Jan 2010 C2

1 i) $2\sin^2 x = 5\cos x - 1$

$\sin^2 x = 1 - \cos^2 x$

$2(1-\cos^2 x) = 5\cos x - 1$

$2 - 2\cos^2 x = 5\cos x - 1$

$0 = 2\cos^2 x + 5\cos x - 3$

ii) let $\cos x = q$

$2\cos^2 x + 5\cos x - 3 = 0$

$2q^2 + 5q - 3 = 0$

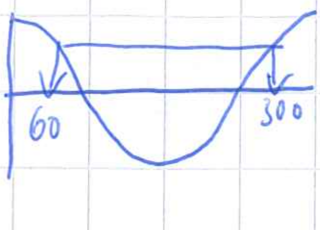
$(2q - 1)(q + 3) = 0$

$2\cos x - 1 = 0$ $\quad\quad$ $\cos x + 3 = 0$

$\cos x = \frac{1}{2}$ $\quad\quad$ $\cos x = -3$

$x = \cos^{-1}\left(\frac{1}{2}\right)$

$x = 60°, 300°$



② $\frac{dy}{dx} = 6x - 4$ $(2, 5)$ $(p, 5)$

i) $y = 3x^2 - 4x + c$

@ $(2, 5)$ ~~$5 = 78 - 20 + c$~~ ~~$c = -1500$~~ $5 = 12 - 8 + c$

$c = 1$

$y = 3x^2 - 4x + 1$

ii) @ $(p, 5)$ $5 = 3p^2 - 4p + 1$

$3p^2 - 4p - 4 = 0$

$(3p + 2)(p - 2) = 0$

$3p = -2$ $\underline{\underline{p = 2}}$ (already given)

$\underline{\underline{p = -2/3}}$

③ i) $(2-x)^7 = {}^7C_0(2)^7 + {}^7C_1(2)^6(-x) + {}^7C_2(2)^5(-x)^2 + {}^7C_3(2)^4(-x)^3$
$+ {}^7C_4(2)^3(-x)^4 + {}^7C_5(2)^2(-x)^5 +$
${}^7C_6(2)(-x)^6 + {}^7C_7(-x)^7$

$= 128 - 448x + 672x^2 - 560x^3 [+ 280x^4 - 84x^5$
$+ 14x^6 - x^7]$ only first 4 needed.

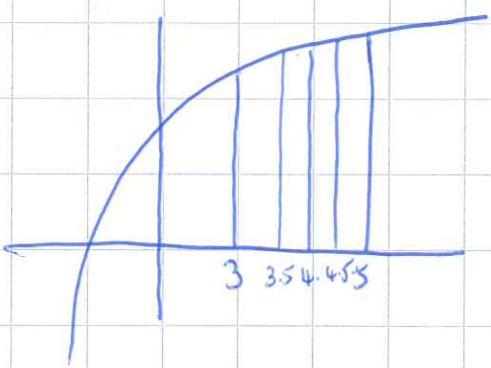
ii) Use the x^3 coefficient to help.

$-560 \times \left(\frac{1}{4}w^2\right)^3$ * negative already considered in $\nearrow -560$

$= -560 \times \frac{1}{64}w^6$

$= -\frac{35}{4}w^6$ Hence $-35/4$ is ~~3~~ w^6 coefficient.

(4) $\int_3^5 \log_{10}(2+x)\,dx$



i) Area $\approx \frac{1}{2} \times \frac{1}{2}\left(\log_{10}(5)+\log_{10}(7)+2\left(\log_{10}(5.5)+\log_{10}(6)+\log_{10}(6.5)\right)\right)$

$= \frac{1}{4}\left(6.206922\ldots\right)$

$= 1.551730659$ ©

$= \underline{1.55}$ (to 3sF).

ii) $\int_3^5 \log_{10}\sqrt{2+x} = \int_3^5 \log_{10}(2+x)^{1/2}$

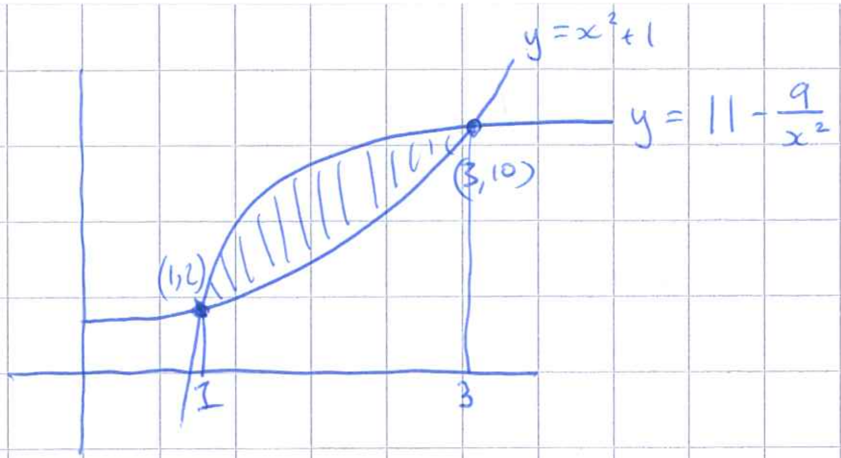
$= \frac{1}{2}\int_3^5 \log_{10}(2+x)$

$= \frac{1}{2} \times 1.55$

$= 0.77586532$[illegible]$\ldots$

$= 0.776$ (3sF)

⑤



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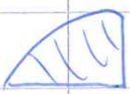
 $= \int_1^3 11 - 9x^{-2}\,dx = \left[11x + 9x^{-1}\right]_1^3$

$= \left(11(3) + 9(3)^{-1}\right) - \left(11(1) + 9(1)^{-1}\right)$

$= 36 - 20$

$= 16$

 $= \int_1^3 x^2 + 1\,dx = \left[\frac{1}{3}x^3 + x\right]_1^3$

$= \left(\frac{1}{3}(3)^3 + 3\right) - \left(\frac{1}{3}(1)^3 + 1\right)$

$= 12 - 1\frac{1}{3}$

$= 10\frac{2}{3}$

$\therefore$ Shaded area $= 16 - 10\frac{2}{3}$

$= \underline{\underline{5\frac{1}{3}}}$

6 $f(x) = 2x^3 + ax^2 + bx + 15$

$(x+3)$ is a factor $\therefore$ $f(-3) = 0$

$(x-2)$ remainder is 35 $\therefore$ $f(2) = 35$

$f(-3) = 2(-3)^3 + a(-3)^2 + b(-3) + 15$

$0 = -54 + 9a - 3b + 15$

$39 = 9a - 3b$

$13 = 3a - b$

$f(2) = 16 + 4a + 2b + 15$

$35 = 31 + 4a + 2b$

$4 = 4a + 2b$

$2 = 2a + b$

$$\begin{array}{l} 13 = 3a - b \\ \underline{2 = 2a + b} \quad + \\ 15 = 5a \\ a = 3 \end{array}$$

$2 = 2(3) + b$

$b = -4$

$f(x) = 2x^3 + 3x^2 - 4x + 15$

$(x+3)(2x^2 + bx + 5)$

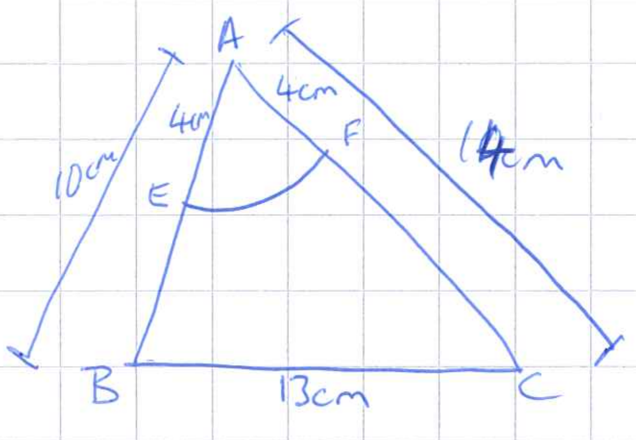
$$\begin{array}{r} 6x^2 \\ \underline{+bx^2} \\ (6+b)x^2 \end{array}$$

$6 + b = 3$

$b = -3$

$\therefore (x+3)(2x^2 - 3x + 5)$

(7)



i)

<u>Cosine Rule</u>

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$A = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right)$$

$$= \cos^{-1}\left(\frac{13^2 - 10^2 - 14^2}{-2(10)(14)}\right) \qquad = \cos^{-1}(0.4535\ldots)$$

$$A = 1.100027\ldots$$

$$\underline{A = 1.1^c}$$

ii) Arc length $= r\theta$

$= 4 \times 1.1^c$

$= 4.4$

Perimeter $= 4.4 + 6 + 10 + 13$

$\underline{= 33.4\text{cm}}$

iii) Area of sector $= \frac{\theta}{2} r^2$

$= \frac{1.1}{2} \times 4^2$

Area of shaded region $= \frac{1}{2}ab\sin C - \frac{\theta}{2}r^2$

$= \frac{1}{2}(10)(14)\sin 1.1 - \left(\frac{1.1}{2} \times 4^2\right)$

$= 53.584515\ldots$

$= \underline{53.6\text{cm}^2}$

(8) i) $U_1 = 8$ $U_{n+1} = U_n + 3$

$U_2 = \cancel{12}\ 11$

$U_3 = \cancel{18}\ 14$

$U_4 = \cancel{18}\ 17$

$U_5 = 20$

ii) $U_n = pn + q$ $p = 3$ $q = 5$

iii) Arithmetic sequence (progression)

iv) $\sum_{n=1}^{2N} U_n - \sum_{n=1}^{N} U_n = 1256$

$S_{2N} = N\left(16 + (2N-1)3\right) = N(13 + 6N)$

$S_N = \frac{N}{2}\left(16 + (N-1)3\right) = \frac{N}{2}(13 + 3N)$

$1256 = 13N + 6N^2 - \left(\frac{13N}{2} + \frac{3N^2}{2}\right)$

$2512 = 26N + 12N^2 - 13N + 3N^2$

$2512 = 9N^2 + 13N$

$9N^2 + 13N - 2512 = 0$

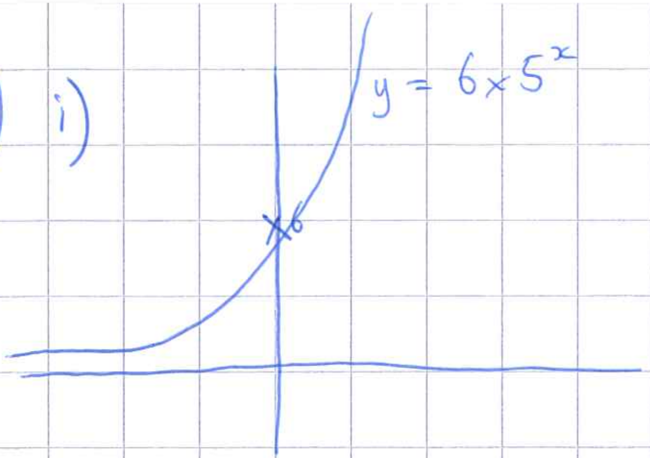
$N = \frac{-13 \pm \sqrt{13^2 - 4(9)(-2512)}}{2 \times 9}$

$\sqrt{90601} = 301$

$= \frac{-13 \pm \sqrt{90601}}{18}$

$\underline{N = 16}$ ~~and~~ (one solution as $N > 0$)

9 i)



ii) $y = 9^x$ $\quad P(x, 150)$

$150 = 9^x$

$\log 150 = x \log 9$

$x = \dfrac{\log 150}{\log 9}$

$x = 2.28$ (3sf)

iii) $y = 9^x \quad y = 6 \times 5^x$

$9^x = 6 \times 5^x$

$x \log_3 9 = \log_3 (6 \times 5^x)$

$2x = \log_3 6 + x \log_3 5$

$\cancel{2x - x\log_3 5 =}$

$2x = \log_3 3 + \log_3 2 + x \log_3 5$

$2x - x\log_3 5 = 1 + \log_3 2$

$x(2 - \log_3 5) = 1 + \log_3 2$

$x = \dfrac{1 + \log_3 2}{2 - \log_3 5}$