**9FM0/3D: Decision Mathematics 01 Mark scheme**

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **1(a)** | 6 1 9 14 18 7 10 4 17 13  9 14 18 10 17 13 7 6 1 4 | M1 | 1.1b |
|  | 14 18 17 13 10 9 7 6 4 1  18 17 14 13 10 9 7 6 4 1 | A1 | 1.1b |
|  | 18 17 14 13 10 9 7 6 4 1 | A1 | 1.1b |
|  |  | **(3)** |  |
| **(b)** | Bin 1: 18 10 1  Bin 2: 17 13  Bin 3: 14 9 7  Bin 4: 6 4 | M1  A1 | 1.1b  1.1b |
|  |  | **(2)** |  |
| **(5 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **M1:** quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot).  **A1:** first two passes correct and correct pivots chosen for third pass  **A1:** cso (correct solution only – all previous marks in this part must have been awarded) – must include a fourth pass  **(b)**  **M1:** must be using ‘sorted’ list in descending order. First five items placed correctly and at least eight values placed in bins  **A1:** cso (so no additional/repeated values) | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **2(a)** | 7 | B1 | 2.2a |
|  |  | **(1)** |  |
| **(b)** | A semi-Eulerian graph requires exactly two odd nodes… | B1 | 1.2 |
|  | …the graph has six odd nodes so only two arcs needs to be added to make the graph semi-Eulerian | B1 | 2.2a |
|  |  | **(2)** |  |
| **(c)** | Creates two lists of arcs | M1 | 2.1 |
|  | e.g. AB BF BE CE  EF EG BG BD | A1 | 1.1b |
|  | Since no arc appears in both lists, the graph is planar (or draws a planar version) | A1 | 2.4 |
|  |  | **(3)** |  |
| **(6 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **B1:** cao  **(b)**  **B1:** accurately recalls the fact that a semi-Eulerian graph contains exactly two odd nodes  **B1:** dependent on previous B mark – cao  **(c)**  **M1:** creates two list of arcs (with at least three arcs in each list) which contain no common arcs  **A1:** cao  **A1:** correct reasoning that no arc appears in both lists + so the graph is therefore planar | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **3(i)(a)** | 22 | M1  A1  A1 | 1.1b  1.1b  1.1b |
|  | Length of quickest route from A to H is 47 minutes | A1ft | 2.2a |
|  |  | **(4)** |  |
| **(b)** | Shortest path from A to F via H: ABGEHF | B1 | 1.1b |
|  | Length: 47 + 12 = 59 minutes | B1ft | 2.2a |
|  |  | **(2)** |  |
| **(c)** | e.g. add 1 to each arc | M1 | 3.5c |
|  | except AB, AD, AC (or EH, GH, FH) | A1 | 2.3 |
|  |  | **(2)** |  |
| **(ii)(a)** | AB + EH = 13 + 10 = 23\*  A(BG)E + B(GE)H = 37 + 34 = 71  A(BGE)H + B(G)E = 47 + 24 = 71 | M1  A1ft  A1 | 2.1  1.1b  1.1b |
|  | Length of the shortest route is 300 + 23 = 323 km | A1ft | 2.2a |
|  |  | **(4)** |  |
| **(b)** | Repeat arcs: AB, EH | B1 | 2.2a |
|  |  | **(1)** |  |
| **(13 marks)** | | | |
| **Notes:**  **(i) (a)**  **M1:** for a larger number replaced by a smaller one in the working values boxes at C, D, E, F or H  **A1:** for all values correct (and in correct order) at A, B, G and C  **A1:** for all values correct (and in correct order) at D, E, F and H  **A1ft:** for 47 or ft their final value at H  **(b)**  **B1:** cao  **B1ft:** for 59 or ft their final at H + 12  **(c)**  **M1:** valid general method – any mention of adding 1 to the weight of the arcs  **A1:** cao – so adding 1 to each arc except {AB, AD, AC} or {EH, GH, FH}  **(ii)(a)**  **M1:** correct three pairings of the required four odd nodes  **A1ft:** at least two pairings and totals correct (ft their values from (a))  **A1:** all three pairings and totals correct  **A1ft:** for 323 or 300 + their shortest repeat  **(b)**  **B1:** selecting the shortest pairing, and stating that these arcs should be repeated | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **4(a)**  **(i)** | G(10) | M1  A1  A1 | 2.1  1.1b  1.1b |
| **(ii)** | Minimum completion time is 85 minutes | A1ft | 2.2a |
| **(iii)** | Critical activities are A, E and I | A1ft | 2.2a |
|  |  | **(5)** |  |
| **(b)** | e.g. | M1  A1  A1 | 1.1b  1.1b  1.1b |
|  |  | **(3)** |  |
| **(c)** | Currently five workers are required between time 20 and 40 and so activities F and H would have to be delayed  If F starts at 35 H could not begin until 55 but the latest start time for H is 40. Therefore the project cannot be completed in the minimum time with only four workers | M1  A1 | 2.4  2.2a |
|  |  | **(2)** |  |
| **(10 marks)** | | | |
| **Notes:** | | | |
| **(a)(i)**  **M1:** All boxes completed, number generally increasing L to R (condone one “rogue”) and decreasing R to L (condone one “rogue”)  **A1:** Cao -Top boxes  **A1:** Cao - Bottom boxes  **(ii)**  **A1ft:** Deduction that result in diagram indicates that project can be completed in 85 minutes  **(iii)**  **A1ft:** Deduction of correct critical activities (from their values at each event)  **(b)**  **M1:** Plausible histogram with no holes or overhangs (must go to at least 70 on the time axis)  **A1:** Histogram correct to time 40  **A1:** Histogram correct from time 40 to time 85  **(c)**  **M1:** Explanation involving the need to delay activities F and H  **A1:** Correct deduction that it is not possible to complete the project with only four workers in the minimum project completion time | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **5(a)** | Maximise | B1 | 2.5 |
|  | Subject to | M1  A1  B1 | 3.3  1.1b  3.3 |
|  |  | **(4)** |  |
| **(b)** | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | b.v. |  |  |  |  |  |  | Value | |  | 3 | 5 | 8 | 1 | 0 | 0 | 400 | |  | 3 | 6 | 10 | 0 | 1 | 0 | 350 | |  | 1 | 1.5 | 1.25 | 0 | 0 | 1 | 75 | |  |  |  |  | 0 | 0 | 0 | 0 | | M1  A1 | 3.4  1.1b |
|  |  | **(2)** |  |
| **(c)** | |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | | b.v. |  |  |  |  |  |  | Value | Row Ops | |  | 0.6 | 0.2 | 0 | 1 |  | 0 | 120 |  | |  | 0.3 | 0.6 | 1 | 0 | 0.1 | 0 | 35 |  | |  | 0.625 | 0.75 | 0 | 0 |  | 1 | 31.25 |  | |  |  |  | 0 | 0 | 6.5 | 0 | 2275 |  | | M1  A1ft  A1  B1ft | 2.1  1.1b  1.1b  2.4 |
|  |  | **(4)** |  |
| **(d)** | so increasing  will decrease profit | B1 | 2.4 |
|  |  | **(1)** |  |
| **(e)** | (i) Make 50 lectern desks, 20 writing desks and no roll top desks | B1 | 3.2a |
|  | (ii) £3300 | B1 | 1.1b |
|  |  | **(2)** |  |
| **(f)** | The 90 is the value of the slack variable  which comes from the constraint | B1 | 2.4 |
|  | Indicating that there is 90 m2 of wood still available | B1 | 3.2a |
|  |  | **(2)** |  |

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| **(g)** | e.g. there is no guarantee that all the desks will be sold | B1 | 3.5b |
|  |  | **(1)** |  |
| **(16 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **B1:** Correct objective function/expression (accept in pence rather than pounds e.g. 4000*x* + 5000*y* + 6500*z*) together with ‘maximise’  **M1:** Correct coefficients and correct right-hand side for at least one inequality – accept any inequality or equals  **A1:** All three correct (non-trivial) inequalities  **B1:**  **(b)**  **M1:** Constructing all four rows including slack variables with at least one negative in *P* row (allow sign/numerical slips)  **A1:** All four rows correct  **(c)**  **M1:** Correct pivot located, attempt to divide row  **A1ft:** Pivot row correct (including change of b.v.) and row operations used at least once, one of columns *x*, *y*,  or Value correct  **A1:** Cao for values (ignore b.v. column and Row Ops)  **B1ft:** The correct Row Operations (on the ft) explained either in terms of the ‘old’ or ‘new’ pivot rows  **(d)**  **B1:** States correct objective function and mention of increasing  will decrease profit  **(e)(i)**  **B1:** Cao – in context so not in terms of *x*, *y* and *z*  **(ii) B1:** Cao  **(f)**  **B1:** Recognises that  and is linked to the wood constraint  **B1:** Evaluates this value in context (so must see both units and mention of ‘wood’)  **(g)**  **B1:** Cao – any suitable limitation to the solution in context | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **6(a)** | Distance table Route table   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | **A** | **B** | **C** | **D** | **E** | | **A** | - | 3 | 7 | 9 |  | | **B** | 3 | - | 2 |  | 8 | | **C** | 7 | 2 | - | 6 | 5 | | **D** | 9 |  | 6 | - | 4 | | **E** |  | 8 | 5 | 4 | - |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | **A** | **B** | **C** | **D** | **E** | | **A** | A | B | C | D | E | | **B** | A | B | C | D | E | | **C** | A | B | C | D | E | | **D** | A | B | C | D | E | | **E** | A | B | C | D | E | | B1  B1 | 1.1b  1.1b |
|  |  | **(2)** |  |
| **(b)** | 1st iteration:  Distance table Route table   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | **A** | **B** | **C** | **D** | **E** | | **A** | - | 3 | 7 | 9 |  | | **B** | 3 | - | 2 | 12 | 8 | | **C** | 7 | 2 | - | 6 | 5 | | **D** | 9 | 12 | 6 | - | 4 | | **E** |  | 8 | 5 | 4 | - |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | **A** | **B** | **C** | **D** | **E** | | **A** | A | B | C | D | E | | **B** | A | A | C | A | E | | **C** | A | B | C | D | E | | **D** | A | A | C | D | E | | **E** | A | B | C | D | E | | M1  A1 | 1.1b  1.1b |
|  | 2nd iteration:  Distance table Route table   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | **A** | **B** | **C** | **D** | **E** | | **A** | - | 3 | 5 | 9 | 11 | | **B** | 3 | - | 2 | 12 | 8 | | **C** | 5 | 2 | - | 6 | 5 | | **D** | 9 | 12 | 6 | - | 4 | | **E** | 11 | 8 | 5 | 4 | - |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | **A** | **B** | **C** | **D** | **E** | | **A** | A | B | B | D | B | | **B** | A | B | C | A | E | | **C** | B | B | C | D | E | | **D** | A | A | C | D | E | | **E** | B | B | C | D | E | | M1  A1ft | 1.1b  1.1b |
|  |  | **(4)** |  |
| **(c)** |  | M1  A1 | 1.1b  1.1b |
|  |  | **(2)** |  |
| **(d)(i)** | NNA: E – D – C – B – A – E | B1 | 1.1b |
| **(ii)** | 4 + 6 + 2 + 3 + 10 = 25 km | B1 | 1.1b |
|  |  | **(2)** |  |
| **(e)** | E – D – C – B – A – B – C – E | B1 | 3.2a |
|  |  | **(1)** |  |
| **(f)** | Prim’s algorithm on reduced network starting at B: BC, CE, DE | B1 | 1.1b |
|  | Lower bound = 11 + 3 + 5 = 19 km | B1ft | 2.2a |
|  |  | **(2)** |  |
| **(g)** |  | M1 | 2.2b |
|  |  | A1 | 1.1b |
|  |  | **(2)** |  |
| **(15 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **B1:** Correct distance table  **B1:** Correct route table  **(b)**  **M1:** No change in the first row and first column of both tables with at least one value in the distance table reduced and one value in the route table changed  **A1:** cao  **M1:** No change in the second row and second column of both tables with at least two values in the distance table reduced and two values in the route table changed  **A1ft:** Correct second iteration follow through from the candidate’s first iteration  **(c)**  **M1:** K5 drawn with at least one shortest distance from the final distance table present  **A1:** cao  **(d)(i)**  **B1:** cao  **(ii)**  **B1:** cao  **(e)**  **B1:** cao  **(f)**  **B1:** correct RMST starting at any node (except A)  **B1ft:** length of their RMST + 3 + 5  **(g)**  **M1:** Their numbers correctly used, accept any inequalities or any indication of interval from their 19 to their 25 (so 19 – 25 can score this mark). Please note that UB > LB for this mark  **A1:** cao (no follow through on their values) including correct inequalities or equivalent set notation (but condone 19 optimal  25 ) | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **7** | Objective line | B1 | 3.4 |
|  |  | B1 | 3.4 |
|  | Line through (0, 12) and (8, 0) is | M1 | 1.1b |
|  | Line through (5, 0) and (10, 10) is | M1 | 1.1b |
|  |  | M1  A1ft  A1 | 2.1  1.1b  1.1b |
|  |  | M1 | 2.2a |
|  | e.g.   |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | b.v. |  |  |  |  |  |  |  |  | Value | |  | 2 |  | 1 | 0 | 0 | 0 | 0 | 0 | 10 | |  | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 10 | |  | 1 | 0 | 0 | 0 |  | 0 | 1 | 0 | 4 | |  | 3 | 2 | 0 | 0 | 0 |  | 0 | 1 | 24 | |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | *A* |  |  | 0 | 0 | 1 | 1 | 0 | 0 |  | | M1  A1 | 2.1  2.2a |
| **(10 marks)** | | | |
| **Notes:** | | | |
| **B1:** cao for objective function (oe e.g. *P* – 3*x* – 4*y* = *k*)  **B1:** cao  **M1:** correct method for finding the equation of the line through (0, 12) and (8, 0)  **M1:** correct method for finding the equation of the line through (5, 0) and (10, 10)  **M1:** translate all 4 inequalities into equations **–** must include all three types of variables (slack, surplus and artificial)  **A1ft:** two correct equations following their inequalities  **A1:** all four correct equations  **M1:** setting up the new objective and substituting for  and  **M1:** setting up tableau – all six lines with four basic variables  **A1:** cao (oe e.g. consistent *P* line with their objective equation) | | | |