\left(\frac{\ln 2}{5}\right) e^{\left(\frac{\ln 2}{5}\right)8}$

$\frac{dN}{dt} = 420.2458658$

420 to 3 S.F.

(c) $M = 500\, e^{1.44t} \qquad t \geq 0$

8 c

$$1000\, e^{\left(\frac{\ln 2}{5}\right)T} = 500\, e^{1.4\left(\frac{\ln 2}{5}\right)T}$$

$$2\, e^{\left(\frac{\ln 2}{5}\right)T} = e^{1.4\left(\frac{\ln 2}{5}\right)T}$$

$$\ln 2 + \frac{\ln 2}{5}T = \frac{1.4T\ln 2}{5}$$

$$5 + T = 1.4T$$

$$5 = 0.4T$$

$$T = \frac{5}{0.4} = \underline{\underline{12.5}}$$

(9) $$\frac{50x^2+38x+9}{(5x+2)^2(1-2x)} \equiv \frac{A}{5x+2} + \frac{B}{(5x+2)^2} + \frac{C}{1-2x}$$

(a)

$$50x^2+38x+9 \equiv A(5x+2)(1-2x) + B(1-2x) + C(5x+2)^2$$

If $x = -2/5$ $\quad \frac{9}{5} = B\frac{9}{5} \Rightarrow B=1$

$x = 1/2$ $\quad \frac{81}{2} = \frac{81}{4}C \Rightarrow C=2$

$x=0$ $\quad 9 = 2A + 1 + 8 \Rightarrow A=0$

b(i) $(5x+2)^{-2} \cancel{=} \cancel{5^{-2}(2/5+x)^{-2}}$

$\cancel{\frac{1}{25}}\{$

$(1+5/2x)^{-2}\ (2)^{-2}$

$$1/4\left(1 - 5x + \frac{(-2)(-3)}{1\times 2}\frac{25}{4}x^2 + \cdots\right.$$

$$= \underline{1/4 - 5/4x + \frac{75}{16}x^2 + \cdots}$$

$$2(1-2x)^{-1} = 2\left(1 + (2x) + \frac{(-1)(-2)}{1\times 2}(-2x)^2 + \cdots\right.$$

$= \underline{2 + 4x + 8x^2}$ If we sum these $\underline{9/4 + 11/4x + 203/16x^2}$

9b (ii) $|x| < 1$ in general

so here $|-2x| < 1$ or $|\frac{5}{2}x| < 1$

$x < \frac{1}{2}$ $\underline{\underline{x < \frac{2}{5}}}$

(10) (a) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos 2A = \cos^2 A - \sin^2 A$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin 2A = 2 \sin A \cos A$

$$\frac{1 - \cos^2 A + \sin^2 A + 2 \sin A \cos A}{1 + \cos^2 A - \sin^2 A + 2 \sin A \cos A}$$

but $1 \equiv \sin^2 A + \cos^2 A$

$$\frac{\sin^2 A + \cancel{\cos^2 A} - \cancel{\cos^2 A} + \sin^2 A + 2 \sin A \cos A}{\cancel{\sin^2 A} + \cos^2 A + \cos^2 A - \cancel{\sin^2 A} + 2 \sin A \cos A}$$

$$\frac{2 \sin^2 A + 2 \sin A \cos A}{2 \cos^2 A + 2 \sin A \cos A}$$

$$\frac{\sin A}{\cos A} \left(\frac{2 \sin A + 2 \cos A}{2 \cos A + 2 \sin A} \right)$$

$\underline{\underline{\tan A}}$

10 b

$$\tan 2x = 3\sin 2x$$

$$0 = 3\sin 2x - \frac{\sin 2x}{\cos 2x}$$

$$0 = \sin 2x\left(3 - \frac{1}{\cos 2x}\right)$$

so $2x = 0,\ 180,\ 360$

$x = 0,\ \underline{\underline{90}},\ 180$

$$0 = 3 - \frac{1}{\cos 2x}$$

$$0 = 3\cos 2x - 1 \Rightarrow \cos 2x = 1/3$$

$2x = \cancel{1.230959441}\ 70.52877937$

or $360 - 70.52877937$

$= \cancel{109.4712206}\ 289.4712206$

so $x = \underline{\underline{35.26438969}}$ or $\cancel{54.73561033}$ $\underline{\underline{144.7356103}}$

11 (a) $h = \frac{b-a}{n} = \frac{4-2}{4} = 1/2$

$1/2 \times 1/2 \left((0.4805 + 1.9218) + 2(0.8396 + 1.2069 + 1.5694) \right)$

$= 2.408525 = 2.41$ to 3 SF

(b) $\int (\ln x)^2 \, dx$ limits 2 – 4

use $\int 1 \times (\ln x)^2 \, dx$ $\quad \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

$u = (\ln x)^2$ $\quad v = x$

$\frac{du}{dx} = 2 \times \frac{1}{x} \ln x$ $\quad \frac{dv}{dx} = 1$

$\left[x (\ln x)^2 \right] - \int 2 \ln x \, dx$

$u = \ln x$ $\quad v = 2x$

$\frac{du}{dx} = \frac{1}{x}$ $\quad \frac{dv}{dx} = 2$

$\left[x (\ln x)^2 \right] - \left[2x \ln x - \int 2 \, dx \right]$

$\left[x (\ln x)^2 - 2x \ln x + 2x \right]_2^4$

$\left(4 (\ln 4)^2 - 8 \ln 4 + 8 \right)$

$- \left(2 (\ln 2)^2 - 4 \ln 2 + 4 \right)$

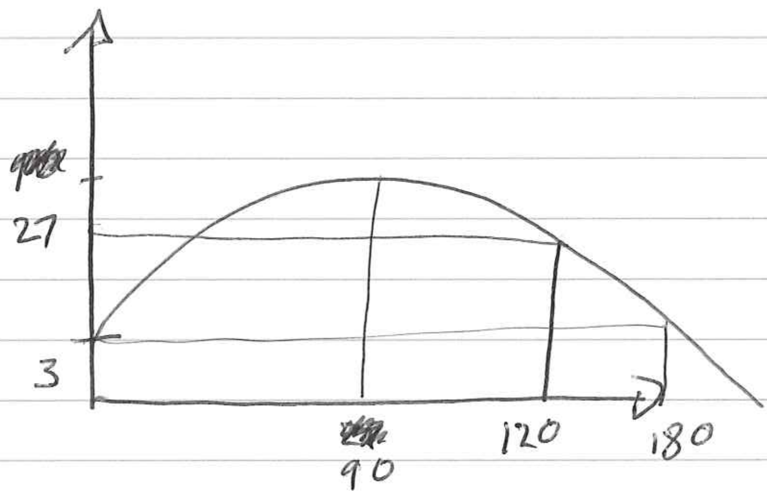
$\boxed{\text{but } (\ln 4)^2 = (\ln 2^2)^2 = (2 \ln 2)^2 = 4 (\ln 2)^2}$

$\Rightarrow 4 \times 4 (\ln 2)^2 - 16 \ln 2 + 8$

$- 2 (\ln 2)^2 + 4 \ln 2 - 4$

$= 14 (\ln 2)^2 - 12 \ln 2 + 4$

12 (a)



$$H = ax^2 + bx + c$$

$x=0 \quad H=3 \quad c=3$

$x=120 \quad H=27 \quad ① \quad 27 = a(120)^2 + b(120) + 3$

$x=180 \quad H=3 \quad ② \quad 3 = a(180)^2 + b(180) + 3$

① $27 = 14400a + 120b + 3$
$24 = 14400a + 120b \Rightarrow b = 1/5 - 120a$

② $3 = 32400a + 180b + 3$
$b = -180a$

$-180a = 1/5 - 120a \Rightarrow 60a = -1/5 \quad a = -1/300$

$b = 3/5$

$$H = \frac{-x^2}{300} + \frac{3x}{5} + 3$$

12 (b) (i) $\frac{dH}{dx} = \frac{-x}{150} + \frac{3}{5}$

if $\frac{dH}{dx} = 0$ $\quad x = 90$

when $x = 90$ $\quad H = \underline{30}$

(ii) if $H = 0$

$$0 = \frac{-x^2}{300} + \frac{3x}{5} + 3$$

$x = 184.8683298 \Rightarrow \underline{185m}$

$x = -4.868329805$ not appropriate

(c) The size and shape of the object may distort its flight.

13. $(x-3)^2+y^2=4$

$$\left(\frac{t^2+5}{t^2+1}-3\right)^2+\left(\frac{4t}{t^2+1}\right)^2=4$$

$$\frac{\left(t^2+5-3(t^2+1)\right)^2}{(t^2+1)^2}+\frac{(4t)^2}{(t^2+1)^2}$$

$$\frac{(-2t^2+2)^2}{(t^2+1)^2}+\frac{16t^2}{(t^2+1)^2}=$$

$$\frac{4t^4-8t^2+4+16t^2}{(t^2+1)^2}=$$

$$\frac{4t^4+8t^2+4}{(t^2+1)^2}$$

$$\frac{4(t^4+2t^2+1)}{(t^2+1)^2}$$

$$\frac{4(t^2+1)^2}{(t^2+1)^2}$$

$$\underline{\underline{4}}$$

(14) $y = \dfrac{x-4}{2+x^{1/2}}$

$\dfrac{f(x)}{g(x)}$ $\qquad \dfrac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$

$f(x) = x - 4 \qquad f'(x) = 1$

$g(x) = 2 + x^{1/2} \qquad g'(x) = \frac{1}{2} x^{-1/2}$

$$\frac{dy}{dx} = \frac{1\left(2+x^{1/2}\right) - (x-4)\ \frac{1}{2} x^{-1/2}}{(2+x^{1/2})^2}$$

$$= \frac{2 + x^{1/2} - \frac{1}{2}x^{1/2} + 2x^{-1/2}}{(2+x^{1/2})(2+x^{1/2})}$$

$$= \frac{2 + \frac{1}{2}x^{1/2} + 2x^{-1/2}}{4 + 4x^{1/2} + x}$$

$$= \frac{\frac{1}{2}\ x^{-1/2}\left(x + 4x^{1/2} + 4\right)}{x + 4x^{1/2} + 4}$$

$$= \frac{1}{2\sqrt{x}}$$

(15) (i) $n \leq 4 \quad n \in N$

$n = 1 \quad 2^3 > 3^1$ ✓

$2 \quad 3^3 > 3^2$ ✓

$3 \quad 4^3 > 3^3$ ✓

$4 \quad 5^3 > 3^4$ ✓

So if $n \leq 4$, $n \in \mathbb{N}$ then $(n+1)^3 > 3^n$

(ii) Assume m is odd

so let $m = 2p + 1$

where $p \in \mathbb{N}$

$m^3 + 5 \qquad (2p+1)(2p+1) = 4p^2 + 4p + 1$

$(2p+1)(4p^2 + 4p + 1)$

$m^3 + 5 \Rightarrow 8p^3 + 8p^2 + 2p$
$\qquad + 4p^2 + 4p + 1 \quad + 5$

$m^3 + 5 \Rightarrow 8p^3 + 12p^2 + 6p + 1 \quad + 5$

$8p^3 + 12p^2 + 6p + 6$

$2(4p^3 + 6p^2 + 3p + 3)$

So if m is odd m^3+5 is even, but $m^3 + 5$ is odd, so there is a contradiction $\therefore$ m must be even.