ANSWERS.

Questions

SGS Mathematics Faculty

Unit 2: Algebra and Number

*Moving from A to A**

Tuesday 6th November 2012

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Indices Powers and Roots

1. (a) Write down the value of $27^{\frac{1}{3}}$

(1 mark)

$\sqrt[3]{27} = 3$

(b) Write down the value of $(4xy)^0$

(1 mark)

1

(c) If $2^x = \frac{1}{32}$ find the value of x.

$2^5 = 32$ So $2^{-5} = \frac{1}{32}$ $x = -5$

(2 marks)

2. (a) Evaluate $49^{0.5} \times 3^{-2}$

Give your answer as a fraction.

$49^{0.5} = \sqrt{49} = 7$

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

So $7/9$

(3 marks)

(b) Work out $27^{\frac{2}{3}}$

$(27^{1/3})^2 = 3^2 = 9$

(2 marks)

3. (a) Work out $81^{\frac{1}{2}} x 2^{-3}$

Give your answer as a mixed number.

$81^{1/2} = \sqrt{81} = 9$

$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

So $9/8$

(3 marks)

(b) Work out $125^{-\frac{2}{3}}$

Give your answer as a fraction.

(2 marks)

$\frac{1}{125^{2/3}} = \frac{1}{(\sqrt[3]{125})^2} = \frac{1}{5^2} = \frac{1}{25}$.

4. Simplify fully

(a) $(3^5)^4$

Give your answer as a power of 3.

$= 3^5 \times 3^5 \times 3^5 \times 3^5$

$= 3^{20}$

(1 mark)

(b) $1000^{-\frac{2}{3}}$

Give your answer as a fraction.

$= \dfrac{1}{1000^{2/3}} = \dfrac{1}{(\sqrt[3]{1000})^2} = \dfrac{1}{10^2} = \dfrac{1}{100}$

(2 marks)

5. (a) Find the value of $64^{\frac{1}{3}}$

$= \sqrt[3]{64} = 4$

(1 mark)

(b) Find the value of $8x^0$

$x^0 = 1.$

$= 8 \times 1 = 8$

(1 mark)

6. Express $32^{-\frac{3}{5}}$ as a fraction.

$= \dfrac{1}{32^{3/5}} = \dfrac{1}{(\sqrt[5]{32})^3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

(2 marks)

7. (a) Work out $8^{\frac{2}{3}}$

$= (\sqrt[3]{8})^2 = 2^2 = 4$

(2 marks)

(b) Work out $64^{-\frac{1}{3}}$

$= \dfrac{1}{64^{1/3}} = \dfrac{1}{\sqrt[3]{64}} = \dfrac{1}{4}$

(2 marks)

8. (a) Find the value of x in $4^x = \frac{1}{16}$ **(1 mark)**

$\frac{1}{16} = \frac{1}{4^2} = 4^{-2} \quad x = -2$

(b) Find the value of y in $8^y = 2$ **(1 mark)**

$2^3 = 8 \quad y = \frac{1}{3}$

(c) Write down the value of $27^{\frac{2}{3}}$ **(2 marks)**

$= (\sqrt[3]{27})^2 = 3^2 = 9$

9. (a) Write down the value of 11^0 **(1 mark)**

$= 1$

(b) Find the value of $8^{\frac{2}{3}}$ **(2 marks)**

$= (\sqrt[3]{8})^2 = 2^2 = 4$

10. (a) If $3^x = \frac{1}{27}$, find the value of x. **(2 marks)**

$\frac{1}{27} = \frac{1}{3^3} = 3^{-3} \quad x = -3$

(b) If $4^y = 64^{\frac{1}{2}}$, find the value of y. **(2 marks)**

$64 = 4^3$

So $64^{1/2} = (4^3)^{1/2} = 4^{3/2}$

So $y = 3/2$

11. (a) Find the value of $(0.25)^{-1}$ **(1 mark)**

$= \frac{1}{0.25} = 4$

(b) Find the value of $81^{-\frac{3}{4}}$ **(2 marks)**

$= \frac{1}{(\sqrt[4]{81})^3}$

$= \frac{1}{3^3} = \frac{1}{27}$

Surds

1. Rationalise the denominator of $\frac{2+\sqrt{3}}{\sqrt{3}}$

Simplify your answer fully.

$$\frac{2+\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}+3}{3}$$

(3 marks)

2. Find values of a and b such that

$$(2+\sqrt{3})(4-\sqrt{3}) = a + b\sqrt{3}$$

$$= 8 - 2\sqrt{3} + 4\sqrt{3} - 3$$
$$= 5 + 2\sqrt{3}$$

(2 marks)

3. Write each of these in the form $p\sqrt{3}$, where p is an integer.

(a) $\sqrt{6} \times \sqrt{50}$

(2 marks)

4

$$= \sqrt{6 \times 50} = \sqrt{300} = \sqrt{3 \times 100}$$

9

$$= \sqrt{100} \times \sqrt{3} = 10\sqrt{3}$$

16

(b) $\sqrt{48} + \sqrt{75}$

25

$$\sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$$

36

$$\sqrt{3 \times 25} = \sqrt{3} \times \sqrt{25} = 5\sqrt{3}$$

$$9\sqrt{3}$$

(2 marks)

(c) $\frac{18}{\sqrt{3}}$

(2 marks)

$$\frac{18}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{18\sqrt{3}}{3} = 6\sqrt{3}$$

4
9
16
25

4. (a) Simplify $\sqrt{18}+\sqrt{32}$

$$\sqrt{2\times 9} = \sqrt{2}\times\sqrt{9} = 3\sqrt{2}$$
$$\sqrt{32} = \sqrt{16\times 2} = \sqrt{16}\times\sqrt{2} = 4\sqrt{2}$$
$$7\sqrt{2}$$

(2 marks)

(b) Rationalise $\frac{1}{\sqrt{6}}$

$$\frac{1}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$

(1 mark)

5. (a) Rationalise the denominator and simplify fully $\frac{1}{\sqrt{12}}$

$$\frac{1}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}} = \frac{\sqrt{12}}{12}$$

But $\sqrt{12} = \sqrt{4\times 3} = 2\sqrt{3}$

$$\rightarrow \frac{2\sqrt{3}}{12} = \frac{\sqrt{3}}{6}$$

(2 marks)

(b) By simplifying $\sqrt{32}-\sqrt{18}$,

write $\sqrt{3}(\sqrt{32}-\sqrt{18})$

in its simplest form.

$$\sqrt{32} = \sqrt{16\times 2} = 4\sqrt{2}$$
$$\sqrt{18} = \sqrt{9\times 2} = 3\sqrt{2}$$

So $\sqrt{32}-\sqrt{18}$
$= 4\sqrt{2}-3\sqrt{2} = \sqrt{2}$

$$\sqrt{3}\times\sqrt{2} = \sqrt{6}$$

(3 marks)

6. (a) Simplify $\sqrt{8}+\sqrt{50}$

$$\sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$$
$$\sqrt{50} = \sqrt{25\times 2} = 5\sqrt{2}$$
$$7\sqrt{2}$$

(2 marks)

(b) Hence simplify

$$(\sqrt{8}+\sqrt{50})(\sqrt{24}+\sqrt{54})$$

4

9

16

25

giving your answer in its simplest surd form.

$\sqrt{8}+\sqrt{50} = 7\sqrt{2}$

But $\sqrt{24} = \sqrt{4\times 6} = 2\sqrt{6}$
$\sqrt{54} = \sqrt{9\times 6} = 3\sqrt{6}$ } $5\sqrt{6}$

$= 7\sqrt{2}(\sqrt{24}+\sqrt{54})$

(3 marks)

$= 7\sqrt{2}\times 5\sqrt{6}$ $(\sqrt{12} = \sqrt{4\times 3} = 2\sqrt{3})$

$= 35\sqrt{12} = 70\sqrt{3}$

7. Rationalise the denominator and simplify fully $\frac{18}{\sqrt{2}}$

$$\frac{18}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{18\sqrt{2}}{2} = 9\sqrt{2}$$

(2 marks)

8. (a) Simplify fully $\sqrt{75}+\sqrt{27}$

You **must** show your working.

$\sqrt{75} = \sqrt{3\times 25} = 5\sqrt{3}$
$\sqrt{27} = \sqrt{9\times 3} = 3\sqrt{3}$ } $8\sqrt{3}$

(2 marks)

(b) Rationalise the denominator and simplify $\frac{21}{\sqrt{7}}$

$$\frac{21}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}} = \frac{21\sqrt{7}}{7} = 3\sqrt{7}$$

(2 marks)

9. Simplify fully $\sqrt{2}(\sqrt{8}-\sqrt{2})$

$= \sqrt{2\times 8} - 2$

(2 marks)

$= \sqrt{16}-2$

$= 4-2 = 2$

10. Show that $\left(\sqrt{50}-\sqrt{2}\right)^2$ is an integer.

$$\left(\sqrt{50}-\sqrt{2}\right)\left(\sqrt{50}-\sqrt{2}\right)$$

$$= 50 - \sqrt{50}.\sqrt{2} - \sqrt{2}.\sqrt{50} + 2$$

$$= 50 - \sqrt{100} - \sqrt{100} + 2$$

$$= 50 - 10 - 10 + 2 \quad = 32$$

(2 marks)

11. Work out $2\sqrt{3}\left(\sqrt{3}+\sqrt{8}\right)$

Give your answer in the form $a + b\sqrt{6}$ where a and b are integers.

$$= 2\times 3 + 2\sqrt{3\times 8}$$

$$= 6 + 2\sqrt{24}$$

$$= 6 + 2\sqrt{4\times 6}$$

$$= 6 + 4\sqrt{6}$$

4
9
16
25

(3 marks)

12. Simplify fully $\dfrac{\sqrt{150}-\sqrt{6}}{\sqrt{12}}$

$$\left.\begin{array}{l}\sqrt{150} = \sqrt{6\times 25} = 5\sqrt{6} \\ \sqrt{12} = \sqrt{4\times 3} = 2\sqrt{3}\end{array}\right\} \quad \frac{5\sqrt{6}-\sqrt{6}}{2\sqrt{3}}$$

(4 marks)

$$= \frac{4\sqrt{6}}{2\sqrt{3}} = \frac{2\sqrt{6}}{\sqrt{3}} = 2\sqrt{\frac{6}{3}} = 2\sqrt{2}$$

13. Express $\sqrt{5} + \sqrt{20}$ in the form $p\sqrt{5}$

$$\sqrt{20} = \sqrt{4\times 5} = 2\sqrt{5}$$

So $\sqrt{5} + 2\sqrt{5} = 3\sqrt{5}$

(2 marks)

14. Show that $\sqrt{12}\left(\sqrt{75}-\sqrt{48}\right)=6$

(3 marks)

$$\left.\begin{aligned}\sqrt{75} &= \sqrt{3\times 25} = 5\sqrt{3}\\ \sqrt{48} &= \sqrt{16\times 3} = 4\sqrt{3}\\ \sqrt{12} &= \sqrt{4\times 3} = 2\sqrt{3}\end{aligned}\right\}$$

So $2\sqrt{3}\left(5\sqrt{3}-4\sqrt{3}\right)$

$= 2\sqrt{3}\times\sqrt{3} = 2\times 3 = 6$

15. (a) By rationalising the denominator, simplify $\dfrac{15}{\sqrt{5}}$

$$\frac{15}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \frac{15\sqrt{5}}{5} = 3\sqrt{5}$$

(2 marks)

(b) Show that $\left(\sqrt{3}+\sqrt{12}\right)^2 = 27$

$\left(\sqrt{3}+\sqrt{12}\right)\left(\sqrt{3}+\sqrt{12}\right)$

(2 marks)

$= 3+\sqrt{36}+\sqrt{36}+12$

$= 3+6+6+12 = 27$

16. Rationalise and simplify $\dfrac{1}{\sqrt{8}}$

$$\frac{1}{\sqrt{8}}\times\frac{\sqrt{8}}{\sqrt{8}} = \frac{\sqrt{8}}{8}$$

(2 marks)

Algebraic Fractions

1. Solve the equations

(a) $\frac{12-y}{3}=5$

$\times 3 \quad 12-y=15$

$+y \quad 12=15+y$

$-15 \quad y=-3$

(3 marks)

(b) $\frac{2x+1}{4}+\frac{4x+1}{6}=1$

$\frac{6(2x+1)}{24}+\frac{4(4x+1)}{24}=1$

$(\times 24) \quad 12x+6+16x+4=24$

$28x+10=24$

$28x=14$

$x=14/28=1/2.$

(4 marks)

2. Solve the equation

$$\frac{x+1}{2}+\frac{x-3}{4}=2$$

You **must** show all your working.

$\frac{4(x+1)}{8}+\frac{2(x-3)}{8}=2$

$(\times 8) \quad 4x+4+2x-6=16$

$(+2) \quad 6x-2=16$

$6x=18$

$x=3.$

(4 marks)

3. Solve the equation

$$\frac{x+1}{2}+\frac{x-3}{4}=2$$

You **must** show all your working.

$$\frac{4(x+1)}{8}+\frac{2(x-3)}{8}=2$$

$(\times 8)$ $4x+4+2x-6=16$

$6x-2=16$

$6x=18$

$x=3$.

(4 marks)

4. Solve the equation $\frac{3x+1}{2}-\frac{2x+5}{3}=1$

$$\frac{3(3x+1)}{6}-\frac{2(2x+5)}{6}=1$$

$(\times 6)$ $3(3x+1)-2(2x+5)=6$

$9x+3-4x-10=6$

$5x-7=6$

$5x=13$

$x=13/5$ or $2\,3/5$

(4 marks)

5. Solve the equation $\frac{x+5}{3}+\frac{2x-1}{4}=1$

$$\frac{4(x+5)}{12}+\frac{3(2x-1)}{12}=1$$

$(\times 12)$ $4(x+5)+3(2x-1)=12$

$4x+20+6x-3=12$

(-17) $10x+17=12$

$10x=-5$

$x=-5/10$

$x=-1/2$.

(4 marks)

6. Solve the equation $\frac{1}{x+1} + \frac{5x}{x-2} = 3$

$$\frac{(x-2)}{(x+1)(x-2)} + \frac{5x(x+1)}{(x+1)(x-2)} = 3$$

$\times\ (x+1)(x-2)$

$$(x-2) + 5x(x+1) = 3(x+1)(x-2)$$
$$x-2+5x^2+5x = 3(x^2-x-2)$$
$$5x^2+6x-2 = 3x^2-3x-6$$

(5 marks)

Solve $2x+1=0 \quad x=-1/2$

Make one side $= 0$

$$2x^2+9x+4 = 0$$

$x+4=0 \quad x=-4$

Factorise

$$(2x+1)(x+4) = 0$$

7. Solve the equation $\frac{x}{x+1} - \frac{2}{x-1} = 1$

$$\frac{x(x-1)}{(x+1)(x-1)} - \frac{2(x+1)}{(x+1)(x-1)} = 1$$

$\times\ (x+1)(x-1)$

$$x(x-1) - 2(x+1) = (x+1)(x-1)$$

(5 marks)

$$x^2 - x - 2x - 2 = x^2 + x - x - 1$$
$$\cancel{x^2} - 3x - 2 = \cancel{x^2} - 1$$

$3x = -1, \quad x = -1/3$

8. Solve this equation $\frac{3}{x+5} - \frac{1}{x+4} = \frac{1}{2}$

$$\frac{3(x+4)}{(x+4)(x+5)} - \frac{1(x+5)}{(x+5)(x+4)} = \frac{1}{2}$$

$\times 2$

$$\frac{6(x+4)}{(x+4)(x+5)} - \frac{2(x+5)}{(x+5)(x+4)} = 1$$

(7 marks)

$(x+4)(x+5)$

$$6(x+4) - 2(x+5) = (x+4)(x+5)$$
$$6x+24-2x-10 = x^2+9x+20$$

Make one side $= 0$

$$4x+14 = x^2+9x+20$$
$$x^2+5x+6 = 0$$

Solve $x = -2$

Factorise

$$(x+2)(x+3) = 0$$

$x = -3$

9. Solve the equation $\frac{x}{x+1} - \frac{2}{x-1} = 1$

$$\frac{x(x-1)}{(x+1)(x-1)} - \frac{2(x+1)}{(x+1)(x-1)} = 1$$

$$x(x-1) - 2(x+1) = (x+1)(x-1)$$

$$x^2 - x - 2x - 2 = x^2 - 1$$

$$x^2 - 3x - 2 = x^2 - 1$$

$$0 = 3x + 1$$

$$3x = -1$$

$$x = -1/3$$

(5 marks)

difference of two squares

10. Simplify fully $\frac{x^2 - 16}{3x^2 + 10x - 8}$

$$\frac{\cancel{(x+4)}(x-4)}{(3x-2)\cancel{(x+4)}} = \frac{x-4}{3x-2}$$

(4 marks)

11. Simplify fully

$$\frac{2x^2 + 5x - 3}{x^2 + 2x - 3}$$

$$= \frac{(2x-1)\cancel{(x+3)}}{(x-1)\cancel{(x+3)}}$$

$$= \frac{2x-1}{x-1}$$

(4 marks)

14. Simplify $\frac{2x^2-9x-18}{x^2-36}$

difference of two squares

$$\frac{(2x+3)\cancel{(x-6)}}{(x+6)\cancel{(x-6)}} = \frac{2x+3}{x+6}$$

(4 marks)

Changing the Subject of a Formula

1. Make x the subject of the formula

$$y = \frac{3x+4}{x-3}$$

$y(x-3) = 3x+4$

$xy - 3y = 3x+4$

collect terms in x

$xy - 3x = 3y+4$

factorise $x(y-3) = 3y+4$

$x = \dfrac{3y+4}{y-3}$.

(4 marks)

2. Make x the subject of the formula

$$a(x-b) = a^2 + bx$$

$ax - ab = a^2 + bx$

collect terms in x

$ax - bx = a^2 + ab$

factorise $x(a-b) = a^2 + ab$

$x = \dfrac{a^2+ab}{a-b}$.

(4 marks)

3. Rearrange $y = \dfrac{xy+2}{3x-4}$

to make x the subject.
Simplify your answer as much as possible.

$y(3x-4) = xy+2$

$3xy - 4y = xy+2$

collect terms in x

$3xy - xy = 4y+2$

factorise $x(3y-y) = 4y+2$

$x(2y) = 4y+2$

$x = \dfrac{4y+2}{2y}$

$x = \dfrac{2y+1}{4}$

(4 marks)

4. Rearrange the formula $3y + 2 = \dfrac{x+3}{x}$ to make x the subject.

$$x(3y+2) = x+3$$
$$3xy + 2x = x+3$$
$$3xy + x = 3$$

Factorise $x(3y+1) = 3$

$$x = \frac{3}{3y+1}$$

(4 marks)

5. Make x the subject of the formula

$$y = \frac{m+x}{x-2}$$

$$y(x-2) = m+x$$
$$xy - 2y = m + x$$

Collect terms in x

$$xy - x = 2y + m$$

Factorise $x(y-1) = 2y+m$

(4 marks)

$$x = \frac{2y+m}{y-1}$$

Completing the Square

1. You are given that $(x + a)^2 + b = x^2 - 6x + 13$.

   Find the values of a and b.

   Expand $x^2 + 2ax + a^2 + b = x^2 - 6x + 13$

   equate lems in x $2a = -6$, $a = -3$

   equate lems w/o x $a^2 + b = 13$

   $9 + b = 13$, $b = 4$.

   (3 marks)

2. Find the values of a and b such that

$$x^2 + 10x + 40 = (x + a)^2 + b$$

$x^2 + 10x + 40 = x^2 + 2ax + a^2 + b$

equate lems in x $2a = 10$, $a = 5$

equate lems w/o x $a^2 + b = 40$, $25 + b = 40$, $b = 15$

(2 marks)

3. Find the values of a and b such that

$$x^2 - 10x + 18 = (x - a)^2 + b$$

$x^2 - 10x + 18 = x^2 - 2ax + a^2 + b$

equate lems in x $-10 = -2a$, $a = 5$

equate lems w/o x $a^2 + b = 18$

$25 + b = 18$, $b = -7$

(2 marks)

4. Find the values of a and b such that

$$x^2 + 6x - 3 = (x + a)^2 + b = x^2 + 2ax + a^2 + b$$

equate lems in x $2a = 6$, $a = 3$

$a^2 + b = -3$

$9 + b = -3$, $b = -12$

(2 marks)

5. Find the values of a and b such that

$$x^2 + 8x - 5 \equiv (x + a)^2 + b$$

expand $x^2 + 8x - 5 \equiv x^2 + 2ax + a^2 + b$

equate terms in x, $2a = 8$, $a = 4$

equate terms w/o x, $a^2 + b = -5$

$16 + b = -5$, $b = -21$

(2 marks)

6. Find the values of a and b such that

$$x^2 + 6x - 11 \equiv (x + a)^2 + b$$

expand $x^2 + 6x - 11 \equiv x^2 + 2ax + a^2 + b$

equate terms in x $6 = 2a$, $a = 3$

equate terms w/o x $-11 = a^2 + b$

$-11 = 9 + b$, $b = -20$

(3 marks)

7. Find the values of a and b.

$$x^2 - 8x + 10 = (x - a)^2 + b$$

Expand $x^2 - 8x + 10 = x^2 - 2ax + a^2 + b$

equate terms in x $-8 = -2a$, $a = 4$

equate terms w/o x $10 = a^2 + b$

$10 = 16 + b$, $b = -6$

(3 marks)

8. Find the values of a and b such that

$$x^2 + 6x - 3 = (x + a)^2 + b$$

Expand $x^2 + 6x - 3 = x^2 + 2ax + a^2 + b$

equate terms in x $2a = 6$, $a = 3$

equate terms w/o x $-3 = a^2 + b$

$-3 = 9 + b$, $b = -12$

(2 marks)

Linear/Non-Linear Simultaneous Equations

1. Solve the simultaneous equations

$$y = x + 2$$

$$y = 3x^2$$

You **must** show your working.
Do **not** use trial and improvement.

Substitute $3x^2 = x + 2$

Make one side = 0 $3x^2 - x - 2 = 0$

Factorise $(3x + 2)(x - 1) = 0$

Solve $3x + 2 = 0, \quad x = -2/3$

$x - 1 = 0, \quad x = 1$

Find y values: $x = -2/3, \quad y = -2/3 + 2 = 4/3$

$x = 1, \quad y = 1 + 2 = 3.$

(5 marks)

2. Solve the simultaneous equations

$$y = 2x - 5$$
$$x^2 + y^2 = 25$$

You **must** show your working.
Do **not** use trial and improvement.

Subs $(2x-5)$ for y

$x^2 + (2x-5)^2 = 25$

$x^2 + 4x^2 - 20x + 25 = 25$

Make oneside = 0 $5x^2 - 20x = 0$

Factorise $5x(x-4) = 0$

Solve $5x = 0, \quad x = 0$

$x - 4 = 0, \quad x = 4$

(6 marks)

Find y values: $x = 0, \quad y = 2\times0 - 5 = -5$

$x = 4, \quad y = 2\times4 - 5 = 3.$

3. Solve the simultaneous equations

$$y = 3x^2$$

$$5x + y = 2$$

Subs. $3x^2$ for y

$$5x + 3x^2 = 2$$

Make one side $= 0$

$$3x^2 + 5x - 2 = 0$$

Factorise $(3x - 1)(x + 2) = 0$

Solve $3x - 1 = 0$, so $x = 1/3$

$x + 2 = 0$, $x = -2$.

Find y values: $y = 3x^2$, $x = \frac{1}{3}$, $y = 3 \times (\frac{1}{3})^2 = \frac{1}{3}$

$x = -2$, $y = 3 \times (-2)^2 = 12$

(5 marks)

4. Solve the simultaneous equations.

$$y = x + 7$$

$$x^2 + y^2 = 25$$

YOU **must** show your working.

Do **not** use trial and improvement.

Subs. $(x+7)$ for y into $x^2 + y^2 = 25$

$$x^2 + (x+7)^2 = 25$$

$$x^2 + x^2 + 14x + 49 = 25$$

Make one side $= 0$ $2x^2 + 14x + 24 = 0$

$\div 2$ $x^2 + 7x + 12 = 0$

Factorise $(x+3)(x+4) = 0$

Solve. $x = -3$ or $x = -4$

Find y values $x = -3$, $y = -3 + 7 = 4$

$x = -4$, $y = -4 + 7 = 3$.

(7 marks)

Algebraic Proof

1.

The sum of the squares of two consecutive integers is one greater than twice the product of the integers.

For example $9^2 + 10^2 = 81 + 100 = 181$ and $2 \times 9 \times 10 = 180$

Prove this result algebraically.

First integer $= n$

Second (consecutive) $= n + 1$

Sum of squares $= (n)^2 + (n+1)^2$

$= n^2 + (n+1)(n+1)$

$= n^2 + n^2 + 2n + 1$

$= 2n^2 + 2n + 1$

Twice product $= 2 \times n \times (n+1)$

$= 2n^2 + 2n$

difference of one.

(5 marks)

2.

Two integers have a difference of 3.
The difference between the squares of the two integers is three times the sum of the integers.

For example, $13 - 10 = 3$, $13^2 - 10^2 = 169 - 100 = 69$
and $3 \times (13 + 10) = 3 \times 23 = 69$

Prove this result algebraically.

Integer $= n$

Second integer $= n + 3$

difference of squares $= (n+3)^2 - (n)^2$

$= (n+3)(n+3) - n^2$

$= n^2 + 6n + 9 - n^2$

$= 6n + 9$

Three times sum $= 3 \times (n + (n+3))$

$= 3 \times (2n+3)$

$= 6n + 9$

same.

(4 marks)

3.

The difference between the squares of two consecutive even numbers is twice the sum of the numbers.

For example $8^2 - 6^2 = 28$

$2 \times (8 + 6) = 28$

Prove this result algebraically.

Even number $= 2n$ consecutive even numbers

Next odd $= 2n+1$

Next Even $= 2n+2$

difference of squares $= (2n+2)^2 - (2n)^2$

$= (2n+2)(2n+2) - 4n^2$

$= 4n^2 + 8n + 4 - 4n^2$

$= 8n+4$

twice the sum $= 2 \times (2n + (2n+2))$ same.

(4 marks)

$= 2 \times (4n+2)$

$= 8n + 4$

Triangle Operations Questions

1. Two integers, a and b, are combined using the operation ▼ in the following way.

$$a \blacktriangledown b = a^2 + a - 4b - b^2$$

(a) Find all solutions to the equation $x \blacktriangledown 2 = 0$

Substitute x for a <u>and</u> b for 2.

$x^2 + x - (4\times2) - (2)^2 = 0$

$x^2 + x - 8 - 4 = 0$

one side $= 0$ $\quad x^2 + x - 12 = 0$

Factorise $\quad (x+4)(x-3) = 0$

So $x = -4$

or $x = 3$

(4 marks)

(b) If a is 4 greater than b, prove that $a \blacktriangledown b$ is always a multiple of 5.

$a = b + 4$

Substitute for a in $a^2 + a - 4b - b^2$

$= (b+4)^2 + (b+4) - 4b - b^2$

$= (b+4)(b+4) + (b+4) - 4b - b^2$

$= \cancel{b^2} + 8b + 16 + b + 4 - 4b - \cancel{b^2}$

$= 5b + 20$

$= 5(b+4)$

5 is a factor so always a multiple of 5.

(4 marks)

2.

Two numbers, a and b, are combined using the operation $\bigtriangledown$ in the following way.

$$a \bigtriangledown b = 2a^2 - 7a - b + b^2$$

Work out **all** solutions of the equation $x \bigtriangledown 3 = 0$

Subs a for x and b for 3 in

$$2x^2 - 7x - 3 + 3^2 = 0$$

$$2x^2 - 7x + 6 = 0$$

Factorise

$$(2x - 3)(x - 2) = 0$$

Solve $2x - 3 = 0 \quad x = 3/2$

$x - 2 = 0 \quad x = 2.$

(4 marks)