Name:

Exam Style Questions

Algebraic Proof

Corbettmaths

Ensure you have: Pencil, pen, ruler, protractor, pair of compasses and eraser

You may use tracing paper if needed

Guidance

1. Read each question carefully before you begin answering it.
2. Don't spend too long on one question.
3. Attempt every question.
4. Check your answers seem right.
5. Always show your workings

Revision for this topic

www.corbettmaths.com/contents

Video 365

1. Prove that the sum of three consecutive integers is divisible by 3.

three consecutive integers; n

$n+1$

$n+2$

sum of;

$n + (n+1) + (n+2)$

$= 3n + 3$

$= 3(n+1)$

$\therefore$ divisible by 3.

(3)

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2. Prove $(n + 6)^2 - (n + 2)^2$ is always a multiple of 8

$(n+6)(n+6)$

$n^2 + 12n + 36$

$(n+2)(n+2)$

$n^2 + 4n + 4$

$= n^2 + 12n + 36 - n^2 - 4n - 4$

$= 8n + 32$

$= 8(n+4)$

$\therefore$ divisible by 8 and is therefore a multiple of 8

(4)

3. Prove $(n + 10)^2 - (n + 5)^2$ is always a multiple of 5

$(n+10)(n+10) - (n+5)(n+5)$

$= n^2+20n+100 - (n^2+10n+25)$

$= n^2+20n+100-n^2-10n-25$

$= 10n+75$

$= 5(2n+15)$

$\therefore$ a multiple of 5

(4)

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4. Prove the sum of two consecutive odd numbers is even.

two consecutive odd numbers; $2n+1$

$2n+3$

Sum of:

$= 2n+1+2n+3$

$= 4n+4$

$= 4(n+1)$

$\therefore$ an even number.

(3)

5. $(2n + 1)(3n - 2) - (6n - 1)(n - 2)$ is always even

$6n^2 + 3n - 4n - 2 - (6n^2 - n - 12n + 2)$

$= 6n^2 - n - 2 - 6n^2 + 13n - 2$

$= 12n - 4$

$= 4(3n - 1)$

$\therefore$ an even outcome.

(3)

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6. Prove that the sum of three consecutive even numbers is always a multiple of 6

three consecutive even numbers; $2n$
$2n+2$
$2n+4$

Sum of:

$= 2n + 2n+2 + 2n+4$

$= 6n+6$

$= 6(n+1)$

$\therefore$ a multiple of 6.

(3)

7. Prove the sum of four consecutive odd numbers is always a multiple of 8

four consecutive odd numbers; $2n+1$

$2n+3$

$2n+5$

$2n+7$

Sum of:

$2n+1+2n+3+2n+5+2n+7$

$= 8n+16$

$= 8(n+2)$

$\therefore$ a multiple of 8.

(4)

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8. Prove $(2n + 9)^2 - (2n + 5)^2$ is always a multiple of 4

$(2n+9)(2n+9) - (2n+5)(2n+5)$

$= 4n^2 + 36n + 81 - (4n^2 + 20n + 25)$

$= 4n^2 + 36n + 81 - 4n^2 - 20n - 25$

$= 16n + 56$

$= 4(4n + 14)$

$\therefore$ a multiple of 4

(4)

9. Prove $(n+1)^2 + (n+3)^2 - (n+5)^2 = (n+3)(n-5)$

$$(n+1)(n+1) + (n+3)(n+3) - (n+5)(n+5)$$

$$= n^2 + 2n + 1 + (n^2 + 6n + 9) - (n^2 + 10n + 25)$$

$$= n^2 + 2n + 1 + n^2 + 6n + 9 - n^2 - 10n - 25$$

$$= n^2 - 2n - 15$$

$$= (n+3)(n-5)$$

(4)

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10. Prove the product of two even numbers is always even

Even numbers: $2n$

$2m$

product: $2n \times 2m = 4mn$

$2(2mn)$

$\therefore$ always even.

(3)

11. Prove the product of three consecutive odd numbers is odd

Three consecutive odd numbers: $2n+1$

$2n+3$

$2n+5$

product: $(2n+1)(2n+3)(2n+5)$

$8n^3 + 36n^2 + 46n + 15$

$\underbrace{2(4n^3 + 18n^2 + 23n)}_{\text{Even}} + 15$

Even + odd = odd

(3)

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12. Prove algebraically that the sum of the squares of two odd integers is always even.

Odd integers: $2n+1$

$2n+3$

$(2n+1)^2 + (2n+3)^3$

$= 4n^2 + 4n + 1 + 4n^2 + 12n + 9$

$= 8n^2 + 16n + 10$

$= 2(4n^2 + 8n + 5)$

$\therefore$ always even.

(4)

13. Prove that when two consecutive integers are squared, that the difference is equal to the sum of the two consecutive integers.

two consecutive integers: n

$n+1$

Sum of: $n + n+1$

$= 2n+1$

Difference of squares:

$= (n+1)^2 - n^2$

$= (n+1)(n+1) - n^2$

$= n^2 + 2n + 1 - n^2$

$= 2n+1$

$\therefore$ equal to the sum of.

(4)

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14. Prove algebraically that

$(4n + 1)^2 - (2n - 1)$ is an even number

for all positive integer values of n.

$(4n+1)(4n+1) - (2n-1)$

$= 16n^2 + 8n + 1 - 2n + 1$

$= 16n^2 + 6n + 2$

$= 2(8n^2 + 3n + 1)$

$\therefore$ even outcome

(4)

15. Prove that $3n(3n + 4) + (n - 6)^2$ is positive for all values of ~~x~~ n

$9n^2 + 12n + (n^2 - 6n - 6n + 36)$

$10n^2 + 36$

as $n > 0$

$n^2 > 0$

$10n^2 > 0$

$10n^2 + 36 > 0$

$\therefore$ $10n^2 + 36$ is always positive.

(4)

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16. The first five terms of a linear sequence are 5, 11, 17, 23, 29 …

(a) Find the nth term of the sequence

$6n - 1$

(2)

A new sequence is generated by squaring each term of the linear sequence and then adding 5.

(b) Prove that all terms in the new sequence are divisible by 6.

$(6n - 1)^2 + 5$

$(36n^2 - 6n - 6n + 1) + 5$

$36n^2 - 12n + 6$

$6(6n^2 - 2n + 1)$ $\therefore$ divisible by 6.

(4)

17. Prove that the product of two consecutive even numbers is a multiple of 4.

Two consecutive even numbers: $2n$

$2n+2$

product: $2n(2n+2)$

$4n^2+4n$

$4n(n+1)$

$\therefore$ a multiple of 4

(3)

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18. Prove that when any odd integer is squared, the result is always one more than a multiple of 8.

odd integer: $2n+1$

odd integer squared: $(2n+1)(2n+1)$

$= 4n^2+4n+1$

$= 4n(n+1)+1$

Since either n or $n+1$ is even, $n(n+1)$ is even.

4 times even is always a multiple of 8.

$\therefore$ $4n(n+1)$ is a multiple of 8

$\therefore$ $4n(n+1)+1$ is one more than a multiple of 8

(4)

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19. Prove that the product of two odd numbers is always odd.

odd numbers: $2K+1$

$2m+1$

product: $(2K+1)(2m+1)$

$4Km+2K+2m+1$

$\underbrace{2(2Km+K+m)}_{\text{Even}} + \underset{\text{odd}}{1} = \text{odd}.$

(3)