

A level Further Devison

October 2021

1.

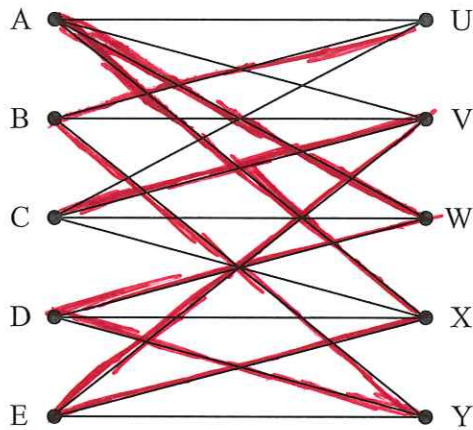
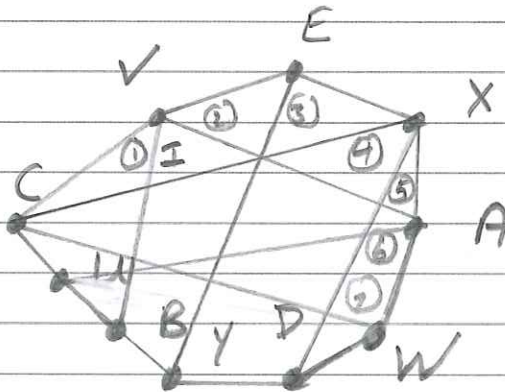


Figure 1

D, Y, B, U



VB is I, so as CW and AU cross each other then this graph is not planar.

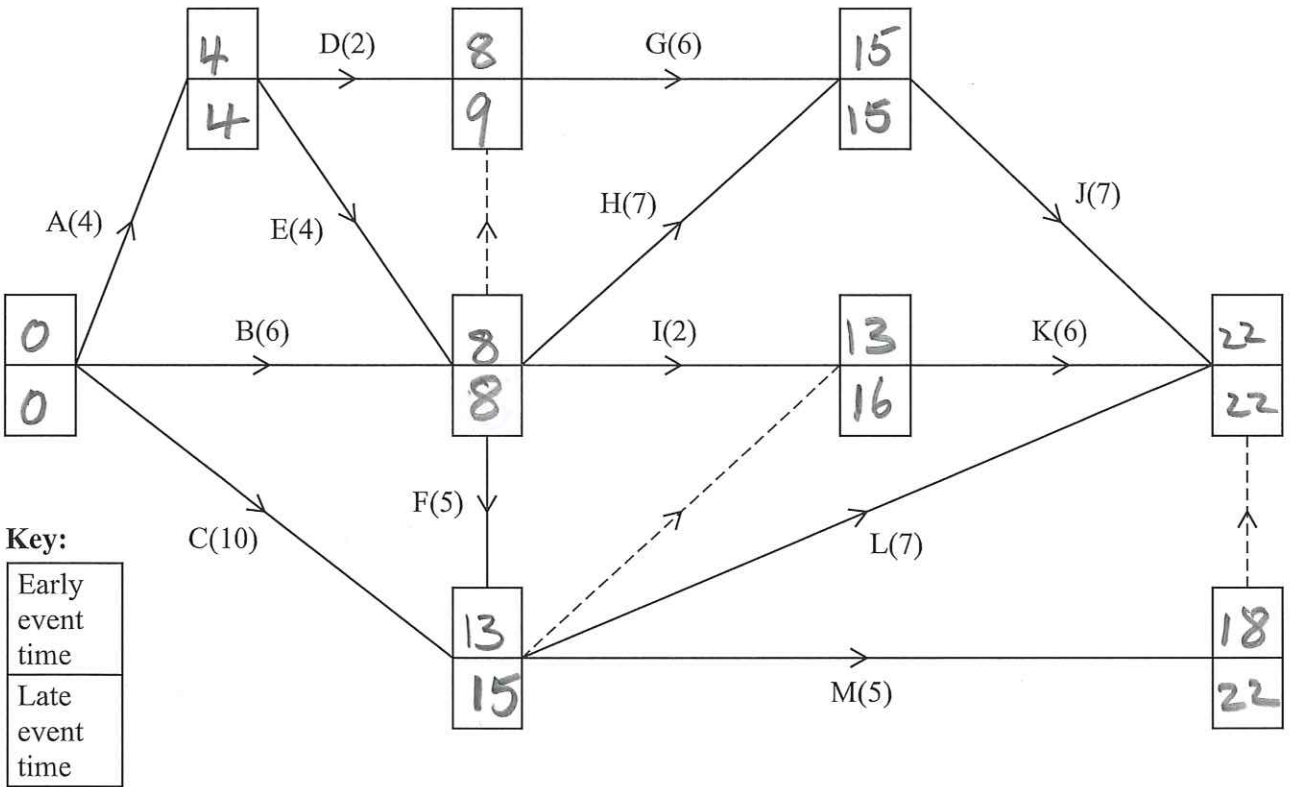
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2.



$$\text{Lower bound} = \frac{\text{Total time taken}}{\text{Min time taken}} = \frac{71}{22} = 3.227\dots$$

$$\underline{\underline{4 \text{ workers}}}$$



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3.

	1	3	5	4	6	7	2	
	A	B	C	D	E	F	G	H
A	-	24	42	48	34	37	32	22
B	24	-	40	35	30	41	39	44
C	42	40	-	21	26	45	38	36
D	48	35	21	-	32	37	29	27
E	34	30	26	32	-	34	40	28
F	37	41	45	37	34	-	43	41
G	32	39	38	29	40	43	-	38
H	22	44	36	27	28	41	38	-

AH AB DH CD CE DG EF

$$24 + 22 + 26 + 21 + 29 + 34 + 27 = 183$$

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### ③ Nearest neighbour algorithm

JF	25
FE	34
EC	26
CD	21
DH	27
HA	22
AB	24
BG	39
GJ	<u>49</u>
	267

3d

④

So with S removed residual minimum spanning tree is 183, need to add in the two shortest arcs to connect S are 27 and 25 so:

$$183 + 27 + 25 = 235$$

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4. (a)

Initial time matrix

	A	B	C	D	E	F
A	-	57	95	150	63	230
B	57	-	72	∞	132	∞
C	95	72	-	289	160	125
D	150	∞	289	-	84	∞
E	63	132	160	84	-	191
F	230	∞	125	∞	191	-

Initial route matrix

	A	B	C	D	E	F
A	A	B	C	D	E	F
B	A	B	C	D	E	F
C	A	B	C	D	E	F
D	A	B	C	D	E	F
E	A	B	C	D	E	F
F	A	B	C	D	E	F

(b) 1st iteration

Time matrix

	A	B	C	D	E	F
A	-	57	95	150	63	230
B	57	-	72	207	120	287
C	95	72	-	245	158	125
D	150	207	245	-	84	380
E	63	120	158	84	-	191
F	230	287	125	380	191	-

Route matrix

	A	B	C	D	E	F
A	A	B	C	D	E	F
B	A	B	C	A	A	A
C	A	B	C	A	A	F
D	A	A	A	D	E	A
E	A	A	A	D	E	F
F	A	A	C	A	E	F

(4c) First consider odd nodes, A, B, C, D, E, F, but start at B and finish at E, so ignore those. Now look at the shortest route between pairs of the remaining nodes.

$AC + DF = 95 + 275 = 370$   
 $AD + CF = 147 + 125 = 272$   
 $AF + CD = 220 + 242 = 462$

→ This is the shortest route, from the diagram, the way that you do this is AE, ED then CF

(d) So total distance is sum of all arcs plus the arcs travelled twice added back in again

$1648 + 272 = 1920$  mins



(5a) 30, 12, 5, 2, 23, (18), 36, 10, 15, 24

$$\frac{n+1}{2} = \frac{11}{2} = 5.5 \quad \text{use item to right}$$

12, 5, (3) 10, 15    18    30, 23, (36), 24

2    12, 5, (10) 15    18    30, (23), 24, 36

2    (5) 10    12    (15)    18    23,    30, (24), 36

2    5    10    (12)    15    18    23    24    (30)    36

Sorted in ascending, descending is the reverse

36, 30, 24, 23, 18, 15, 12, 10, 5, 2

(b) Bin one has  $30+12+2=44$ , so

must be 44 or more

but 5 not go in here, so less than 47.

Also Bin 2 could not fit 18 in, so

less than  $5+23+18=46$ .

$\therefore$  must be 44 or 45



(5c) First fit decreasing bin packing algorithm

~~36~~, ~~30~~, ~~24~~, ~~23~~, ~~18~~, ~~15~~, ~~12~~, ~~8~~, ~~5~~, ~~2~~

(1) 36, 5, 2

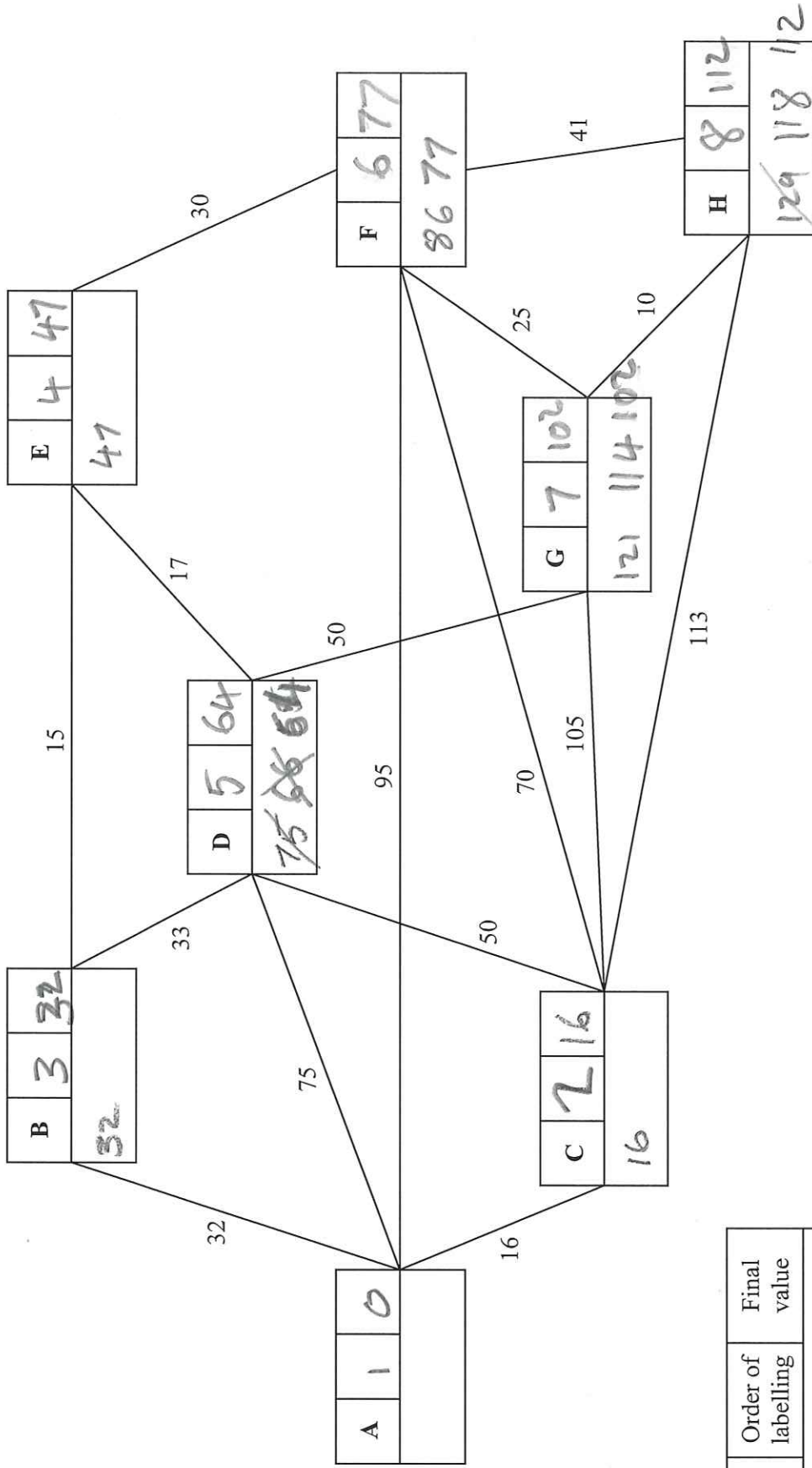
(2) 30, 15

(3) 24, 10,

(4) 23, 12, 10

(5)

6.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path from A to H:

A B E F G H

Length of shortest path from A to H:

112

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P 6 6 8 0 1 A 0 1 2 2 0

b(i)

$$n^2 \times n = n^3$$

$$10^3 \quad 0.082$$

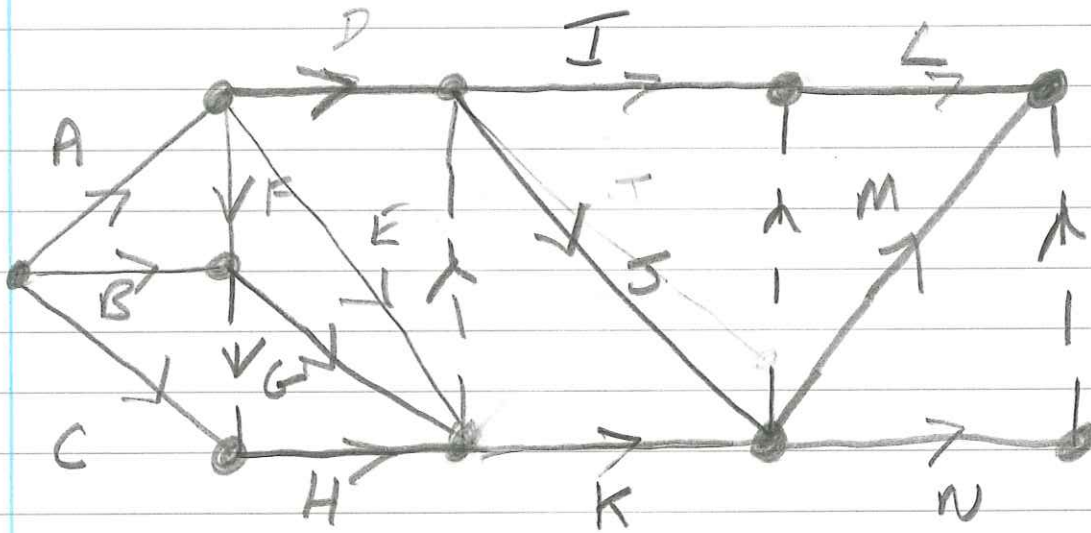
$$\frac{200}{10} = 20$$

$$20^3 \times 0.082 = 656 \text{ s}$$

or 10 min 56 s

(c) The cubic is the largest order, but this does not mean that the function is exactly cubic.

7.	Activity	Preceding
	A	-
	B	-
	C	-
	D	A
	E	A
	F	A
	G	B, F
	H	C, B, F



(c) To do activity I must have done both AD as well as AFG, since all take the same time D must not be a critical activity.

⑧

$$y + z = P$$

maximise

$$x + y + z \leq 39 \quad (1)$$

$$0.4(x + y + z) \leq x$$

$$0.4x + 0.4y + 0.4z \leq x$$

$$-0.6x + 0.4y + 0.4z \leq 0$$

$$-3x + 2y + 2z \leq 0 \quad (2)$$

$$x + z \geq 28 \quad (3)$$

Question 8 continued

$$\text{So } x + y + z + s_1 = 39$$

$$-3x + 2y + 2z + s_2 = 0$$

$$x + y - s_3 + a_1 = 28$$

$$P - y - z = 0$$

$$I = -a_1 = +x + y - s_3 - 28$$

$$\text{So } I - x - y + s_3 = -28$$

b.v.	x	y	z	s <sub>1</sub>	s <sub>2</sub>	s <sub>3</sub>	a <sub>1</sub>	Value
S <sub>1</sub>	1	1	1	1	0	0	0	39
S <sub>2</sub>	-3	2	2	0	1	0	0	0
S <sub>3</sub>	1	0	1	0	0	-1	1	28
P	0	-1	-1	0	0	0	0	0
I	-1	-1	0	0	0	1	0	-28



8c

The only negative in the objective row is in the  $Z$  column  $-1$ .

The smallest value is  $62/5$ , so the 5 is the pivot.

Question 8 continued

b.v.	x	y	z	$s_1$	$s_2$	$s_3$	Value
y	0	1	0	1	0	1	11
$s_2$	0	0	5	-2	1	-5	62
x	1	0	1	0	0	-1	28
P	0	0	-1	1	0	1	11

b.v.	x	y	z	$s_1$	$s_2$	$s_3$	Value	Row Ops
y	0	1	0	1	0	1	11	R1
z	0	0	1	-2/5	1/5	-1	62/5	R2 ÷ 5
x	1	0	0	2/5	-1/5	0	78/5	R3 - 1/5 R2
P	0	0	0	3/5	1/5	0	117/5	R4 + 1/5 R2

$\frac{117}{5}$

Spare copy

b.v.	x	y	z	$s_1$	$s_2$	$s_3$	Value	Row Ops
P								

total time

x (swimming)  $78/5 = 15.6$  hrs

y (cycling) 11 hrs

z (running)  $62/5 = 12.4$  hrs.



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