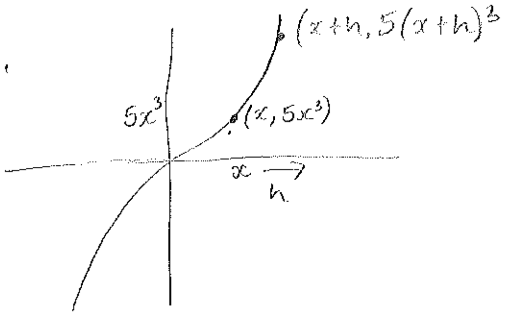
AS Practice Paper C

1.



$$\frac{dy}{dx} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{5(x+h)^3 - 5x^3}{x+h-x}$$

$$= \frac{5[(x+h)(x+h)(x+h)] - 5x^3}{h}$$

$$= \frac{5[(x+h)(x^2+2xh+h^2)] - 5x^3}{h}$$

$$= \frac{5[x^3+2x^2h+xh^2+x^2h+2xh^2+h^3] - 5x^3}{h}$$

$$= \frac{5[x^3+3x^2h+3xh^2+h^3] - 5x^3}{h}$$

$$= \frac{\cancel{5x^3}+15x^2h+15xh^2+5h^3-\cancel{5x^3}}{h}$$

$$= \frac{15x^2h+15xh^2+5h^3}{h}$$

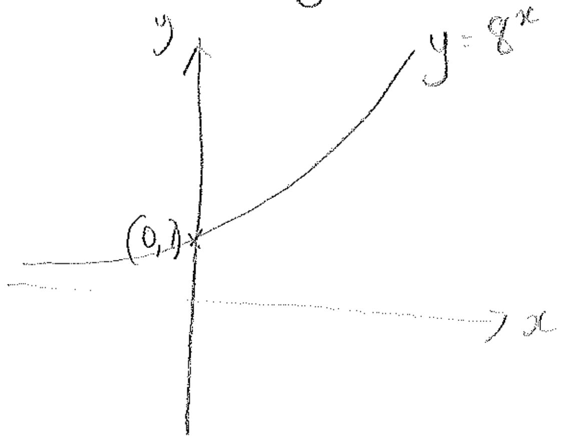
$$= 15x^2+15xh+5h^2$$

As $h \to 0$, $\frac{dy}{dx} = 15x^2$

2. a) $y = 8^x$

$x = 0$, $y = 8^0$

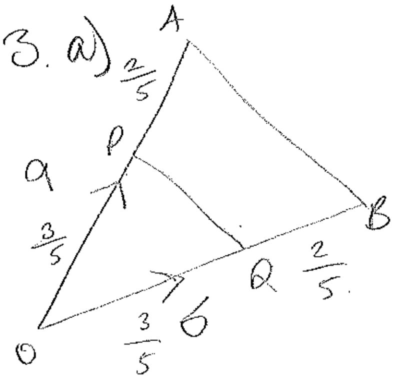
$y = 1$



b) Translation in the positive x direction by $+1$

c) Translation in the positive y direction by $+5$.

3. a)



$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$= -\frac{3}{5}a + \frac{3}{5}b = \frac{3}{5}(-a+b)$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -a + b$

Scalar multiple of $\overrightarrow{AB}$ $\therefore$ parallel

b) $|\overrightarrow{AB}| = 10\text{cm}$.

$|\overrightarrow{PQ}| = \frac{3}{5}|\overrightarrow{AB}|$

$= 6\text{cm}$

4. $g(x) = \frac{4}{x-6} + 5$

$g(0) = \frac{4}{-6} + 5 = \frac{13}{3}$

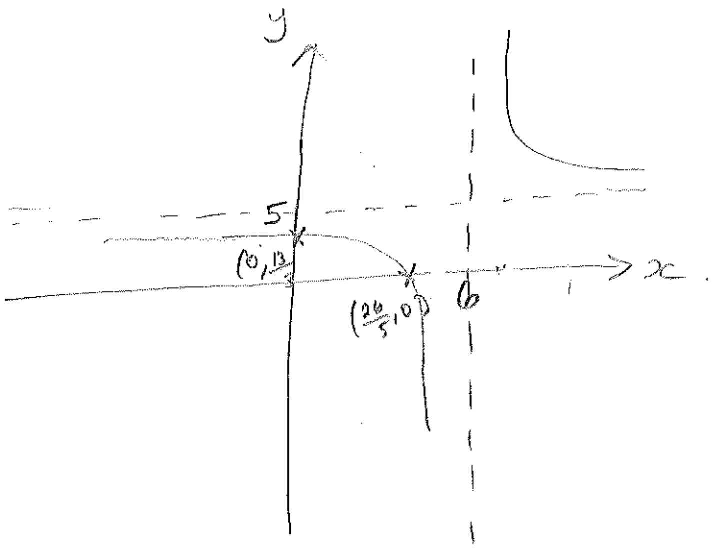
$0 = \frac{4}{x-6} + 5$

$-5(x-6) = 4$

$-5x + 30 = 4$

$26 = 5x$

$\frac{26}{5} = x$



5. $f(x) = 2x^3 - x^2 - 13x - 6$

$-3 \times 2 \times 1$
$-6 \times 1 \times 1$ etc.
$3 \times -2 \times 1$

$f(3) = 2(3)^3 - (3)^2 - 13(3) - 6 = 0$ $(x-3)$ is a factor

5 continued....

$$\begin{array}{r|l} & 2x^2+5x+2 \\ \hline x-3 & 2x^3-x^2-13x-6 \\ & -\underline{2x^3-6x^2} \\ & \quad 5x^2-13x \\ & -\underline{\quad 5x^2-15x} \\ & \qquad 2x-6 \\ & -\underline{\qquad 2x-6} \\ & \qquad\quad 0 \end{array}$$

$2x^2+5x+2$

$(2x+1)(x+2)$

$$f(x)=2x^3-x^2-13x-6$$
$$=(2x+1)(x+2)(x-3).$$

6.a) $(p+q)^5=p^5+\binom{5}{1}p^4q+\binom{5}{2}p^3q^2+\binom{5}{3}p^2q^3+\binom{5}{4}pq^4+\binom{5}{5}q^5$

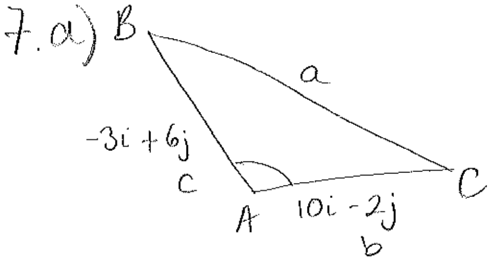
$=p^5+5p^4q+10p^3q^2+10p^2q^3+5pq^4+q^5$

b) $p=\frac{1}{4}$ $q=\frac{3}{4}$

$p^5+5p^4q+10p^3q^2$

$\left(\frac{1}{4}\right)^5+5\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)+10\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2=\frac{1}{1024}+\frac{15}{1024}+\frac{90}{1024}=\underline{\underline{\frac{53}{512}}}$

7.a)



$|\vec{AB}|=\sqrt{(-3)^2+6^2}=\sqrt{45}=3\sqrt{5}$

$|\vec{AC}|=\sqrt{10^2+(-2)^2}=\sqrt{104}=2\sqrt{26}.$

$|BC|=13i-8j$

$|\vec{BC}|=\sqrt{13^2+(-8)^2}=\sqrt{233}$

$a^2=b^2+c^2-(2bc\cos A)$

$(\sqrt{233})^2=(\sqrt{104})^2+(\sqrt{45})^2-(2\times\sqrt{104}\times\sqrt{45}\times\cos A)$

$233=104+45-(2\times\sqrt{104\times45}\times\cos A)$

$\cos A=\frac{-7}{\sqrt{103}}$ $\underline{A=127.9^\circ}$

7b) $A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times \sqrt{45} \times \sqrt{104} \times \sin 127.9$

$= \underline{27 \text{ units}^2}$

8a) $A(3k-4, -2)$

$B(1, k+1)$

Gradient $= \dfrac{k+1-(-2)}{1-(3k-4)} = \dfrac{-3}{2}$

$\dfrac{k+3}{5-3k} = \dfrac{-3}{2}$

$2(k+3) = -3(5-3k)$

$2k+6 = -15+9k$

$21 = 7k$

$\underline{k = 3}$

b) $B(1, 4)$

$m = \dfrac{-3}{2}$

$y - y_1 = m(x - x_1)$

$y - 4 = \dfrac{-3}{2}(x-1)$

$2y - 8 = -3(x-1)$

$2y - 8 = -3x + 3$

$2y = -3x + 11$

$y = \dfrac{-3}{2}x + \dfrac{11}{2}$

c) $A(5,-2)$ Midpoint AB: $\left(\dfrac{5+1}{2}, \dfrac{4+-2}{2}\right) = (3,1)$

Gradient of perpendicular $= \dfrac{2}{3}$

$y - y_1 = m(x - x_1)$

$y - 1 = \dfrac{2}{3}(x-3)$

$3y - 3 = 2x - 6$

$3y - 2x + 3 = 0$

9. a) $h(t) = 115 + 12 \cdot 25t - 4 \cdot 9t^2$

At $t = 0$, the stone is 115 m above ground level, this is therefore the height of the cliff plus the height of the person.

b) $h(t) = -4 \cdot 9t^2 + 12 \cdot 25t + 115$

$= -4 \cdot 9\left[t^2 - 2 \cdot 5t\right] + 115$

$= -4 \cdot 9\left[\left(t - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 115$

$= -4 \cdot 9\left[\left(t - \frac{5}{4}\right)^2 - \frac{25}{16}\right] + 115$

$= -4 \cdot 9\left(t - \frac{5}{4}\right)^2 + \frac{245}{32} + 115$

$= 122 \cdot 65 - 4 \cdot 9(t - 1 \cdot 25)^2$

c) i) $h(t) = 0$

$0 = 122 \cdot 65 - 4 \cdot 9(t - 1 \cdot 25)^2$

$4 \cdot 9(t - 1 \cdot 25)^2 = 122 \cdot 65$

$(t - 1 \cdot 25)^2 = 25 \cdot 0318$

$t - 1 \cdot 25 = \pm 5 \cdot 0031$

$\underline{t = 6 \cdot 25 s}$ or $t = \cancel{-3 \cdot 75}$

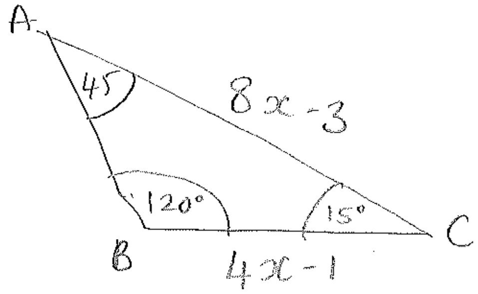
ii) From completed square form.

max at $(1 \cdot 25, 122 \cdot 65)$

$t = 1 \cdot 25 s$.

$h = 122 \cdot 65 m$

10. a)



$$\frac{4x-1}{\sin 45} = \frac{8x-3}{\sin 120}$$

$$4x-1(\sin 120) = 8x-3(\sin 45)$$

$$4x-1\left(\frac{\sqrt{3}}{2}\right) = 8x-3\left(\frac{\sqrt{2}}{2}\right)$$

$$\sqrt{3}(4x-1) = \sqrt{2}(8x-3)$$

$$4\sqrt{3}x - \sqrt{3} = 8\sqrt{2}x - 3\sqrt{2}.$$

$$3\sqrt{2} - \sqrt{3} = 8\sqrt{2}x - 4\sqrt{3}x.$$

$$3\sqrt{2} - \sqrt{3} = x(8\sqrt{2} - 4\sqrt{3})$$

$$\frac{3\sqrt{2} - \sqrt{3}}{8\sqrt{2} - 4\sqrt{3}} = x$$

$$= \frac{3\sqrt{2} - \sqrt{3} \times (8\sqrt{2} + 4\sqrt{3})}{8\sqrt{2} - 4\sqrt{3} \times (8\sqrt{2} + 4\sqrt{3})}$$

$$= \frac{48 + 12\sqrt{6} - 8\sqrt{6} - 12.}{128 - 48}$$

$$= \frac{36 + 4\sqrt{6}}{80} \begin{matrix} \div 4 \\ \div 4 \end{matrix}$$

$$= \frac{9 + \sqrt{6}}{20}$$

10.b) $A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 8x-3 \times 4x-1 \times \sin 15$

$= (32x^2 - 12x - 8x + 3)\left(\frac{\sqrt{6}-\sqrt{2}}{6}\right)$

$= (32x^2 - 20x + 3)\left(\frac{\sqrt{6}-\sqrt{2}}{6}\right)$

$= \left[32\left(\frac{9+\sqrt{6}}{20}\right)^2 - 20\left(\frac{9+\sqrt{6}}{20}\right) + 3\right]\left(\frac{\sqrt{6}-\sqrt{2}}{6}\right)$

$= \underline{0.26}$

11. a) $\int_a^{2a} (10-6x)\,dx = 1$

$\left[10x - \frac{6x^2}{2}\right]_a^{2a} = 1$

$\left[10x - 3x^2\right]_a^{2a} = 1$

$\left(10(2a) - 3(2a)^2\right) - \left(10(a) - 3(a)^2\right) = 1$

$20a - 12a^2 - (10a - 3a^2) = 1$

$-9a^2 + 10a = 1$

$9a^2 - 10a + 1 = 0$

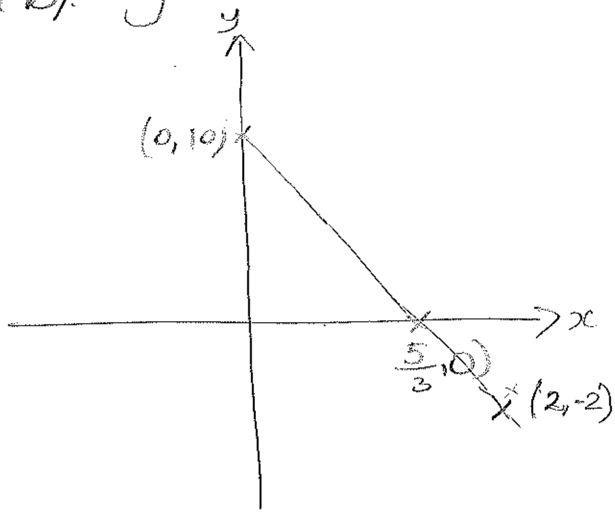
$(9a-1)(a-1) = 0$

$9a - 1 = 0 \qquad a - 1 = 0$

$9a = 1 \qquad a = 1$

$a = \frac{1}{9}$

11 b). $y = 10 - 6x$



$y = 0, \quad 0 = 10 - 6x$

$6x = 10$

$x = \frac{10}{6} = \frac{5}{3}$

$x = 0, \quad y = 10 - 6(0)$

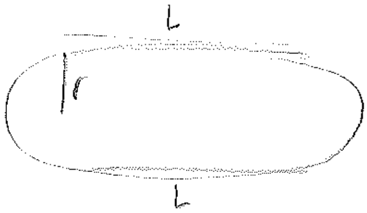
$y = 10$

$x = 2, \quad y = 10 - 6(2)$

$y = -2.$

c) Definite integral will only equal the area (1) if the function is above the x-axis between the defined limits.
When $a = 1$, $2a = 2$, so part of the area will be above the x-axis and part will be below.
The area would be greater than 1.

12. a)



Perimeter $= 300\text{m}$

$= 2L + 2\pi r$

$300 = 2L + 2\pi r$

$150 = L + \pi r$

$150 - \pi r = L$

$C = \pi d$

$C = 2\pi r$

Area $= (2r \times L) + \pi r^2$

$= 2r(150 - \pi r) + \pi r^2$

$= 300r - 2\pi r^2 + \pi r^2$

$= 300r - \pi r^2$

b) $\frac{dA}{dr} = 300 - 2\pi r$

max when $\frac{dA}{dr} = 0$, $\quad 300 - 2\pi r = 0$

$2\pi r = 300$

$r = \frac{150}{\pi}$

max $A = 300r - \pi r^2$

$= 300\left(\frac{150}{\pi}\right) - \pi\left(\frac{150}{\pi}\right)^2 = \frac{22500}{\pi}$

13. a) $V = ab^t$

$m = -\frac{1}{10}$

$(0, \log_4 40{,}000)$

$y - y_1 = m(x - x_1)$

$y - \log_4 40000 = -\frac{1}{10}(x - 0)$

$10y - 10\log_4 40000 = -1(x - 0)$

$10y - 10\log_4 40000 = -x$

$y - \log_4 40000 = -\frac{1}{10}x$

$y = \frac{-1}{10}x + \log_4 40000$

$\log_4 V = -\frac{1}{10}t + \log_4 40000$

b) $V = ab^t$

$\log_4 V = \log_4 (ab^t)$

$\log_4 (ab^t) = -\frac{1}{10}t + \log_4 40000$

$\log_4 a + \log_4 b^t$

$\log_4 a + t\log_4 b$

$\frac{-1}{10} = \log_4 b$

$4^{-\frac{1}{10}} = b$

$\log_4 a = \log_4 40000$

$a = 40000$

$\boxed{3^2 = 9 \quad \log_3 9 = 2}$

c) a = Initial value of the car

b = Annual proportional decrease in the value of the car.

d) $t = 7$, $V = 40000 \times 4^{-\frac{7}{10}}$

$V = £15{,}157$

e) $\log_4 V = -\frac{1}{10}t + \log_4 40000$

$\log_4 10000 = -\frac{1}{10}t + 40000$

$t = 10$ years.

f) Milage will also affect the value of the car.