(1) i) $x^{\frac{1}{3}} = 2$

$\sqrt[3]{x} = 2$

$\underline{x = 8}$

ii) $10^t = 1$

$\underline{t = 0}$

iii) $\left(\frac{1}{y^2}\right)^2 = \frac{1}{81}$

$\frac{1}{y^4} = \frac{1}{81}$

$81 = y^4$

$y \underline{= 3}$

C1 Jan 2006.

(2) i) $9x^2 + 6x + 1 - 8x^2 + 24x - 18$

$x^2 + 30x - 17$

ii) $(2x^3 - 3x^2 + 4x - 3)(x^2 - 2x + 1)$

$2x^3 + 6x^3 + 4x^3 = 10x^3 + 2x^3$

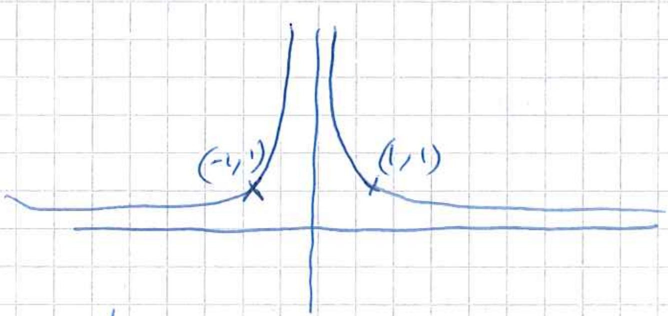
$= 12x^3$ $\quad \underline{\underline{12}}$

(3) $y = 3x^5 - x^{1/2} + 15$

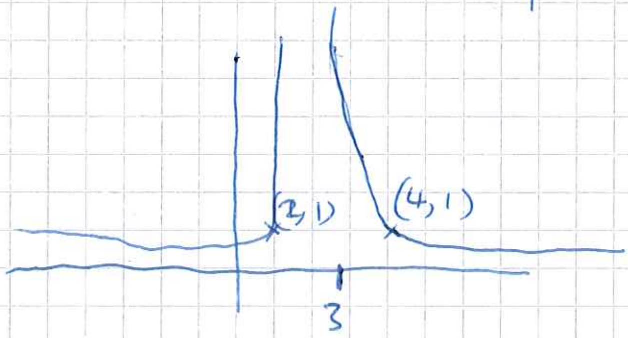
$\frac{dy}{dx} = 15x^4 - \frac{1}{2}x^{-1/2}$

$\frac{d^2y}{dx^2} = 60x^3 + \frac{1}{4}x^{-3/2}$

(4) i) $y = \frac{1}{x^2}$



ii)



iii) $y = \frac{1}{x^2} \longrightarrow y = \frac{2}{x^2}$

stretch s.F. 2 parallel to y-axis

⑤ i) $\cancel{x}$ $\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}$

$\frac{4}{\cancel{2}} = \frac{16}{4}$

ii) $(y-2)^2 - 4 - \frac{11}{4}$

$= (y-2)^2 - \cancel{\frac{8}{4}}\ \frac{27}{4}$

circle : $x^2 + y^2 + 3x - 4y - \frac{11}{4} = 0$

$\left(x + \frac{3}{2}\right) - \frac{9}{4} + (y-2)^2 - \frac{27}{4} = 0$

iii) Centre $= \left(-\frac{3}{2}, 2\right)$

iv) radius $= \sqrt{\frac{36}{4}} = \frac{6}{2} = 3$

⑥ $y = x^3 - 3x^2 + 4$

i) $\frac{dy}{dx} = 3x^2 - 6x$

@ stationary point $3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ $\quad$ $x - 2 = 0$

$\underline{x = 0}$ $\quad$ $\underline{x = 2}$

$y = 4$ $\quad$ $y = 8 - 12 + 4$

$(0, 4)$ $\quad$ $y = 0$

$(2, 0)$

ii) $\frac{d^2y}{dx^2} = 6x - 6$

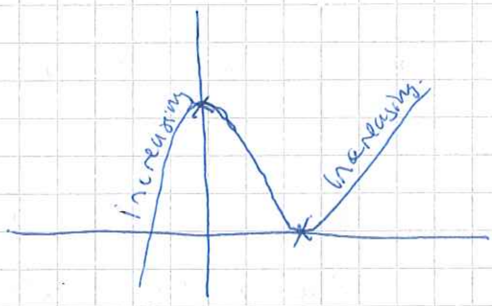
@ $x = 0$ $\quad 6x - 6 = -6$

hence max point

@ $x = 2$ $\quad 6x - 6 = 6$

hence min point.

iii) $x^3 - 3x^2 + 4$

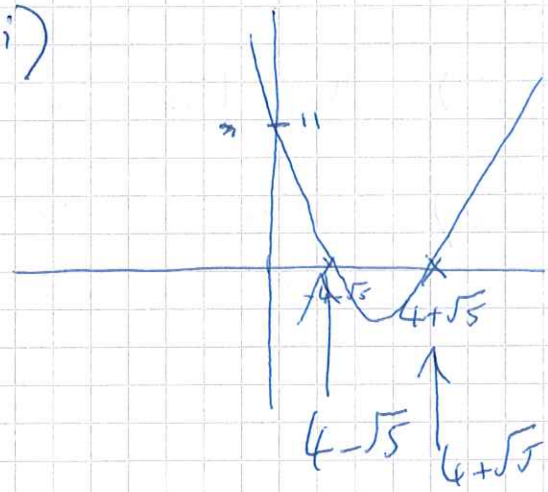


increases when

$x < 0 \quad x > 2$

(7) $x^2 - 8x + 11 = 0$

ii)



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{8 \pm \sqrt{64 - 4(1)(11)}}{2}$$

$$= \frac{8 \pm \sqrt{20}}{2}$$

$$= 4 \pm \sqrt{\frac{20}{4}}$$

$$= 4 \pm \sqrt{5}$$

$\sqrt{5} \approx 2. \ldots$

iii) $y = x^2$

$y = \cancel{\sqrt{}}(4 \pm \sqrt{5})^2$

$= (4 \pm \sqrt{5})(4 \pm \sqrt{5})$

$= 16 + 5 \pm 8\sqrt{5}$

$= 21 \pm 8\sqrt{5}$

8 i) $y = x^2 - 5x + 15$ $\quad 5x - 10 = y$

$x^2 - 5x + 15 = 5x - 10$

$x^2 - 10x + 25 = 0$

ii) $b^2 - 4ac = 100 - 4(1)(25)$
$= 0$

iii) the line is a tangent to the curve.

iv) $x^2 - 10x + 25 = 0$

$(x - 5)(x - 5) = 0$

$\underline{x = 5}$ $\quad y = 5(5) - 10$

$\underline{y = 15}$

v) $y = x^2 - 5x + 15$

$\frac{dy}{dx} = 2x - 5$ @ $x = 5$ $\frac{dy}{dx} = 10 - 5$
$\underline{= 5}$

gradient of tangent $= 5$

gradient of normal $= -1/5$

$y = mx + c$

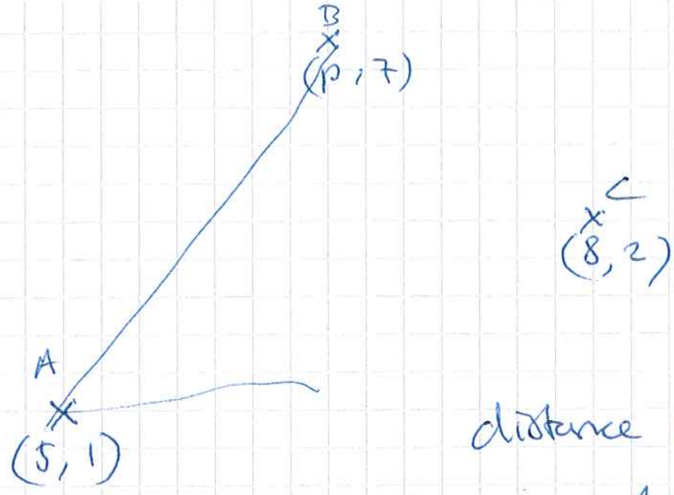
$15 = -1/5(5) + c$

$15 = -1 + c$

$c = 16$

$\boxed{y = -1/5x + 16}$

9 i)



distance AB $= \overset{2\times}{\cancel{\tfrac{4}{7}}}$ distance of AC

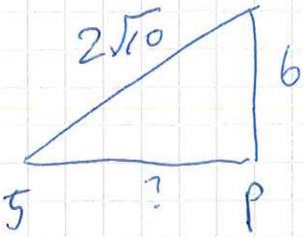
distance AC =

$$AC = \sqrt{3^2 + 1^2}$$
$$= \sqrt{10}$$

distance AB $= 2\sqrt{10}$ $\cancel{\frac{}{7}}$

$= \cancel{\sqrt{\frac{10}{4}}}$

$= \cancel{\sqrt{2.5}}$



$$? = \sqrt{(2\sqrt{10})^2 + 6^2}$$
$$= \sqrt{40+36}$$
$$= \sqrt{76}$$

$$\sqrt{(p-5)^2 + 6^2} = 2\sqrt{10}$$
$$\sqrt{(p-5)^2 + 6^2} = \sqrt{40}$$
$$(p-5)^2 + 6^2 = 40$$
$$p^2 - 10p + 25 + 36 = 40$$
$$p^2 - 10p + 21 = 0$$
$$(p-7)(p-3) = 0$$
$$\underline{p = 7} \quad \underline{p = 3}$$

ii) $y = 3x - 14$

$x = 7$ $\quad y = 21 - 14$

$\underline{y = 7}$

$x = 3$ $\quad y = 9 - 14$

$\neq -5$

hence $(7, 7)$ is the co-ordinate

mid point $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

$= \left(\frac{5+7}{2}, \frac{1+7}{2} \right)$

$= (6, 4)$