

Please write clearly in block capitals.

Centre number

Candidate number

Surname _____

Forename(s) MISS FORREST WORKED SOLUTIONS

Candidate signature _____

GCSE MATHEMATICS

H

Higher Tier Paper 2 Calculator

Monday 6 November 2017

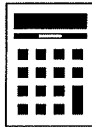
Morning

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

- In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2-3	
4-5	
6-7	
8-9	
10-11	
12-13	
14-15	
16-17	
18-19	
20-21	
22-23	
24-25	
26-27	
28-29	
TOTAL	



N 0 V 1 7 8 3 0 0 2 H 0 1

Answer **all** questions in the spaces provided

- 1 Circle the fraction that is equivalent to 3.875

[1 mark]

$$\frac{15}{4} \quad \frac{29}{8} \quad \frac{31}{8} \quad \frac{15}{8}$$

$$3.75 \quad 3.625 \quad 3.875$$

- 2 What is 50 as a percentage of 20?
-
- Circle your answer.

[1 mark]

$$\frac{50}{20} \times 100 \quad 10\% \quad 40\% \quad 150\% \quad 250\%$$

- 3 Circle the point that does
- not**
- lie on the curve
- $y = x^3$

[1 mark]

$$\left(-\frac{1}{2}, -\frac{1}{8}\right) \quad (5, 125) \quad \left(\frac{1}{3}, \frac{1}{9}\right) \quad (-1, -1)$$

$$-\frac{1}{8} = \left(-\frac{1}{2}\right)^3 \quad 125 = 5^3 \quad \frac{1}{27} = \left(\frac{1}{3}\right)^3 \quad -1 = (-1)^3$$

$$-1 \times -1 \times -1$$



- 4 Which **one** of these is a unit of density?
Circle your answer.

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

[1 mark]

kg/m^2

m^2/kg

kg/m^3

m^3/kg

- 5 Solve $4(3x - 2) = 2x - 5$

[3 marks]

$$12x - 8 = 2x - 5$$

$$-2x \quad -2x$$

$$10x - 8 = -5$$

$$+8 \quad +8$$

$$10x = 3$$

$$x = \frac{3}{10}$$

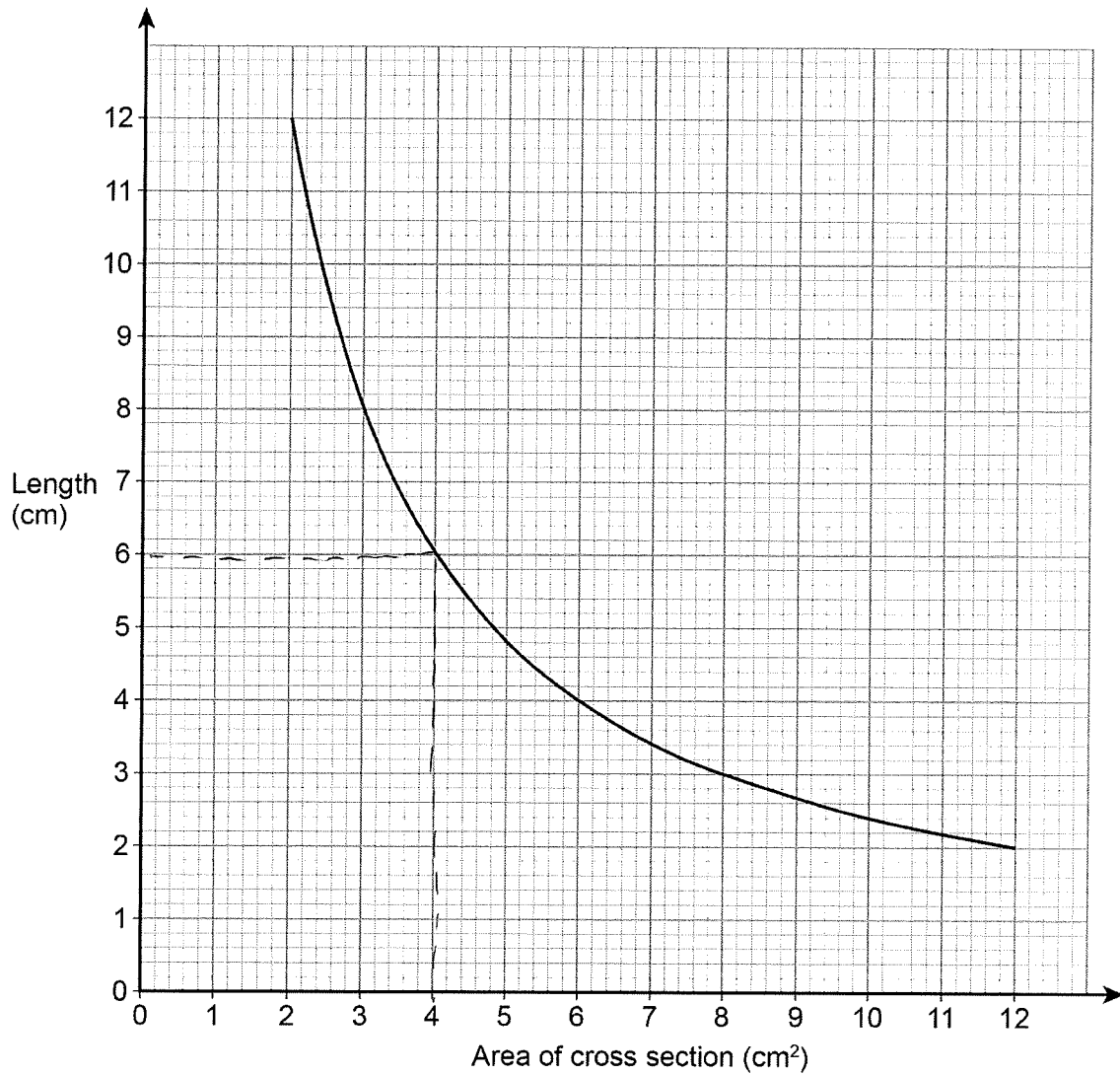
Turn over for the next question

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6

The graph shows information about prisms with the same volume.

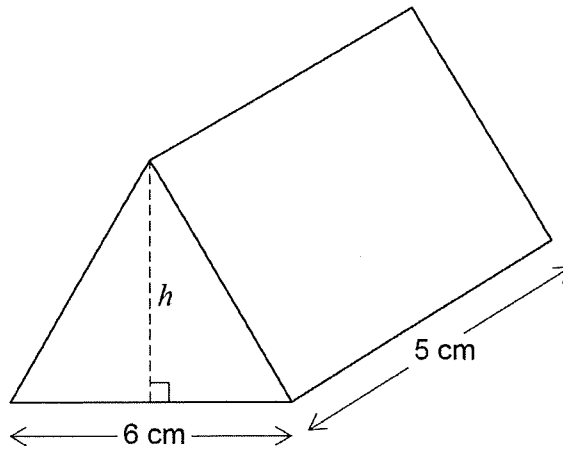
6 (a) Give one example to show the volume is 24 cm^3

[1 mark]

$$\begin{aligned}
 V &= \text{Area of cross section} \times \text{length} \\
 &= 4 \times 6 \\
 &= 24 \text{ cm}^3
 \end{aligned}$$



- 6 (b) The diagram shows a prism with volume 24 cm^3
The height of the triangular cross section is h .



Work out the height, h .

[3 marks]

$$V = \text{Area of cross section} \times \text{Length}$$

$$V = \frac{1}{2} \times b \times h \times L$$

$$V = \frac{1}{2} \times 6 \times h \times 5$$

$$24 = 15h$$

$$h = \frac{24}{15}$$

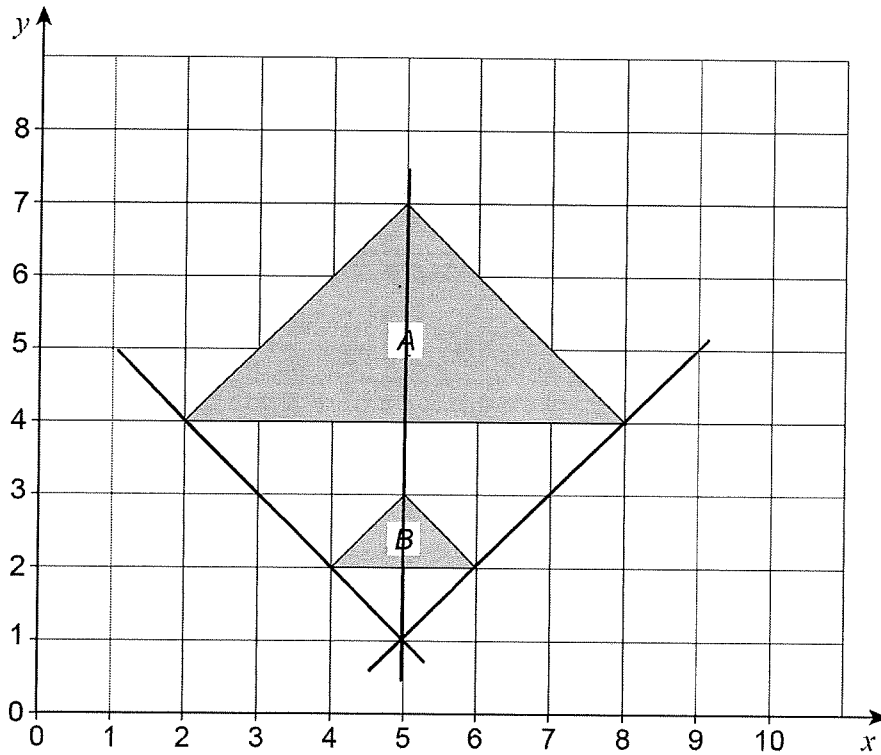
Answer 1.6 cm

$$h = 1.6 \text{ cm}$$

Turn over for the next question



- 7 Describe fully the **single** transformation that maps triangle A to triangle B.



[3 marks]

- Enlargement
- Scale factor $\frac{1}{3}$
- Centre of enlargement (5, 1)



- 8 The table shows information about the distances walked by 120 students on their way to school one week.

Distance, x (miles)	f Frequency	m Midpoint	$f \times m$
$0 < x \leq 5$	20	2.5	50
$5 < x \leq 10$	48	7.5	360
$10 < x \leq 15$	30	12.5	375
$15 < x \leq 20$	22	17.5	385
	Total = 120		1170

Work out an estimate for the mean distance.

[3 marks]

$$\frac{\text{Total } f \times m}{\text{Total } f} = \frac{1170}{120} = 9.75$$

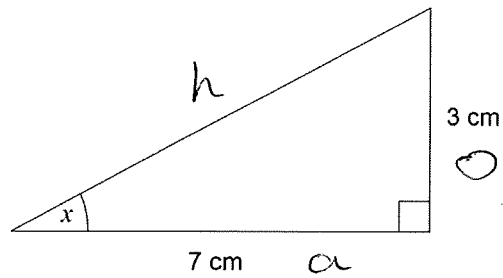
Answer 9.75 miles

Turn over for the next question

Turn over ►



9

Work out the size of angle x .Not drawn
accurately

[2 marks]

$$\text{SOH CAH (TOA)} \quad \tan x = \frac{3}{7}$$

$$\tan x = \frac{3}{7}$$

$$x = \tan^{-1}(3 \div 7) = 23.1985$$

Answer 23.2 degrees



10 Work out the next term of this quadratic sequence.

[2 marks]

$$\begin{array}{ccccccc}
 5 & & 8 & & 14 & & 23 & & 35 \\
 & \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} & \\
 & +3 & & +8 & & +9 & & +12 & \\
 & & \underbrace{\hspace{1cm}} & & \underbrace{\hspace{1cm}} & & \underbrace{\hspace{1cm}} & & \\
 & & +3 & & +3 & & +3 & &
 \end{array}$$

Answer 35

11 Circle the expression that is equivalent to

$$\frac{3x^2}{6x^2+3} \stackrel{\div 3}{=} \frac{x^2}{2x^2+1}$$

[1 mark]

$$\frac{x^2}{2x^2+3}$$

$$\frac{x^2}{6x^2+1}$$

$$\frac{x^2}{2x^2+1}$$

$$\frac{1}{2} + x^2$$

Turn over for the next question

Turn over ►



12

The table shows information about the UK and Germany.

	Population	Area (square miles)
UK	64 000 000	95 000
Germany	82 000 000	140 000

$$\text{Population density} = \frac{\text{population}}{\text{area}}$$

Compare the population densities of the UK and Germany.

[3 marks]

$$\text{UK} = \frac{64000000}{95000} = 673.68$$

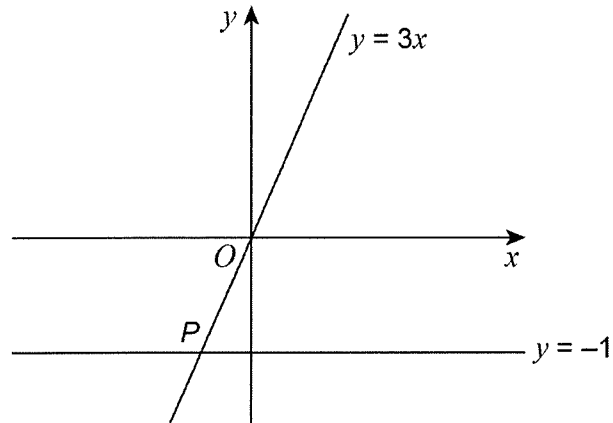
~~Germany~~

$$\text{Germany} = \frac{82000000}{140000} = 585.71$$

The UK is more densely populated compared to Germany ie the population is compacted more than Germany.



- 13 Two straight lines intersect at point P .



Not drawn
accurately

Circle the coordinates of P .

$(-3, -1)$

$(-1, -\frac{1}{3})$

$(-1, -3)$

$(-\frac{1}{3}, -1)$

[1 mark]

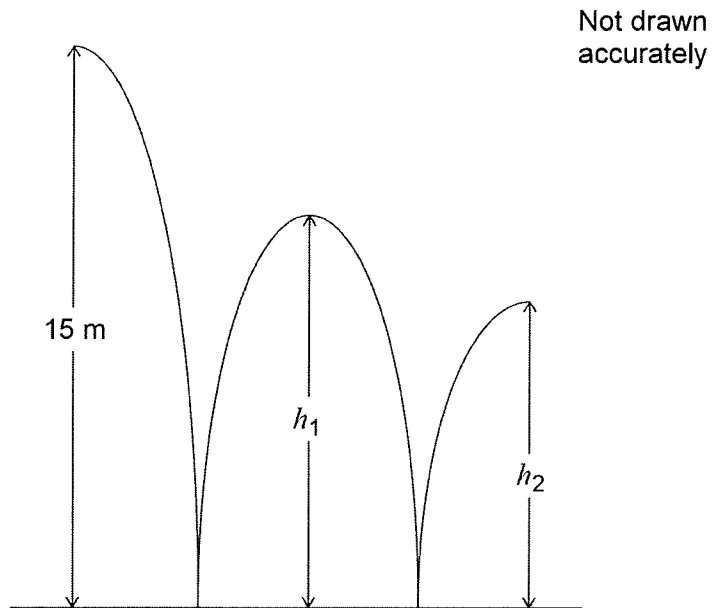
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$$\begin{aligned}
 y &= 3x & y &= -1 \\
 y &= y \\
 3x &= -1 \\
 x &= -\frac{1}{3} & y &= -1
 \end{aligned}$$

Turn over ►



- 14 A ball is thrown from a height of 15 metres.
It bounces to height h_1 , then to height h_2 as shown.



h_1 is three quarters of the original height.

- 14 (a) Jack expects h_2 to be three quarters of h_1

Work out the value of h_2 that he expects.

[2 marks]

$$h_1 = 0.75 \times 15$$

$$h_2 = 0.75 \times h_1 = 0.75 \times 0.75 \times 15 \\ = 8.4375$$

Answer 8.4375 metres



14 (b) In fact, h_2 is two thirds of h_1

$$\frac{3}{4} > \frac{2}{3}$$

How does this affect the answer to part (a)?

Tick a box.

The ball bounced higher than he expected

The ball bounced lower than he expected

Show working to support your answer.

[2 marks]

$$h_2 = \frac{2}{3} \times \frac{3}{4} \times 15$$

$$= 7.5$$

$$8.4375 > 7.5.$$

Turn over for the next question

Turn over ►



15

Mirek invests £6000 at a compound interest rate of 1.5% per year.
He wants to earn more than £1000 interest.

Work out the **least** time, in whole years, that this will take.

[3 marks]

$$\text{Yr } 1 = 6000 \times 1.015 = 6090$$

$$\text{Yr } 4 = 6000 \times 1.015^4 = \del{6336} 6368.18$$

$$\text{Yr } 10 = 6000 \times 1.015^{10} = \del{693} 6963.24$$

$$\text{Yr } 11 = 6000 \times 1.015^{11} = 7067.69$$

Answer 11 years



16 (a) Factorise fully $9y^3 - 6y$

[2 marks]

$$3y(3y^2 - 2)$$

Answer $3y^2(3y - 2)$

16 (b) Factorise $3x^2 - 22x + 7$

[2 marks]

$$(3x - 1)(x - 7)$$

Answer $(3x - 1)(x - 7)$

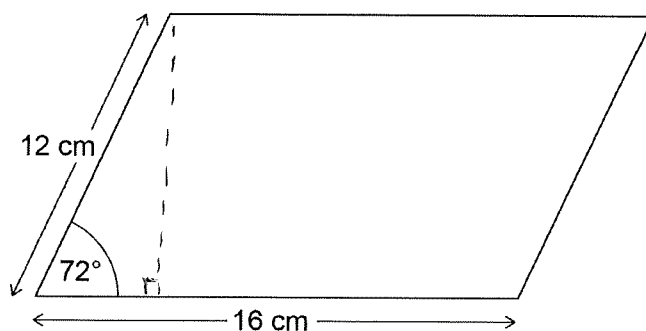
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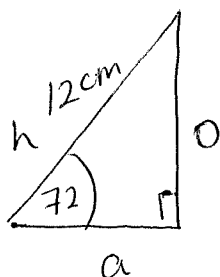
17

Work out the area of the parallelogram.

Not drawn
accurately

[3 marks]

$$\begin{aligned} \text{Area of parallelogram} &= b \times \text{perpendicular } h \\ &= 16 \times 11.4126782 \\ &= 182.60 \end{aligned}$$

Answer 182.6 cm²

(SOH) CAH TOA

$$\sin \alpha = \frac{\text{Opp}}{\text{hyp}}$$

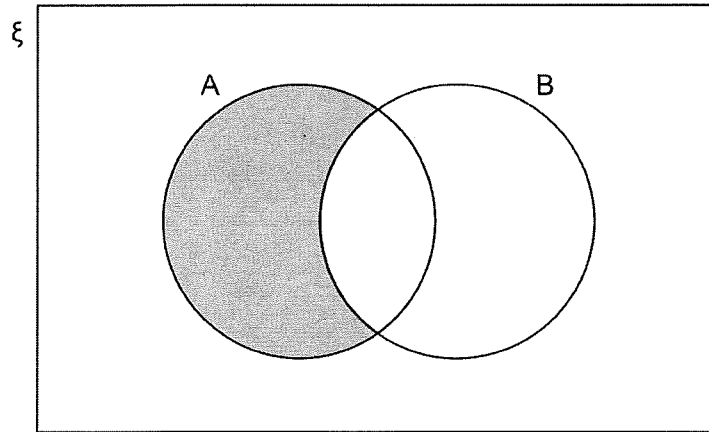
$$\sin 72 = \frac{\text{Opp}}{12}$$

$$12 \times \sin 72 = \text{Opp}$$

$$\text{opp} = 11.4126782$$



18 (a)



Which of these represents the shaded region?

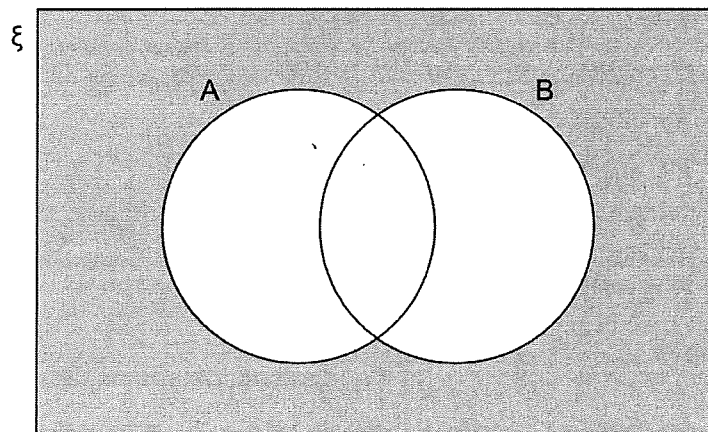
Circle your answer.

[1 mark]

A

 B' $A \cap B'$ $A \cup B'$

18 (b)



Which of these represents the shaded region?

Circle your answer.

[1 mark]

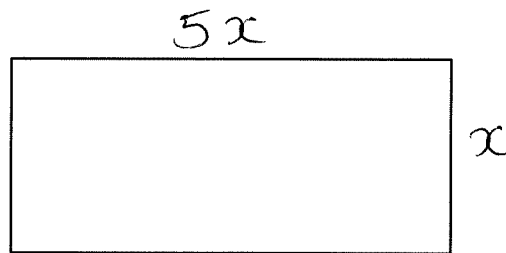
 $(A \cup B)'$ $(A \cap B)'$ $A' \cap B$ $A' \cup B'$

Turn over ►



- 19 The length of a rectangle is five times the width.
The area of the rectangle is 1620 cm^2

Not drawn
accurately



Work out the width of the rectangle.

[3 marks]

$$5x \times x = 1620$$

$$5x^2 = 1620$$

$$x^2 = 324$$

$$x = \sqrt{324}$$

$$x = 18$$

Answer 18 cm



20

A stone is thrown upwards with a speed of v metres per second.

The stone reaches a maximum height of h metres.

h is directly proportional to v^2

When $v = 10$, $h = 5$

Work out the maximum height reached when $v = 24$

[4 marks]

$$h \propto v^2$$

$$h = kv^2$$

$$5 = 10^2 k$$

$$5 = 100k$$

$$\frac{5}{100} = k$$

$$0.05 = k$$

$$\frac{1}{20} = k$$

$$h = 0.05v^2$$

$$h = 0.05 \times 24^2$$

$$h = 28.8$$

Answer 28.8 m

Turn over for the next question

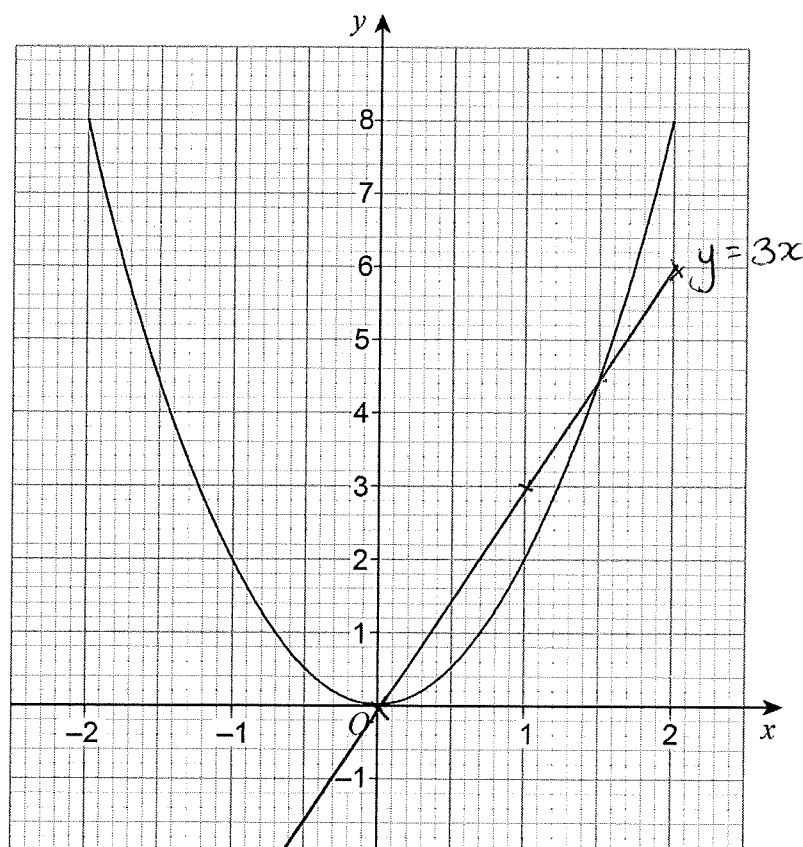
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21 (a) Meera is using a **graphical** method to solve $2x^2 - 3x = 0$

She draws the graph of $y = 2x^2$ and a straight line graph on the same grid.

Here is the graph of $y = 2x^2$



Complete her method to solve $2x^2 - 3x = 0$

[2 marks]

$$2x^2 = 3x$$

x	-2	-1	0	1	2
$y = 2x^2$	8	2	0	2	8
$y = 3x$	-6	-3	0	3	6

$y = y$

Answer $x = 0$ and $x = 1.5$
 $y = 0$ $y = 4.5$



- 21 (b) Levi is solving $2x^2 + 5x = 0$
He uses this method.

$$2x^2 + 5x = 0 \quad \text{subtract } 5x \text{ from both sides}$$

$$2x^2 = -5x \quad \text{divide both sides by } x$$

$$2x = -5 \quad \text{divide both sides by 2}$$

$$x = -2.5$$

Evaluate his method and his answer.

[2 marks]

- Levi should have factorised the quadratic.
- By dividing both sides by x he is not doing the inverse operation of x^2 .

Turn over for the next question

Correct Answer :

$$2x^2 + 5x = 0$$

$$x(2x + 5) = 0$$

$$x = 0 \quad 2x + 5 = 0$$

$$2x = -5$$

$$x = -\frac{5}{2}$$

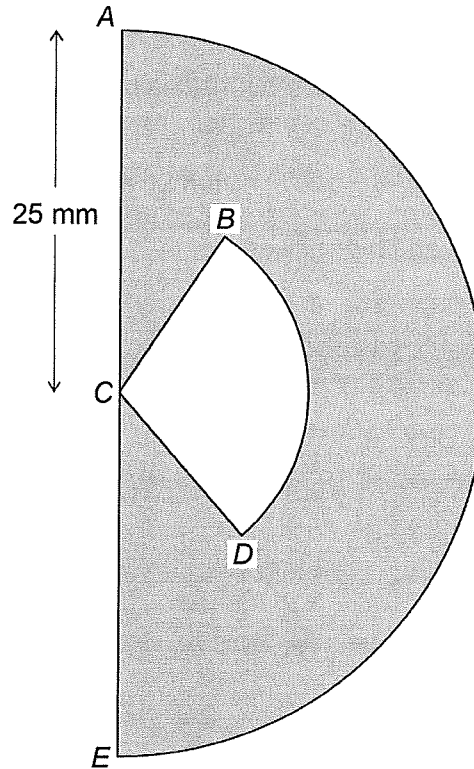
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22

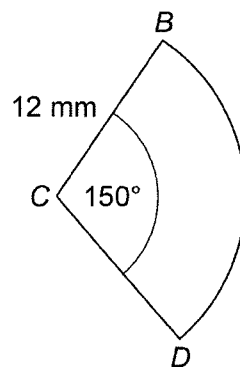
The cross section of an earring is a semicircle, centre C , radius 25 mm
The earring is black and white.

The shaded area is black.



Not drawn
accurately

Sector BCD is white and has radius 12 mm



Not drawn
accurately



Is more than 20% of the semicircle white?

You **must** show your working.

[5 marks]

$$\text{Area of semi-circle} = \frac{\pi r^2}{2} = \frac{\pi \times 25^2}{2} = 981.75 \text{ mm}^2$$

$$\text{Area of white sector} = \frac{150}{360} \times \pi r^2$$

$$= \frac{150}{360} \times \pi \times 12^2$$

$$= 188.5 \text{ mm}^2$$

$$\frac{188.5}{981.75} \times 100 = 19.2\%$$

Answer No, only 19.2% of the semicircle is white.

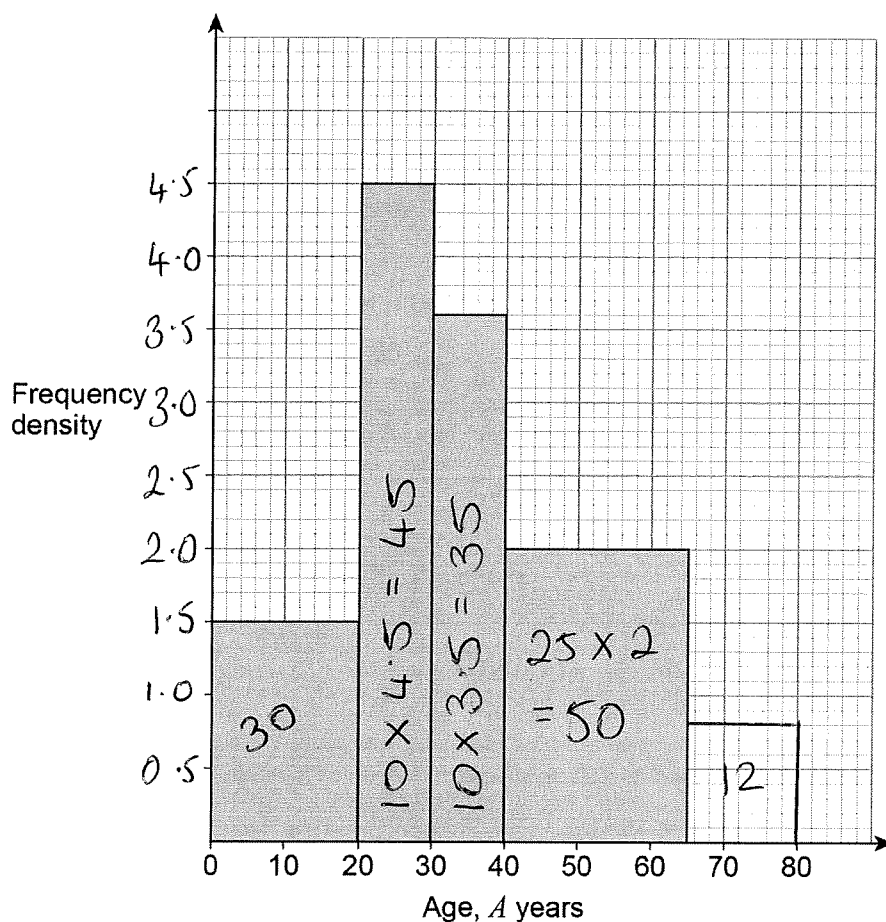
Turn over for the next question



23

Here is some information about a tennis club.

Members of a tennis club

There are 30 members with $A < 20$ There are 12 members with $65 \leq A < 80$ There are no members with $A \geq 80$

23 (a) Complete the histogram.

[3 marks]

$$\begin{aligned} \text{freq Density} &= \frac{\text{freq}}{\text{class width}} \\ &= \frac{30}{20} \\ &= 1.5 \\ \text{freq Density for } 65 \leq A < 80 &= \frac{12}{15} = 0.8 \end{aligned}$$



23 (b) Work out the total number of members of the club.

[2 marks]

$$30 + 45 + 35 + 50 + 12 = 172$$

Answer 172

Turn over for the next question

5

Turn over ►



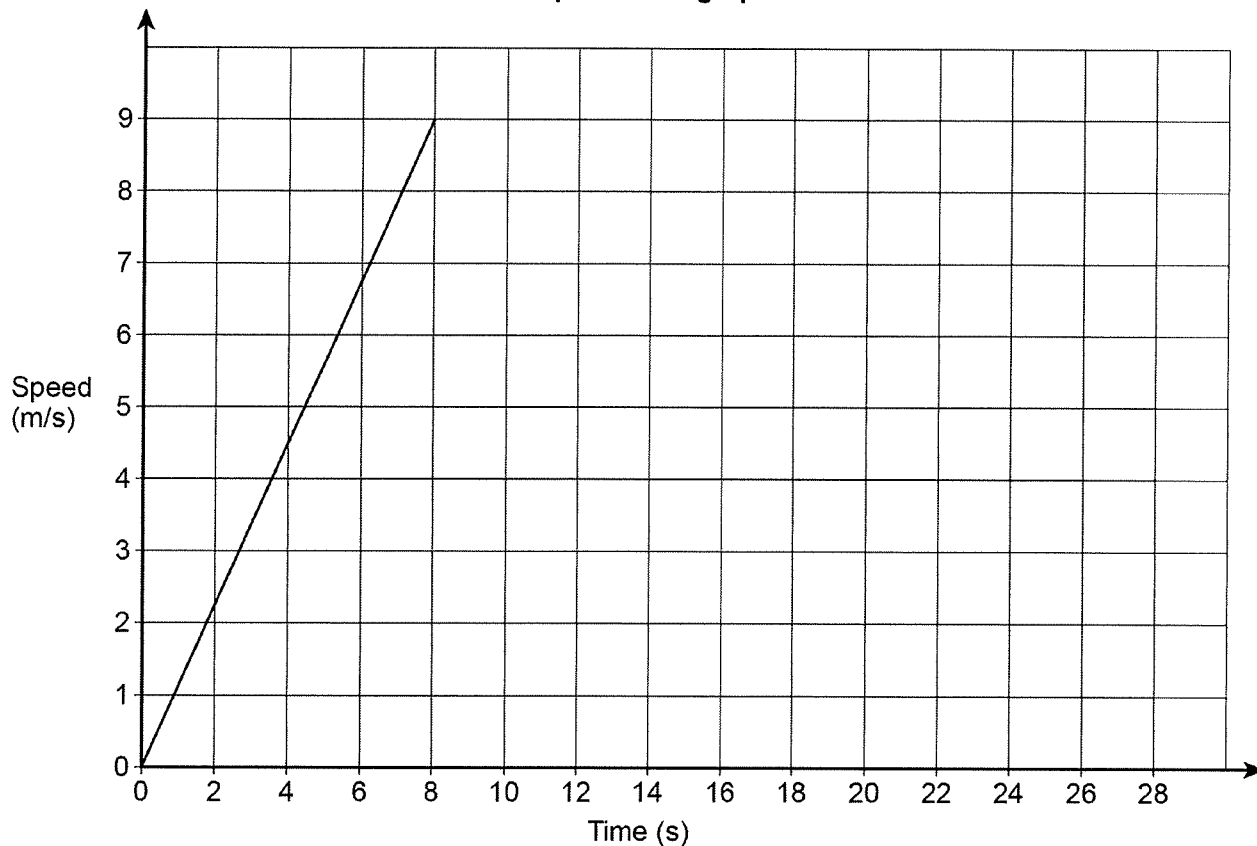
24

Beth ran a 200 metre race.

Here is a graph of the first 8 seconds of her race.

She completed the race at a constant speed of 9 m/s

Speed-time graph for Beth



Amy completed the race in 27 seconds.

Did Beth finish before Amy?

You **must** show your working.**[3 marks]**

$$\text{Beth} \Rightarrow S = \frac{D}{T}$$

+

$$\text{Beth's total time} = 8 + 18 \cdot 2$$

$$9 = \frac{D}{8}$$

$$= 26.2 \text{ s}$$

8

$$36 = D$$

$$\text{Rest of race} \Rightarrow 200 - 36 = 164$$

$$S = \frac{D}{T}$$

Answer

Yes, Beth finished before Amy.

$$9 = \frac{164}{T}$$

$$9T = 164$$

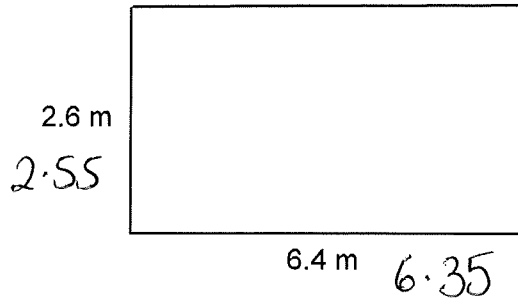
$$T = \frac{164}{9} = 18.2$$



2 6

25

The dimensions of a rectangular floor are to the nearest 0.1 metres.



Not drawn
accurately

A force of 345 Newtons is applied to the floor.

The force is to the nearest 5 Newtons.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the upper bound of the pressure.

Give your answer to 4 significant figures.

You **must** show your working.

[5 marks]

$$\begin{aligned} \text{Max pressure} &= \frac{\text{Max force}}{\text{min area.}} \\ &= \frac{347.5}{2.55 \times 6.35} \\ &= 21.46 \end{aligned}$$

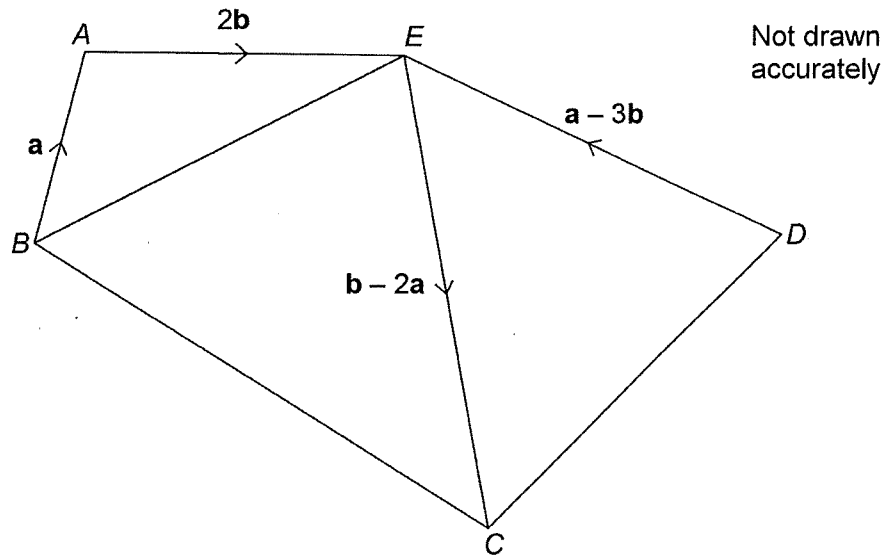
Answer 21.46 N/m²

8

Turn over ►



26

 $ABCDE$ is a pentagon.Show that $BCDE$ is a parallelogram.

[3 marks]

$$\begin{aligned} \vec{BE} &= a + 2b & \vec{BC} &= \vec{BE} + \vec{EC} \\ \vec{CD} &= -(b-2a) - (a-3b) & &= a+2b+b-2a \\ &= -b+2a-a+3b & &= -a+3b \\ &= a+2b. & \vec{ED} &= -(a-3b) \\ & & &= -a+3b. \end{aligned}$$

$$\vec{BE} = \vec{CD} \text{ and } \vec{BC} = \vec{ED}$$

Therefore $BCDE$ is a parallelogram

