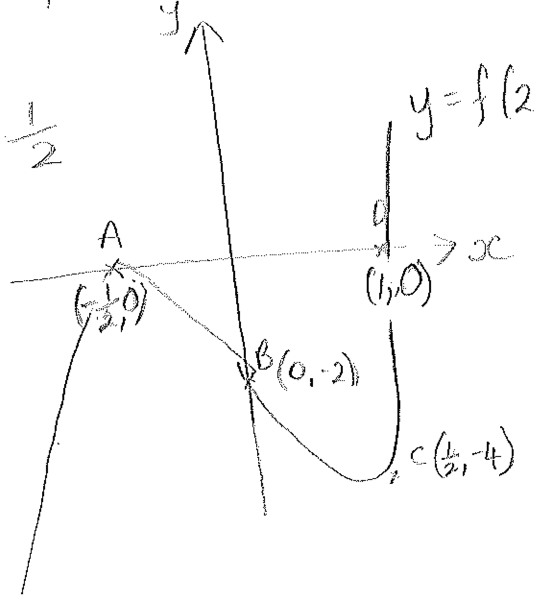
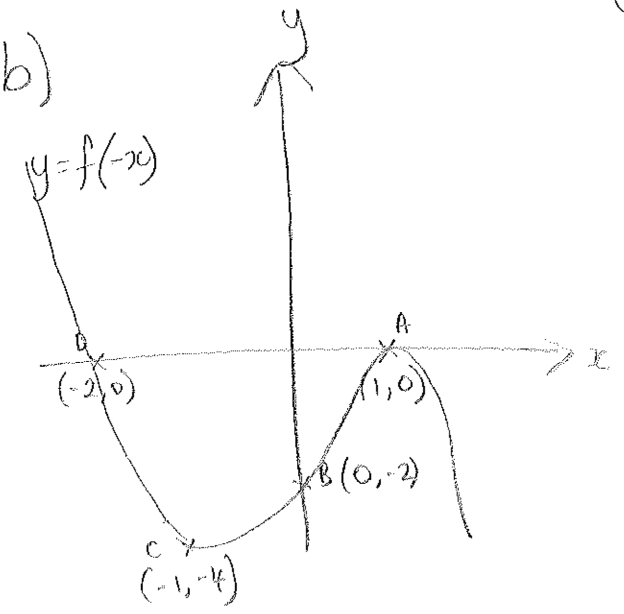
AS Practice Paper D.

1 a) $f(x) = x^3 - 3x - 2$

$SF = \frac{1}{2}$



b)



2. $\int (5 - 3\sqrt{x})^2 \, dx$

$\int (5 - 3\sqrt{x})(5 - 3\sqrt{x}) \, dx$

$\int 25 - 30\sqrt{x} + 9x \, dx$

$\int 25 - 30x^{1/2} + 9x \, dx$

$= 25x - \frac{2 \times 30x^{3/2}}{3} + \frac{9x^2}{2} + C$

$= 25x - 20x^{3/2} + \frac{9}{2}x^2 + C.$

3. $(8^{x-1})^2 - 18(8^{x-1}) + 32 = 0$

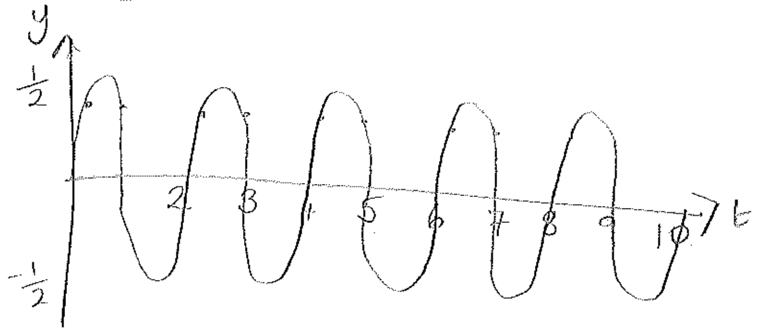
$t = 8^{x-1}$, $\quad t^2 - 18t + 32 = 0$

$(t - 2)(t - 16) = 0$

$t = 2, \quad t = 16$

$8^{x-1} = 2$	$8^{x-1} = 16$
$(2^3)^{x-1} = 2^1$	$(2^3)^{x-1} = 2^4$
$2^{3x-3} = 2^1$	$2^{3x-3} = 2^4$
$3x - 3 = 1$	$3x - 3 = 4$
$3x = 4$	$3x = 7$
$x = \frac{4}{3}$	$x = \frac{7}{3}.$

4. a) $y = \frac{1}{2} \sin 180t$



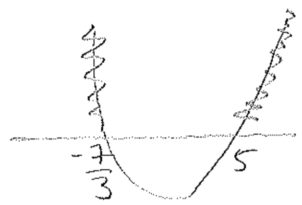
b) 10 times

c) Waves in the sea are not uniform.

5. $f(x) = x^3 - 4x^2 - 35x + 20$

$f'(x) = 3x^2 - 8x - 35.$

$f'(x) > 0, \quad 3x^2 - 8x - 35 > 0.$



$(3x+7)(x-5) > 0$

$x = -\frac{7}{3} \quad x = 5.$

$\{x : x < -\frac{7}{3}\} \cup \{x : x > 5\}$

6. $v(t) = \frac{1}{20}(50\sqrt{t} + 20t^2 - t^3)$

$s = \int_0^{20} v(t)\, dt$

$= \int_0^{20} \frac{1}{20}(50\sqrt{t} + 20t^2 - t^3)\, dt.$

$= \frac{1}{20}\int_0^{20} 50t^{\frac{1}{2}} + 20t^2 - t^3\, dt.$

$= \frac{1}{20}\left[\frac{2 \times 50 t^{\frac{3}{2}}}{3} + \frac{20t^3}{3} - \frac{t^4}{4}\right]_0^{20}$

$= \frac{1}{20}\left[\frac{100}{3}t^{3/2} + \frac{20}{3}t^3 - \frac{t^4}{4}\right]_0^{20}$

6. continued...

$$S = \frac{1}{20}\left[\left(\frac{100}{3}(20)^{3/2} + \frac{20}{3}(20)^3 - \frac{(20)^4}{4}\right) - \left(\frac{100}{3}(0)^{3/2} + \frac{20}{3}(0)^3 - \frac{0^4}{4}\right)\right]$$

$$S = \frac{1}{20}(16314.7573 - 0)$$

$$S = 815.7379$$

$$S = 816\text{m } (3sf).$$

7a) $f(x) = x^2 - (k+8)x + 8k + 1$

$a = 1$
$b = -(k+8) = -k-8$
$c = 8k+1$

$b^2 - 4ac$

$(-k-8)^2 - (4 \times 1 \times (8k+1))$

$(-k-8)(-k-8) - (4(8k+1))$

$k^2 + 16k + 64 - 32k - 4$

$k^2 - 16k + 60$

b) $b^2 - 4ac = 0 \rightarrow$ Equal roots/one root.

$k^2 - 16k + 60 = 0$

$(k-10)(k-6) = 0$

$k = 10, \quad k = 6$

c) $k = 8, \; f(x) = x^2 - 16x + 65$

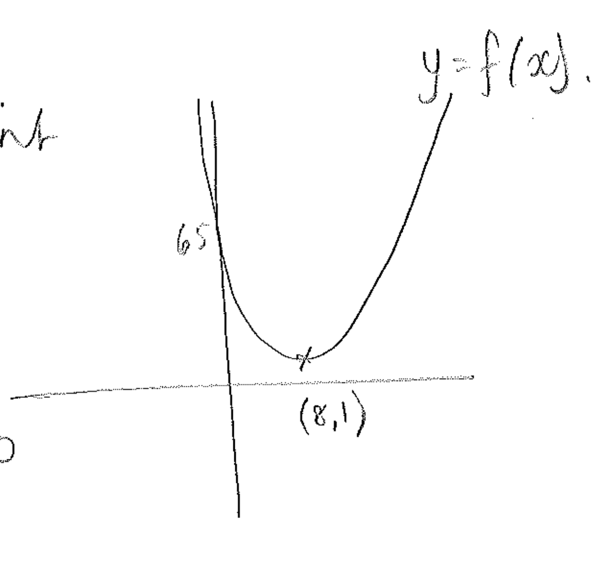
$f'(x) = 2x - 16$

$f'(x) = 0 \rightarrow$ min point

$2x - 16 = 0$

$2x = 16$

$x = 8, \; y = 1$



Therefore when $k = 8$, $f(x) > 0$ for all values of x.

8. a) $x^2 + 10x + y^2 - 12y = 3.$

$(x+5)^2 - 25 + (y-6)^2 - 36 = 3.$

$(x+5)^2 + (y-6)^2 = 64$

Centre $= (-5, 6)$

Radius $= \sqrt{64} = 8.$

$x^2 - 6x + y^2 - 2qy = 9$

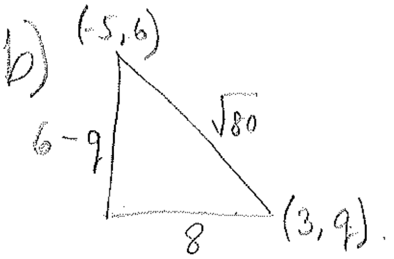
$(x-3)^2 - 3^2 + (y-q)^2 - q^2 = 9$

$(x-3)^2 + (y-q)^2 = 18 + q^2$

Centre $= (3, q)$

Radius $= \sqrt{18+q^2}$

b)



$(\sqrt{80})^2 = 8^2 + (6-q)^2$

$80 = 8^2 + (6-q)(6-q)$

$80 = 64 + 36 + q^2 - 12q.$

$q^2 - 12q + 20 = 0$

$(q-10)(q-2) = 0$

$q = 10, \quad q = 2.$

9. a) $y = ab^x \quad (2, 400), \ (5, 50)$

$400 = ab^2 \quad 50 = ab^5$

$\dfrac{ab^5}{ab^2} = \dfrac{50}{400}$

$b^3 = \dfrac{1}{8}$

$\underline{b = \dfrac{1}{2}}$

$400 = ab^2$

$400 = a\left(\dfrac{1}{2}\right)^2$

$400 = \dfrac{a}{4}$

$\underline{1600 = a}$

9b) $ab^x < k \quad k > 0$

$1600 \times \left(\frac{1}{2}\right)^x < k.$

$\left(\frac{1}{2}\right)^x < \frac{k}{1600}$

$\log\left(\frac{1}{2}\right)^x < \log\left(\frac{k}{1600}\right).$

$x \log\left(\frac{1}{2}\right) < \log\left(\frac{k}{1600}\right).$

$\div \log\left(\frac{1}{2}\right)$ minus $\therefore$ flip sign

$x > \dfrac{\log\left(\frac{k}{1600}\right)}{\log\left(\frac{1}{2}\right)}.$

$x > \dfrac{\log\left(\frac{1600}{k}\right)}{\log 2}.$

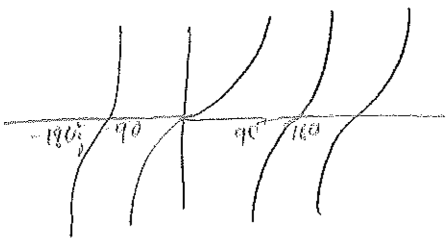
$\log\left(\frac{1}{2}\right) = \log(2^{-1}) = -1\log 2$

$\log\left(\frac{k}{1600}\right) = \log\left(\left(\frac{1600}{k}\right)^{-1}\right)$

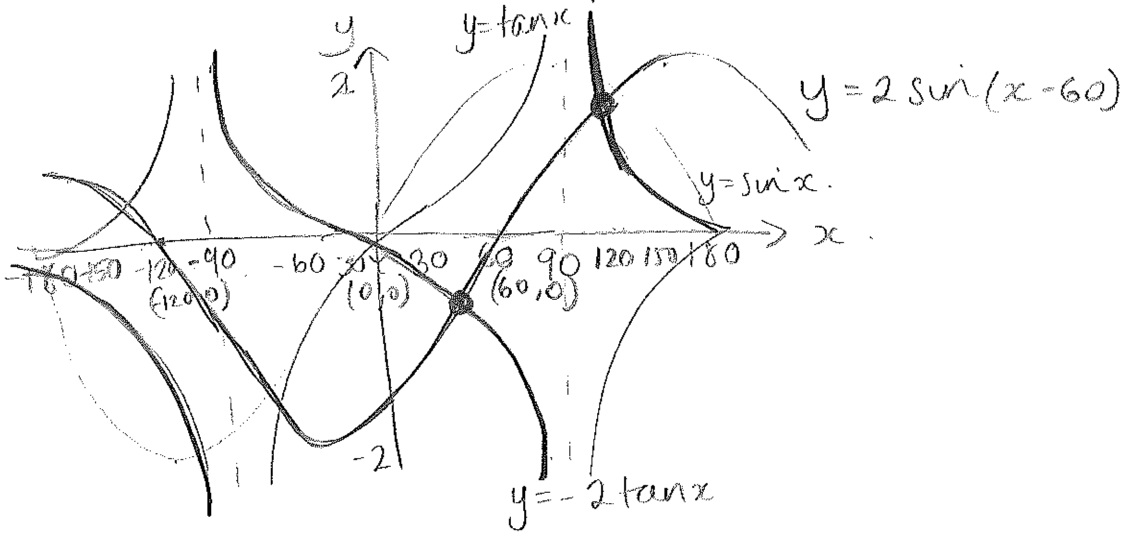
$= -\log\left(\frac{1600}{k}\right).$

10. a) $-2\tan(-120)$

$-2 \times \sqrt{3} = \underline{-2\sqrt{3}}.$



b)



c) $2\sin(x-60) = -2\tan x.$

$y = 2\sin(x-60) + 2\tan x.$ Solutions will occur where the two lines intersect.

d) Two solutions

11. a) $y = x^3 - x^2 - x + 2$

$\frac{dy}{dx} = 3x^2 - 2x - 1$

b) $P(2, y)$

At $x = 2$, $\frac{dy}{dx} = 3(2)^2 - 2(2) - 1$

$= 12 - 4 - 1$

$m = 7$

$x = 2$, $y = 2^3 - 2^2 - 2 + 2$

$y = 4$

$y - y_1 = m(x - x_1)$

$y - 4 = 7(x - 2)$

$y - 4 = 7x - 14$

$y = 7x - 10$

c) Gradient of normal $= -\frac{1}{7}$

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{1}{7}(x - 2)$

$7y - 28 = -x + 2$

At A, $y = 0$, $0 - 28 = -x + 2$

$x = 30$

$A(30, 0)$

12. a) $f(x) = x^3 + x^2 + px + q$

$f(5) = 0 \quad f(-3) = 8.$

$f(5) = 5^3 + 5^2 + 5p + q$

$0 = 125 + 25 + 5p + q$

$-150 = 5p + q$

$f(-3) = (-3)^3 + (-3)^2 - 3p + q$

$8 = -27 + 9 - 3p + q$

$26 = -3p + q$

$$\begin{array}{rl} & 5p + q = -150 \\ - & -3p + q = 26 \\ \hline & 8p = -176 \\ & \underline{p = -22} \end{array}$$

$5p + q = -150$

$5(-22) + q = -150$

$-110 + q = -150$

$\underline{q = -40}$

b) $f(x) = x^3 + x^2 + px + q$

$f(x) = x^3 + x^2 - 22x - 40$

$f(5) = 0$, $(x-5)$ is a factor

$$\begin{array}{r|l} & x^2 + 6x + 8 \\ \hline x - 5 & x^3 + x^2 - 22x - 40 \\ & -\ x^3 - 5x^2 \\ \hline & \quad 6x^2 - 22x \\ & -\ 6x^2 - 30x \\ \hline & \quad\quad 8x - 40 \\ & -\ 8x - 40 \\ \hline & \quad\quad\quad 0 \end{array}$$

$x^2 + 6x + 8$

$(x+2)(x+4)$

$f(x) = (x-5)(x+2)(x+4)$

13. a) $\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{OA} + \overrightarrow{AN}$

$= -\frac{1}{5}b + a + \frac{4}{5}b$

$= a + \frac{3}{5}b.$

b) $\overrightarrow{ST} = \overrightarrow{SB} + \overrightarrow{BO} + \overrightarrow{OT}$

$= -\frac{1}{5}a - b + \frac{4}{5}a$

$= \frac{3}{5}a - b.$

c) Question not possible.